Underground pillar stability: A probabilistic approach

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ABSTRACT: The majority of geotechnical analyses are deterministic, in that the inherent variability of the materials is not modeled directly, rather some "factor of safety" is applied to results computed using "average" properties. In the present study, the influence of randomly distributed shear strength is assessed via numerical experiments involving the compressive strength and stability of pillars typically used in underground construction and mining operations. The model involves combining random field theory with an elasto-plastic finite element algorithm in a Monte-Carlo framework. It is found that the "average" shear strength of the rock is not a good indicator of the overall strength of the pillar. The results of this study enable traditional approaches involving "factors of safety" to be re-interpreted in the context of reliability based design.

1 INTRODUCTION

A review and assessment of existing design methods for estimating the factor of safety of coal pillars based on statistical approaches was covered recently by Salamon (1999). This paper follows this philosophy by investigating in a rigorous way, the influence of rock strength variability on the overall compressive strength of rock pillars typically used in mining and underground construction. The technique merges elasto-plastic finite element analysis (e.g. Smith and Griffiths 1998) with random field theory (e.g. Vanmarcke 1984, Fenton 1990) within a Monte-Carlo framework. The rock strength is characterized by an elastic-perfectly plastic Tresca failure criterion, in which the variable cohesion c is defined by a lognormal distribution with three parameters as shown in Table 1.

Table 1. Input parameters for rock strength

		Units
Mean	μ_c	kN/m^2
Standard Deviation	σ_c	kN/m^2
Spatial Correlation Length	$\theta_{\ln c}$	m

The Spatial Correlation Length describes the distance over which the spatially random values will tend to be correlated in the underlying Gaussian field. Thus, a large value will imply a smoothly varying field, while a small value will imply a ragged field. Initial studies on a similar problem were reported by Paice and Griffiths (1999).

In order to non-dimensionalize the input, the rock strength variability is expressed in terms of the Coefficient of Variation $C.O.V._c = \sigma_c/\mu_c$, and a normalized spatial correlation length $\Theta_c = \theta_{\ln c}/B$ where B is the side length of the pillar. Typical $C.O.V._c$ values for rock are thought to be of the order of 0.4 (see e.g. Savely 1987, Hoek and Brown 1997).

A typical finite element mesh is shown in Figure 1 and consists of 400 8-node plane strain quadrilateral elements. Each element is assigned a different c-value based on the underlying lognormal distribution. At each Monte-Carlo simulation, the block is compressed by incrementally displacing the top surface vertically downwards. Following each displacement increment, the nodal reaction loads are summed and divided by the width of the block B to give the average axial stress. The maximum value of this axial stress q_f, is then defined as the compressive strength of the block.

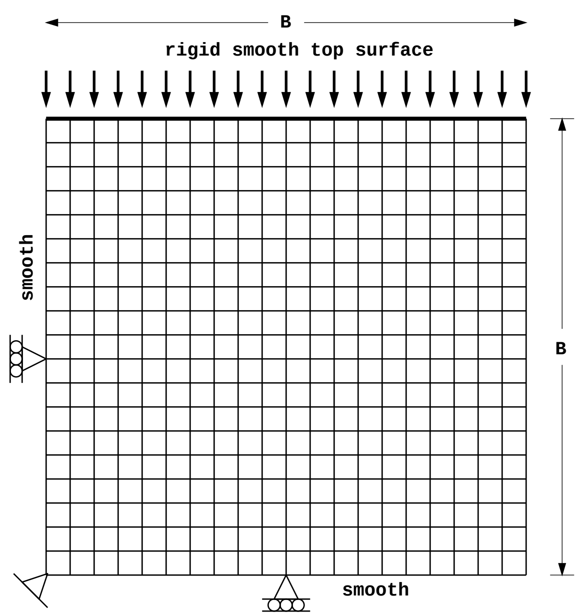


Figure 1. Mesh used for FE pillar analysis.

This study focuses on the dimensionless "bearing capacity factor" N_c defined at each of the n_{sim} Monte-Carlo simulation as:

$$N_c^i = q_f^i / \mu_c, \quad i = 1, 2, ..., n_{sim} \tag{1}$$

The N_c^i values are then analysed statistically to enable probabilistic statements to be made about the compressive strength of the pillar.

For a homogeneous rock, $N_c = 2$, so for a given level of rock strength variability, it will be important for design to estimate the factor of safety required to reduce the probability of failure to acceptable levels.

2 PARAMETRIC STUDIES

Analyses were performed with input parameters within the following ranges:

$$0.01 < \Theta_c < 10$$

$$0.05 < C.O.V._c < 1.6$$

For each pair of values of $C.O.V._c$ and Θ_c, n_{sim} (=2500) Monte-Carlo simulations were performed, and from these, the *estimated* statistics of the bearing capacity factor were computed leading to a mean m_{N_c} and standard deviation s_{N_c}.

