Flow Through Earth Dams with Spatially Random Permeability

by Gordon A. Fenton[1]*, Member, ASCE, and D. V. Griffiths*[2]*, Member, ASCE*

Abstract

Even though soil shows the highest variability of any engineering material, the analysis of flow through earth dams typically proceeds deterministically and results can sometimes be quite misleading. In fact it is well known that soil permeability varies randomly in space and an improved earth dam model should incorporate this variation. In this paper the soil permeability in an earth dam of typical geometry is viewed as a spatially random field following a lognormal distribution with prescribed mean, variance, and spatial correlation structure. The statistics of flow and free surface drawdown through the dam are computed using Monte Carlo simulations and observations are made on the statistics of the flow rate and downstream exit-point.

Introduction

Many water retaining structures in North America are earth dams and the study of flow through such structures is of considerable interest to planners and designers. Although it is well known that soils are highly variable materials, the prediction of flow rates through earth dams is generally performed using deterministic models. This paper introduces a stochastic model of an earth dam and investigates the effects of spatially random soil properties on two quantities of general interest. These are the total flow rate through the dam and the amount of drawdown of the free surface at the downstream face of the dam. The drawdown is defined as the elevation of the point on the downstream face at which the water begins to exit the dam.

A Monte Carlo analysis approach has been adopted, that is a sequence of realizations (1000 in this paper) of spatially varying soil properties with prescribed mean and variance are generated and then analyzed separately to obtain a sequence of flow rates and free surface profiles. The mean and variance of the flow rate and drawdown statistics can then be estimated directly from the sequence of computed results. Because the analysis is Monte Carlo in nature, the results are strictly only applicable to the particular earth dam geometry and boundary conditions studied, however the general trends and observations may be extended to a range of earth dam boundary value problems.

---

[1] Associate Professor, Department of Applied Mathematics, Technical University of Nova Scotia, Halifax, Nova Scotia, Canada B3J 2X4

[2] Professor, Geomechanics Research Center, Colorado School of Mines, Golden, Colorado 80401-1887, U.S.A.

The earth dam considered in this study is illustrated in Fig. 1, from which it can be seen that the free surface typically lies some distance below the top of the dam. Because the position of the surface is not known a-priori, the flow analysis necessarily proceeds iteratively. Under the free surface, flow is assumed to be governed by Darcy's Law characterized by an isotropic permeability, $K(\boldsymbol{x})$, where $\boldsymbol{x}$ is the spatial location. The permeability is a spatially random field governed by three parameters: its mean, μ_k, its variance σ_k^2, and its so-called scale of fluctuation, θ_k. The scale of fluctuation may be loosely defined as the distance over which soil properties are substantially correlated. In fact the correlation, $\rho(\tau)$, between log-permeability at two points, $\ln K(\boldsymbol{x})$ and $\ln K(\boldsymbol{x}+\tau)$, separated by τ, follows a Gauss-Markov model which is an exponentially decaying function of separation distance, $\rho(\tau) = \exp\{-2|\tau|/\theta_k\}$.

Flow rate and drawdown statistics for the earth dam are evaluated over a range of the statistical parameters of K. Specifically the mean and standard deviation of the total flow rate, m_Q and s_Q, and the drawdown, m_Y and s_Y, are estimated for $\sigma_k/\mu_k = \{0.1, 0.5, 1.0, 2.0, 8.0\}$ and $\theta_k = \{0.1, 0.5, 1.0, 2.0, 8.0\}$. The mean permeability, μ_k, is held fixed at 1.0. The drawdown elevations Y are normalized by expressing them as a fraction of the overall dam height, in this case 3.2.

The Stochastic Model

Simulations of the soil permeability field proceeds in two steps; first an underlying Gaussian random field, $G(\boldsymbol{x})$, is generated with mean zero, unit variance, and scale of fluctuation θ_k using the Local Average Subdivision method (Fenton and Vanmarcke, 1990). The permeability itself is assumed to be lognormally distributed so that the values of $K(\boldsymbol{x})$ are obtained by the transformation $K(\boldsymbol{x}_i) = \exp\{\mu_{\ln k} + \sigma_{\ln k} G(\boldsymbol{x}_i)\}$, where $\boldsymbol{x}_i$ is the centroid of the i'th finite element and $\sigma_{\ln k}^2 = \ln(1 + \sigma_k^2/\mu_k^2)$ and $\mu_{\ln k} = \ln(\mu_k) - \frac{1}{2}\sigma_{\ln k}^2$ are the mean and variance of log-permeability respectively. The lognormal assumption for permeability is consistent with the findings of other researchers (e.g. Hoeksema and Kitanidis, 1985, Sudicky, 1986) and is commonly used.

