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Finite element modeling of seepage beneath a sheet pile wall in spatially random soil

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ABSTRACT: The effect of random and spatially correlated soil permeability on confined seepage beneath a single sheet pile wall has been studied. Random field theory for the generation of soil permeability properties with a fixed mean, standard deviation and spatial correlation structure, have been combined with finite element methods to perform 'Monte Carlo' simulations of the seepage problem. The results of parametric studies to gauge the effect of the standard deviation and correlation structure of the permeability on the output statistics relating to seepage quantities and exit gradients are presented. In all cases, comparison is made with results that would have been achieved on a deterministic basis.

1 INTRODUCTION

This work presented in this paper brings together Finite Element Analysis and Random Field Theory in the study of a simple boundary value problem of steady seepage. The aim of the investigation is to observe the influence of soil variability on the expected value of 'output' quantities such as the flow rate and exit gradient. Smith and Freeze (1979, Pts. 1 and 2) were among the first to study the problem of confined flow through a stochastic medium using finite differences, in which examples of flow between parallel plates and beneath a single sheet-pile were presented. Recent developments in random field and finite element methodology have led to further studies of steady seepage problems for a range of boundary value problems (Fenton and Griffiths 1993, Griffiths and Fenton 1993).

A conference on probabilistic methods in geotechnical engineering (Li and Lo 1993) highlighted some of the recent advances in this field. For example Mostyn and Li (1993) emphasised the importance of taking account of the spatial correlation of soil properties in probabilistic analyses. It was pointed out that the "vast majority of existing models do not do this", and although their particular application was the analysis of slope stability in which the random soil properties in question were the shear strength parameters, the same arguments could be applied to soil permeability in a seepage problem. White (1993) also described how early probabilistic analyses typically represented soil property uncertainty by the use of a single 'perfectly correlated' random variable which was varied from one realization to the next.

The use of *random fields* (Vanmarcke 1984, Fenton and Vanmarcke 1990) was considered to be an important refinement, in that the soil property at each location within the soil mass was itself considered to be a random variable. An important feature of the random field approach is that it appropriately takes into account the positive correlation that is observed between soil properties measured at locations that are 'close' together.

2 BRIEF DESCRIPTION OF THE FINITE ELEMENT MODEL

In this paper a random field generator known as the Local Average Subdivision Method (LAS) devised by Fenton (1990) is combined with the power of the Finite Element Method for modeling spatially varying soil properties. The problem chosen

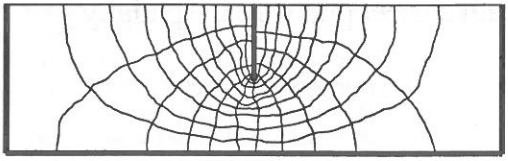

Figure 1: Flow net through stochastic soil for the 1–wall problem

for study is a simple boundary value problem of steady seepage beneath a single sheet pile wall penetrating a layer of soil. The variable soil property in this case is the soil permeability k.

A typical flow net through a stochastic soil is shown in Figure 1, which also indicates the general boundaries of the problem under consideration. The finite element program used for the solutions of Laplace's equation presented in this paper is published in full in the text by Smith and Griffiths (1988). In all analyses, a uniform mesh of square 4-node elements was used with 60 elements in the x-direction (30 on each side of the wall) and 20 elements in the y-direction. A time-saving feature of square (or rectangular) elements is that their conductivity matrices are easily computed explicitly without the need for numerical integration. In this case assuming the permeability of the i^{th} element is k_i, the symmetrical element conductivity matrix is given by:

$$\mathbf{k_i} = \frac{k_i}{6}\begin{bmatrix} 4 & -1 & -2 & -1 \\ & 4 & -1 & -2 \\ & & 4 & -1 \\ & & & 4 \end{bmatrix} \qquad (1)$$

During assembly of the global conductivity matrix, a 'skyline' storage strategy was used together with a Cholesky factorization approach (see e.g. Griffiths and Smith 1991). The skyline approach ran faster than conventional (constant band-width) methods as well as giving substantial savings on memory requirement.

3 BRIEF DESCRIPTION OF THE RANDOM FIELD MODEL

Field measurements of permeability have indicated an approximately lognormal distribution (see e.g. Hoeksema and Kitanidis 1985, and Sudicky 1986). The same distribution has therefore been adopted for the simulations generated in this paper.