Figure 2 shows a typical deformed mesh at failure with a superimposed greyscale in which lighter regions indicate stronger rock and darker regions indicate weaker soil. It is clear in this case that the weak (dark) region has triggered a quite irregular failure mechanism. In general, the mechanism is attracted to the weak zones and "avoids" the strong zones.

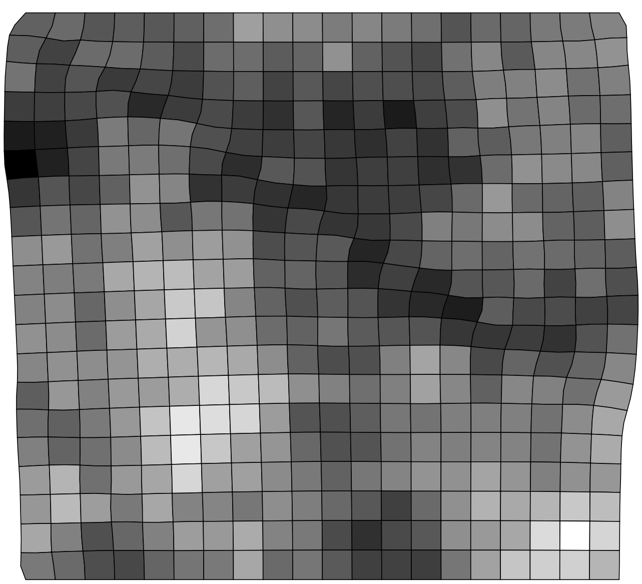
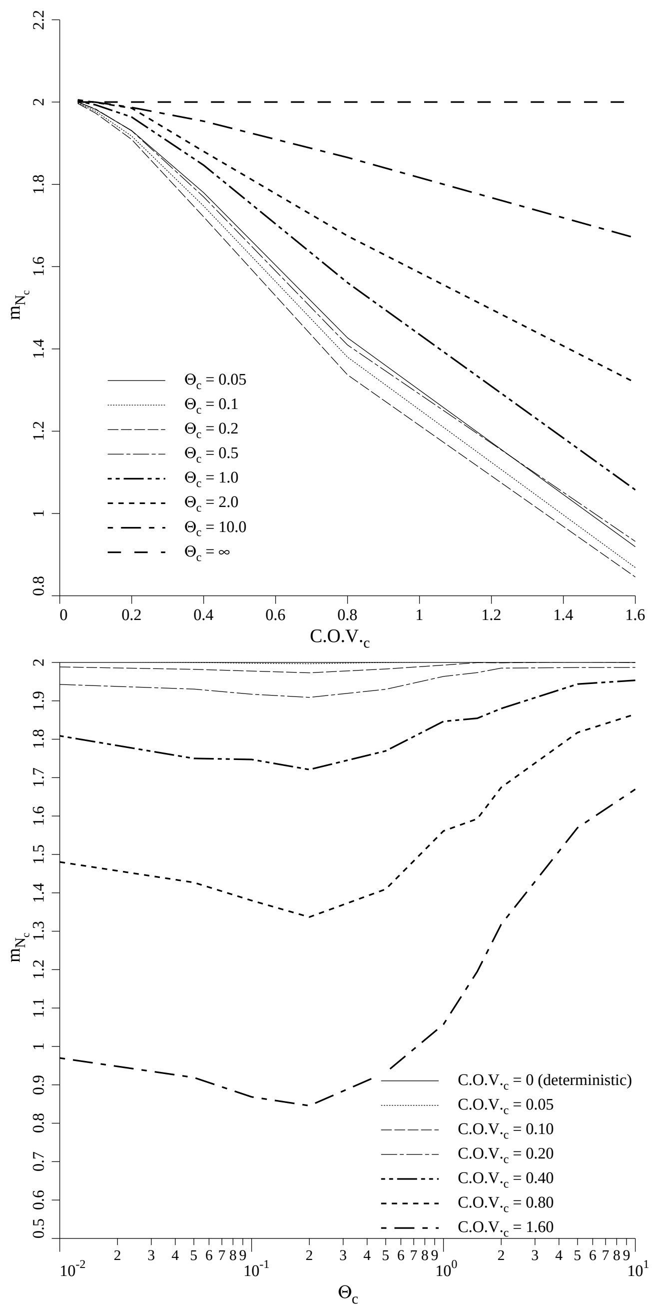


Figure 2. Typical deformed mesh and grey scale at failure for $C.O.V._c = 0.4$ and $\Theta_c = 0.2$.