Both permeability and scale of fluctuation are assumed to be isotropic in this study. Although layered construction of an earth dam may lead to some anisotropy relating to the scale of fluctuation and permeability, this is not thought to be a major feature of the reconstituted soils typically used in earth dams. In contrast, however, natural soil deposits can exhibit quite distinct layering and stratification in which anisotropy can not be ignored.

The model itself is two-dimensional, which is equivalent to assuming that the stream-lines remain in the plane of analysis. This will occur if the dam ends are impervious and if the scale of fluctuation in the out-of-plane direction is infinite (implying that soil properties are constant in the out-of-plane direction). Clearly the latter condition will be false, however a full three-dimensional analysis is beyond the scope of the present study. It is believed that the two-dimensional analysis will still yield valuable insights.

For a given permeability field realization, the free surface location and flow through the earth dam is computed using an iterative finite element model derived from Smith and Griffiths (1988), Program 7.1. The elements are 4-node quadrilaterals and the mesh is deformed on each iteration until the total head along the free surface approaches its elevation head above a pre-defined horizontal datum. Convergence is obtained when the maximum relative change in the free surface elevation at the surface nodes becomes less than 0.005. Fig. 1 illustrates two possible free surface profiles corresponding to different permeability field realizations with the same input statistics.

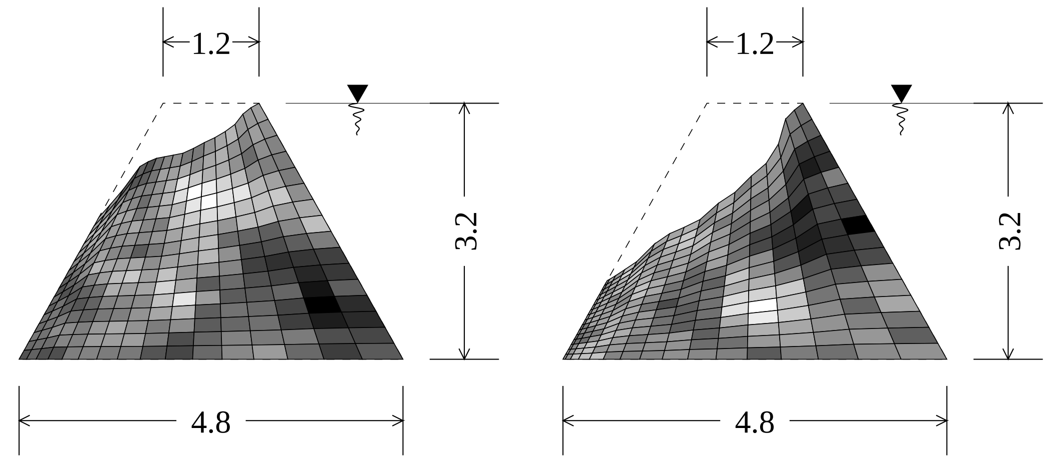

Figure 1. FEM discretization of earth dam; two possible realizations.

Discussion of Results

The estimated mean and standard deviation of the total flow rate, denoted here as m_Q and s_Q respectively, are shown in Figure 2 as a function of the variance of log-permeability, $\sigma^2_{\ln k}$ = ln(1 + σ^2_k/μ^2_k), and the scale of fluctuation, θ_k. Clearly the mean flow rate tends to decreases from the deterministic value of Q_{μ_k} = 1.51 (obtained by assuming $K = \mu_k = 1.0$ everywhere) as the permeability variance increases. The effect is more pronounced at shorter scales of fluctuation. As the scale of fluctuation increases to infinity, the mean flow rate becomes equal to Q_{μ_k}, independent of $\sigma^2_{\ln k}$. For very short scales of fluctuation, the standard deviation of the flow rate is small, as shown by s_Q, increasing dramatically as the scale of fluctuation and $\sigma^2_{\ln k}$ increases.