Essentially, the permeability field is obtained through the transformation

$$k_i = \exp\{\mu_{\ln k} + \sigma_{\ln k}\, g_i\} \qquad (2)$$

in which k_i is the permeability assigned to the i^{th} element, g_i is the local average of a standard Gaussian random field, g, over the domain of the i^{th} element, and $\mu_{\ln k}$ and $\sigma_{\ln k}$ are the mean and standard deviation of the logarithm of k (obtained from the 'target' mean and standard deviation μ_k and σ_k).

The LAS technique (Fenton 1990, Fenton and Vanmarcke 1990) generates realizations of the local averages g_i which are derived from the random field g having zero mean, unit variance, and a spatial correlation controlled by the scale of fluctuation, θ_k. As the scale of fluctuation goes to infinity, g_i becomes equal to g_j for all elements i and j – that is the field of permeabilities tends to become uniform on each realization. At the other extreme, as the scale of fluctuation goes to zero, g_i and g_j become independent for all $i \neq j$ – the soil permeability changes rapidly from point to point.

In the two dimensional analyses presented in this paper, the scales of fluctuation in the vertical and horizontal directions are taken to be equal (isotropic) for simplicity. Although beyond the scope of this paper, it should be noted that for a layered soil mass the horizontal scale of fluctuation is generally larger than the vertical scale due to the natural stratification of many soil deposits. The 2-d model used herein implies that the out-of-plane scale of fluctuation is infinite – soil properties are constant in this direction – which is equivalent to specifying that the streamlines remain in the plane of the analysis. This is clearly a deficiency of the present model, however it is believed that useful information regarding the variability of flow quantities is still to be gained from the 2-d model.

4 SUMMARY OF THE RESULTS FROM SEEPAGE ANALYSES

A Monte-Carlo approach to the seepage problem was adopted in which for each set of input statistics $(\mu_k, \sigma_k, \theta_k)$ 2000 realizations were performed. An extensive set of parametric studies for the single wall seepage problem has been reported by Paice (1993) of which a summary will be presented here.

The main output quantities of interest in this problem are the total flow rate through the system Q and the exit gradient i_e. Following the Monte-Carlo procedure, the mean and standard deviation of both these quantities were computed. The flow rate results were presented in non-dimensional form by representing it in terms of a normalized flow rate $\bar{Q}$ where:

$$\bar{Q} = Q/(H\mu_k) \tag{3}$$

and H is the total head loss across the wall, typically set to unity.

As the coefficient of variation of the input permeability increases, Figure 2 indicates a consistent fall in the expected value of the flow rate from its deterministic value of $\bar{Q} \approx 0.5$. This is especially true for smaller values of the scale of fluctuation θ_k, however as θ_k is increased the variation of $\mu_{\bar{Q}}$ is clearly tending towards the deterministic result that would be expected for a strongly correlated permeability field.

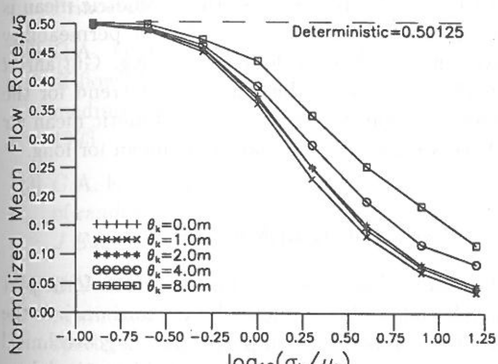

Figure 2: Influence of random permeability on mean flow rate

Figure 3 shows the standard deviation of the normalized flow rate $\sigma_{\bar{Q}}$. For small θ_k very little variation in $\bar{Q}$ was observed, even for high coefficients of variation. This can be explained by the fact that the poorly correlated field has little influence on the flow rate computed from one realization to the next - the global response being almost deterministic in character. For higher values of θ_k a more variable flow rate was observed which tended to the limiting value indicated for $\theta_k = \infty$ given by equation (4). The maximum point observed in the

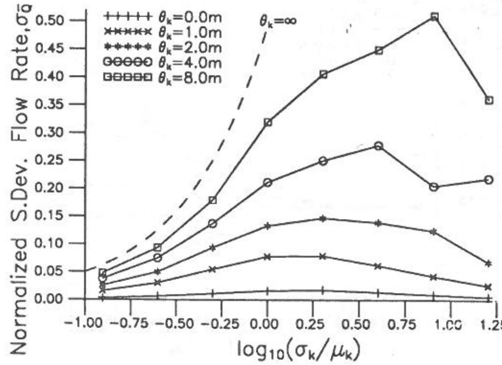

Figure 3: Influence of random permeability on standard deviation of flow rate

$\sigma_{\bar{Q}}$ vs. $\log_{10}(\sigma_k/\mu_k)$ relationship for all values of θ_k remains an interesting and as yet unexplained result.