2.1 *Mean of N_c*

A summary of the mean bearing capacity factor (m_{N_c}) computed using the values provided by equation (1) for each simulation is shown in Figures 3a and 3b. The plots confirm that for low values of $C.O.V._c$, m_{N_c} tends to the deterministic value of 2. As the $C.O.V._c$ of the rock increases, the mean bearing capacity factor falls quite rapidly, especially for smaller values of Θ_c. As shown in Figure 3b, however, m_{N_c} reaches a minimum at about $\Theta_c = 0.2$ and starts to climb again. It could be speculated that in the limit of $\Theta_c = 0$, there are no "preferential" paths the mechanism can follow, and the mean bearing capacity factor will return once more to the deterministic value of 2. This hypothesis can only be tested with an extremely fine mesh and is currently under further investigation.



Figures 3a,b. Variation of m_{N_c} with $C.O.V._c$ and Θ_c

Also included on Figure 3a is the horizontal line corresponding to the solution that would be obtained for

$\Theta_c = \infty$. This hypothetical case implies that each realization of the Monte-Carlo process involves essentially homogeneous soil, albeit with properties varying from one realization to the next. In this case, the distribution of q_f will be statistically similar to the underlying distribution of c but magnified by 2, thus $m_{N_c} = 2$ for all values of $C.O.V._c$.

2.2 *Coefficient of Variation of N_c*

Figure 4 shows the influence of Θ_c and $C.O.V._c$ on the coefficient of variation of the estimated bearing capacity factor, $C.O.V._{N_c} = s_{N_c}/m_{N_c}$. The plots indicate that $C.O.V._{N_c}$ is positively correlated with both $C.O.V._c$ and Θ_c, with the limiting value of $\Theta_c = \infty$ giving the straight line $C.O.V._c = C.O.V._{N_c}$.

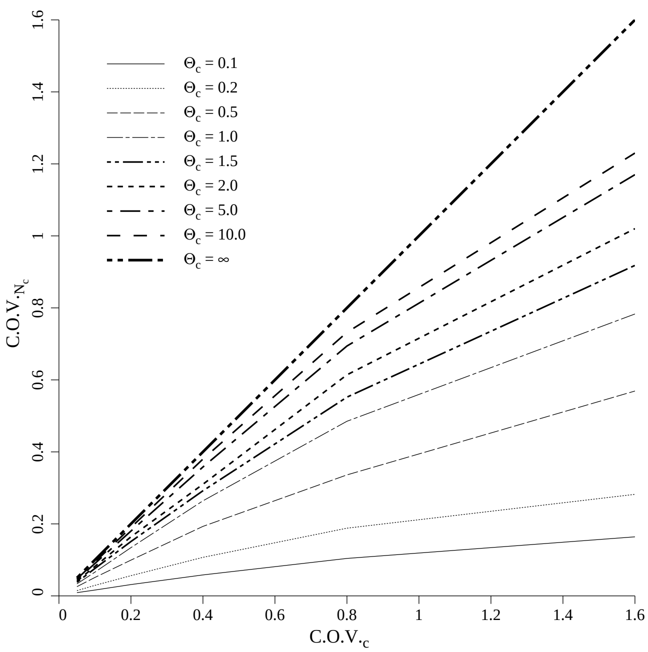


Figure 4. Variation of $C.O.V._{N_c}$ with $C.O.V._c$ and Θ_c

3 PROBABILISTIC INTERPRETATION

Following Monte-Carlo simulations for each parametric combination of input parameters (Θ_c and $C.O.V._c$), the suite of computed bearing capacity factor values from equation (1) was plotted in the form of a histogram, and a "best-fit" lognormal distribution superimposed. An example of such a plot is shown in Figure 5 for the case where $\Theta_c = 0.2$ and $C.O.V._c = 0.4$.

Since the lognormal fit has been normalized to enclose an area of unity, areas under the curve can be directly related to probabilities. From a practical viewpoint, it would be of interest to estimate the probability of "design failure", defined here as occurring when the computed compressive strength is less than the deterministic value based on the mean strength divided by a "factor of safety" F, i.e.

$$\text{"Design failure" if } q_f < 2\mu_c/F \tag{2}$$

In the interests of brevity, only the case corresponding to F=1.5 will be presented here. Let the probability of "design failure" be $p(N_c < 2/F)$, hence from the properties of the underlying normal distribution we get:

$$p(N_c < 2/F) = \Phi\left(\frac{\ln 2/F - m_{\ln N_c}}{s_{\ln N_c}}\right) \tag{3}$$

where Φ is the cumulative normal function.