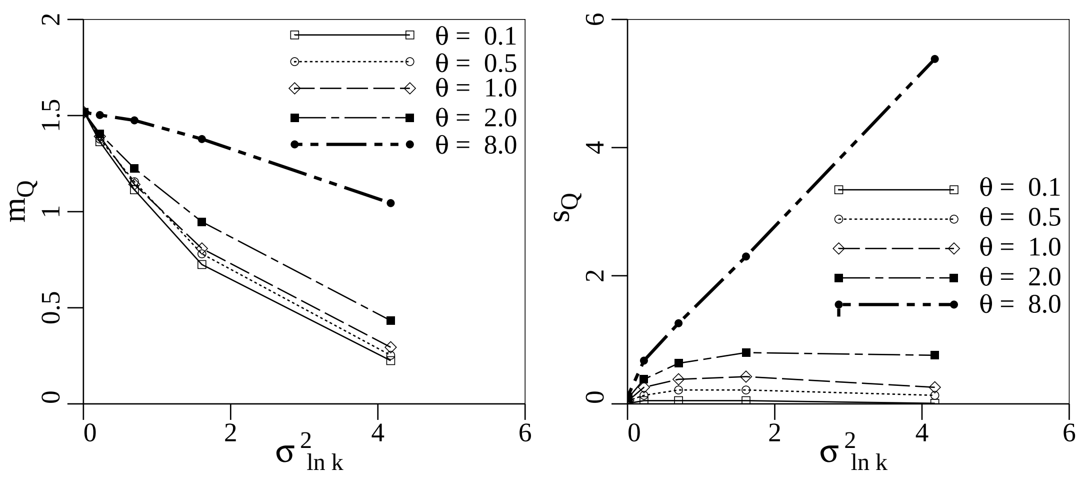

Figure 2. Estimated mean and standard deviation of flow rate through the dam.

Figure 3 shows the estimated mean and standard deviation of the normalized drawdown, m_Y and s_Y respectively. It can be seen that although some clear patterns exist for the mean drawdown with respect to the scale of fluctuation and $\sigma^2_{\ln k}$, the magnitude of the mean drawdown is little affected by these parameters and remains close to Y = 0.57 of the total dam height obtained in the deterministic case with $K = \mu_k = 1.0$. The variability of the drawdown, however, is significantly effected by θ_k and $\sigma^2_{\ln k}$. For small scales of fluctuation relative to the dam size, the drawdown shows little variability even for high permeability variance. This suggests that, under these conditions, using a fixed free surface to model the dam may be acceptable. For larger scales of fluctuation, the drawdown shows more variability and the stochastic nature of the free surface location must be included in any analysis.

In summary, for scales of fluctuation which are small relative to the size of the dam, the results indicate that the flow through the dam is well represented using only the estimated mean flow rate m_Q; the free surface profile will be relatively static and can be estimated from the deterministic case. As the scale of fluctuation becomes larger, the mean flow rate does not fall as rapidly with increasing $\sigma^2_{\ln k}$ but the variability of the flow rate and free surface location from one realization to the next does increase significantly.

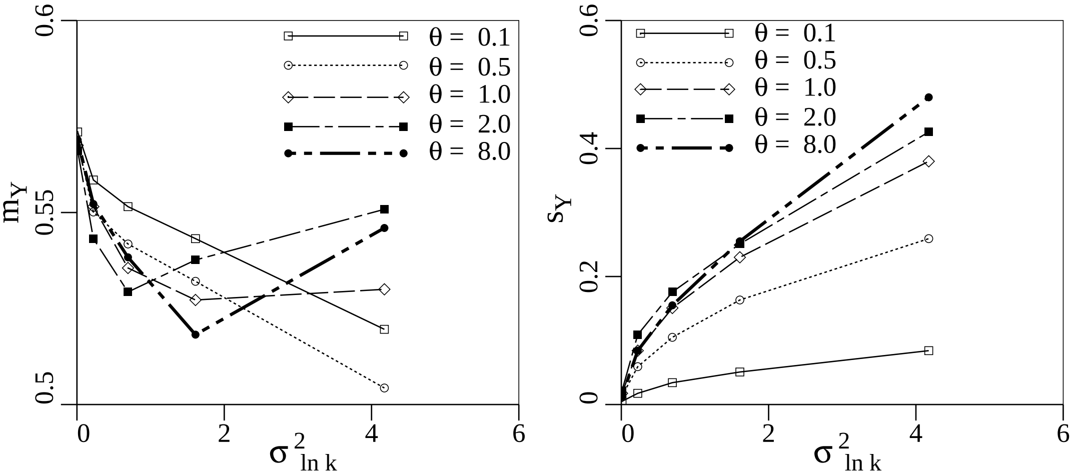

Figure 3. Estimated mean and standard deviation of free surface drawdown.

References

Fenton, G. A. and Vanmarcke, E. H. (1990). "Simulation of Random Fields via Local Average Subdivision," *ASCE J. Engrg. Mech.*, **116**(8), 1733–1749.

Hoeksema, R.J. and Kitanidis, P.K. (1985). "Analysis of the spatial structure of properties of selected aquifers," *Water Resources Res.*, **21**(4), 563–572.

Smith, I.M. and Griffiths, D.V. (1988). *Programming the Finite Element Method*, John Wiley & Sons, New York, NY.

Sudicky, E.A. (1986). "A natural gradient experiment on solute transport in a sand aquifer: Spatial variability of hydraulic conductivity and its role in the dispersion process," *Water Resources Res.*, **22**(13), 2069–2083.