$$\sigma_{\bar{Q}} = \frac{\sigma_k}{\mu_k}\bar{Q}_{det} \tag{4}$$

Figures 4 and 5 show the corresponding mean and standard deviation of i_e. As shown in Figure 4, low values of the coefficient of variation retrieve the deterministic value of $i_e = 0.128$, however at higher values the expected exit gradient tends to increase for the majority of values of θ_k. The exception to this rule occurs for the uncorrelated case ($\theta_k = 0$) when the expected exit gradient appears to *decrease*. This rather surprising result is currently under further investigation although it may relate to the 'differentiation length' used by the backward-difference differentiation formula in the calculation of i_e. It is not too surprising however, that a quantity based on a first derivative is going to be particularly sensitive to fluctuations in potential caused by random soil properties. For $\theta_k = \infty$, μ_{i_e} should return to the deterministic value.

Figure 5 confirms that the standard deviation of the exit gradient steadily increases with the coefficient of variation of the input permeabilities. For $\theta_k = \infty$ however, σ_{i_e} should return to zero for all coefficients of variation, implying that there exists a 'worst-case' scale of fluctuation in relation to the reliability of exit gradient predictions.

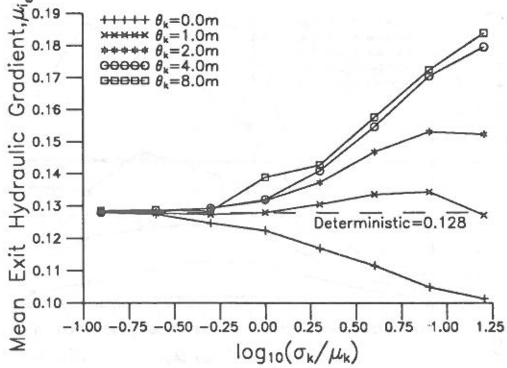

Figure 4: Influence of random permeability on mean exit gradient

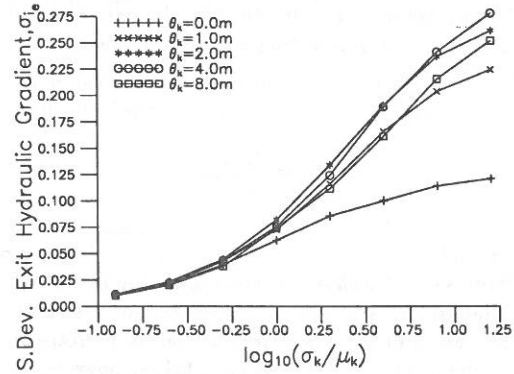

Figure 5: Influence of random permeability on standard deviation of exit gradient

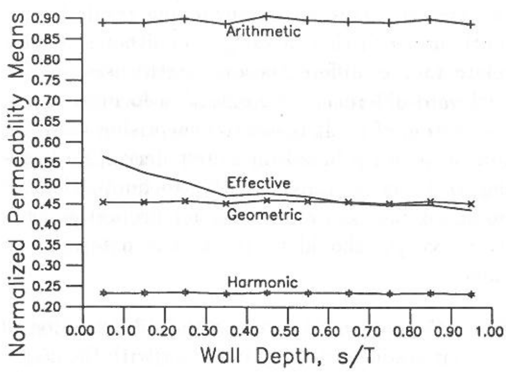

Figure 6: Comparison of means and the effective permeability ($\sigma_k/\mu_k = 2, \theta_k = 2$)

5 EFFECTIVE PERMEABILITY

A useful measure of the expected flow rate though a random soil is to express it in terms of the effective permeability $\bar{k}$. This quantity is defined as the permeability which would give the expected flow rate in a deterministic analysis with a constant or perfectly correlated permeability field.

For a given set of n values, three mean permeabilities and the circumstances under which they are appropriate measures of the effective permeability can be defined as follows:

Arithmetic	$k^a = \frac{1}{n}\sum_{i=1}^{n} k^i$	1-d parallel
Geometric	$k^g = (\prod_{i=1}^{n} k^i)^{1/n}$	2-d unbounded
Harmonic	$k^h = n(\sum_{i=1}^{n} 1/k^i)^{-1}$	1-d series

Figure 6 shows the relationship between the three means and the effective permeability for the single wall problem over a range of embedment depths. The permeabilities have been normalized by dividing by μ_k. It appears that the geometric mean is an excellent predictor of the effective permeability even in 2-d bounded domains (see e.g. Gutjahr *et al* 1978), although there is a slight trend for the effective mean to tend to the arithmetic mean for short walls and to the harmonic mean for long.