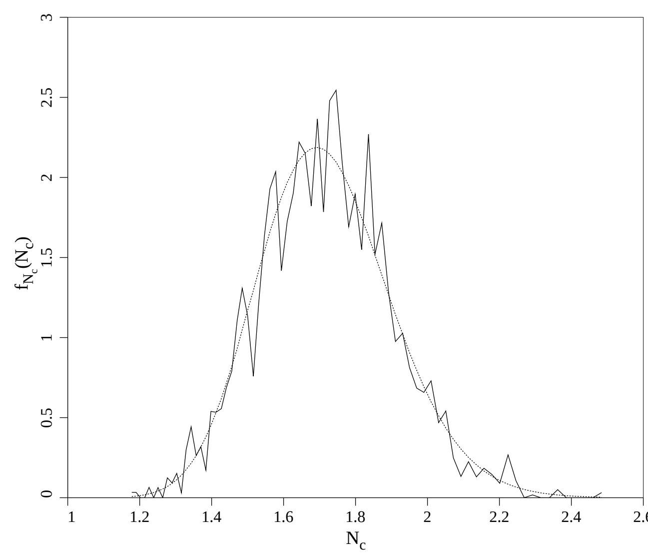


Figure 5. Histogram and lognormal fit for the computed bearing capacity factors with $C.O.V._c = 0.4$ and $\Theta_c = 0.2$.

For the particular case shown in Figure 5, the fitted lognormal distribution has the properties $m_{N_c} = 1.721$ and $s_{N_c} = 0.185$, hence the underlying normal distribution (see e.g. Griffiths and Fenton 1997) is defined by $m_{\ln N_c} = 0.537$ and $s_{\ln N_c} = 0.107$. Equation (3) therefore gives $p(N_c < 2/F) = 0.00997$, indicating an 0.997% probability of "design failure" as defined above.

Figure 6 shows the effect of Θ_c and $C.O.V._c$ on the probability of failure with a factor of safety of 1.5. The complex trends for the probability of failure are due to the interaction of the individual influences of m_{N_c} and s_{N_c}. Although the probability of failure is influenced by m_{N_c} and s_{N_c}, m_{N_c} is the greater factor. Assuming constant s_{N_c}, increasing m_{N_c} decreases the probability of failure. Assuming constant m_{N_c}, the influence of s_{N_c} depends on the value of m_{N_c}. When m_{N_c} is greater than $2/F$, increasing s_{N_c} increases the probability of failure; however, when m_{N_c} is less than $2/F$, increasing s_{N_c} decreases the probability of failure.

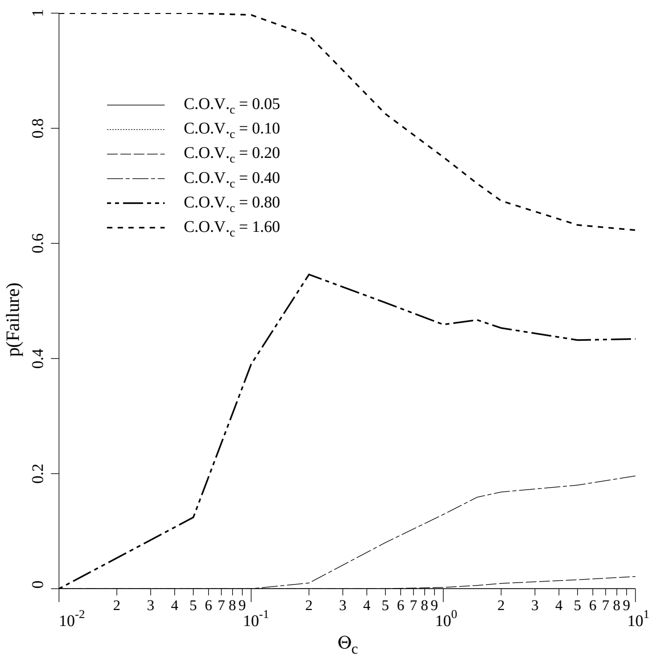


Figure 6. Probability of failure resulting from varying $C.O.V._c$ and Θ_c with $F = 1.5$.

4 CONCLUDING REMARKS

The paper has shown that rock strength variablility in the form of a spatially varying lognormal distribution can significantly reduce the compressive strength of an axially loaded rock pillar.

The following more specific conclusions can be made:

1. As the coefficient of variation of the rock strength increases, the expected compressive strength decreases. The decrease in compressive strength is greatest for small correlation lengths.

2. As the correlation length is further decreased however, the compressive strength appears to reach a minimum and start to increase. It is speculated that as the correlation length becomes vanishingly small and approaches the limiting value of zero (white noise), the compressive strength tends to approach the deterministic value once more.

3. The coefficient of variation of the compressive strength is observed to be positively correlated with both the spatial correlation length and the coefficient of variation of the rock strength.

4. By interpreting the Monte-Carlo simulations in a probabilistic context, a direct relationship between factors of safety and probability of failure can be established.

ACKNOWLEDGEMENT

The writers acknowledges the support of NSF Grant No. CMS-9877189

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