6 CONCLUDING REMARKS

The paper has presented results which form part of a broad study conducted by the authors into the influence of random soil properties on geotechnical design. In this paper, random field methodology has been combined with the finite element method to study the flow rate and exit gradient due to steady seepage beneath a sheet-pile wall embeded in a layer of random soil. The influence of spatial correlation of soil properties has been fully incorporated through a scale of fluctuation parameter θ_k, which has been varied across a wide range of values.

For moderate values of the scale of fluctuation, the expected value of the flow rate was found to fall consistently as the coefficient of variation of the input permeability was increased. For higher values of the scale of fluctuation, the normalised flow rate tended to the deterministic value. The geometric mean permeability was shown to model closely the effective permeability over the range of confined 2-d problems considered. The exit gradient, being a first derivative of the potential field at the exit point, was found to be particularly sensitive to fluctuations caused by the random field. This raised questions about the numerical differentiation formula used in the calculation of i_e, and this is currently a topic of further investigation.

Increased computer sophistication and recent developments in random field generation means that this work will soon be extended to three-dimensions, thus removing the need to assume perfect correlation in the out-of-plane direction.

References

[1] G.A. Fenton. *Simulation and analysis of random fields.* PhD thesis, Department of Civil Engineering and Operations Research, Princeton University, 1990.

[2] G.A. Fenton and D.V. Griffiths. Statistics of flow through a simple bounded stochastic medium. *Water Resour Res*, 29(6):1825–1830, 1993.

[3] G.A. Fenton and E.H. Vanmarcke. Simulation of random fields via local average subdivision. *J Eng Mech, ASCE*, 116(8):1733–1749, 1990.

[4] D.V. Griffiths and G.A. Fenton. Seepage beneath water retaining structures founded on spatially random soil. *Géotechnique*, 43(4), 1993.

[5] D.V. Griffiths and I.M. Smith. *Numerical Methods for Engineers.* Blackwell Scientific Publications Ltd., Oxford, 1991.

[6] A.L. Gutjahr, L.W. Gelhar, A.A. Bakr, and J.R. MacMillan. Stochastic analysis of spatial variability in subsurface flows, 2, Evaluation and application. *Water Resour Res*, 14(5):953–959, 1978.

[7] R.J. Hoeksema and P.K. Kitanidis. Analysis of the spatial structure of properties of selected aquifers. *Water Resour Res*, 21(4):563–572, 1985.

[8] K.S. Li and S-C.R. Lo (eds). *Probabilistic methods in geotechnical engineering.* A.A. Balkema, Rotterdam, 1993.

[9] G.R. Mostyn and K.S. Li. Probabilistic slope stability – State of play. In K.S. Li and S-C.R. Lo, editors, *Proc. Conf. Probabilistic Meths. Geotech. Eng.*, pages 89–110. A.A. Balkema, Rotterdam, 1993.

[10] G.M. Paice. Finite element analysis of stochastic soils. Transfer Report, University of Manchester, Department of Engineering, 1993.

[11] I.M. Smith and D.V. Griffiths. *Programming the Finite Element Method.* John Wiley and Sons, Chichester, New York, 2nd edition, 1988.

[12] L. Smith and R.A. Freeze. Stochastic analysis of steady state groundwater flow in a bounded domain, 1. One-dimensional simulations. *Water Resour Res*, 15(3):521–528, 1979.

[13] L. Smith and R.A. Freeze. Stochastic analysis of steady state groundwater flow in a bounded domain, 2. Two-dimensional simulations. *Water Resour Res*, 15(6):1543–1559, 1979.

[14] E.A. Sudicky. A natural gradient experiment on solute transport in a sand aquifer: Spatial variability of hydraulic conductivity and its role in the dispersion process. *Water Resour Res*, 22(13):2069–2083, 1986.

[15] E.H. Vanmarcke. *Random fields: Analysis and synthesis.* The MIT Press, Cambridge, Mass., 1984.

[16] W. White. Soil variability: Characterisation and modelling. In K.S. Li and S-C.R. Lo, editors, *Proc. Conf. Probabilistic Meths. Geotech. Eng.*, pages 111–120. A.A. Balkema, Rotterdam, 1993.

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