

Reliability of site investigations using different reduction techniques for foundation design

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ABSTRACT: The scope of a site investigation, with an aim to accurately estimate the response of a foundation design, typically involves using the results of several geotechnical tests, located at various locations around a site. However, many of the traditional settlement relationships only use a single set of soil properties, generally varying with depth, to obtain a settlement estimate of a loaded area. It is therefore necessary, to reduce the results of a series of tests into a single set of values, applicable for use in the settlement model. As such, the authors have developed and implemented a framework to measure the effectiveness of several common methods used to reduce the results of multiple tests into a single set of values. Results display the ability of reduction methods to provide a reliable footing design compared to a design utilising the complete and exact knowledge of the site.

1 INTRODUCTION

Traditional settlement models such as those proposed by Janbu et al. (1956) and Schmertmann (1970), as well as other models based on the theory of elasticity (Perloff, 1975; Timoshenko and Goodier, 1951), require knowledge of Young's modulus at various depths in a soil profile. These methods, however, do not accommodate horizontal variation in Young's modulus. Yet, it is widely accepted that soil properties, including Young's modulus, vary significantly from one point to another (Vanmarcke, 1983) in both vertical and horizontal directions. Therefore, there is likelihood that results of geotechnical tests at different locations provide contradicting modulus values.

Analyses were undertaken to evaluate the reliability of the methods used to reduce the results of a sampling programme consisting of more than one sample location and the impact on a pad foundation design for settlement. The pad design utilises one of several commonly used settlement models, which are also evaluated.

The results are displayed in a reliability framework where the performances of each sampling programme, reduction method and settlement model is compared to an optimal design. Uncertainties due to the spatial variability of the soil profile, limited knowledge of the site and simplification in the settlement models are incorporated.

2 METHODOLOGY

Jaksa et al. (2003) introduced a methodology to investigate the reliability of site investigation strategy with respect to the resulting foundation design. Briefly, the framework involves simulating virtual soil profiles, with spatial statistics resembling properties of an actual soil profile, using a three-dimensional random field generator as discussed by Fenton and Vanmarcke (1990). This allows the unattainable scenario in practice of knowing all soil properties completely at all locations within the soil deposit. A comprehensive and rigorous design is therefore made possible to provide the optimal or best design for the given soil profile. On the other hand, if the simulated soil profile is sampled as would be done in a site investigation, a design based on the results of the site investigation data mimics the actual design process. By comparing the two designs, conclusions can be drawn regarding the effectiveness of the site investigation strategy and the design model used. To generate probabilities and hence a reliability of the site investigation strategy and design model, this process is repeated on a thousand soil profiles, which are generated conforming with the same second-order statistics. Jaksa et al. (2005), Goldsworthy et al. (2004a) and Goldsworthy et al. (2004b) have used this framework to investigate the effectiveness of site investigation scope

with respect to the number of tests in the strategy. However, the research in this paper extends these results into comparing the method to reduce the results of a sampling programme.

The implementation of the framework introduced by Jaksa et al. (2003) adopted in this paper, involves using the Local Average Subdivision (LAS) method (Fenton and Vanmarcke, 1990) to generate three-dimensional random fields to simulate soil profiles and three-dimensional finite element analysis (3DFEA) developed by Smith and Griffiths (2004). The results in this paper include two “complete knowledge” designs utilising the complete and exact knowledge of the simulated soil profile. The first uses the same settlement model as used in the limited knowledge design, but incorporates the complete knowledge of the soil profile. This enables direct comparisons between the results without the influence of model error. The second complete knowledge design uses 3DFEA, where the properties generated by the LAS are used as the element properties in the 3DFEA mesh. As with similar studies concerning stochastic 3DFEA (Fenton et al., 1996; Fenton and Griffiths, 2002; Fenton et al., 2003), only the serviceability of the foundation is considered. This allows the 3DFEA to be restricted to a linear-elastic analysis, reducing the required computational time. The comparison of 3DFEA design with the limited knowledge design using a common settlement model, allows the investigation of the inherent model error. Such model errors are a result of simplifications and assumptions, similar to the idealised strain profile used in the Schmertmann (1970) model.

The problem is to design a single pad footing located in the centre of a 50 m × 50 m site with a soil profile 30 m in depth (underlain by incompressible bedrock). The pad supports a single point load of 1500 kN and the footing is assumed to be rigid and unable to rotate. A maximum allowable settlement of 25 mm is set as the serviceability criterion. Several of the settlement models considered estimate the settlement of a flexible footing; therefore, a correction has been implemented to estimate the rigid footing displacement. This correction involves estimating the settlement under the corner and centre of the flexible footing and then uses a parabolic average to estimate the corresponding rigid footing displacement.

The settlement models, including 3DFEA, only provide an estimate of the footing settlement for a given footing size. As a result, the footing sizes are increased until the model estimates a settlement less than the 25 mm limit. Footings are increased incrementally in each direction allowing both rectangular

and square footings. When dealing with only the common settlement models, footings are increased in increments of 0.1 m in each direction. However, when using 3DFEA, discretisation and computational time restrictions do not allow such a small increment. Therefore, an increment size of 1 m in each direction, signifying a mesh element size of 0.5 m, is adopted. To maintain consistency, when 3DFEA is compared with the results of a common settlement model, the larger increment size (1 m) is used for both methods.

Several soil profiles are investigated by varying the spatial statistics used to simulate them. Poisson's ratio is set to a constant value of 0.3, representative of a cohesionless soil (Bowles, 1997). It is held constant, as its variability has little effect on settlement (Fenton et al., 1996). Three statistical properties: the mean; coefficient of variation and the scale of fluctuation describe the variability of the elastic modulus. The coefficient of variation (COV) is a measure of the spread of values about the mean and is defined as the ratio of the standard deviation to the mean. COV values of 20%, 50% and 100% are used to simulate increased soil variability, as shown in Table 1. For the purposes of this paper, a profile with COV = 20% represents a relatively uniform soil profile, while profiles with COV = 50% and 100%, represent moderately and highly variable profiles, respectively. The scale of fluctuation (SOF) is a measure of the spatial dependence of points within the profile, where a small scale of fluctuation gives a soil profile where properties vary rapidly about the mean over short distances, while longer scales of fluctuation show properties which vary slower and more continuously. SOF values of 1 m, 4 m and 16 m have been used in this paper, as shown in Table 1.

Profiles are simulated with isotropic correlation where the SOF is the same in all three directions. It is common, however, that soils possess anisotropic correlation, where the SOF in the horizontal direction is greater than the vertical. Jaksa et al. (2005) have investigated these effects on the reliability of site investigations. A finite Markov correlation structure has been adopted to generate the elastic modulus field, as soils do not explicitly exhibit fractal behaviour (Jaksa and Fenton, 2002). Elastic moduli are selected from a lognormal distribution to ensure non-negative values and all profiles are generated to satisfy wide-sense homogeneity, showing no trend in the vertical direction.

The site investigation strategy is defined by three variables: the quantity of samples; sampling frequency in the vertical direction and the reduction method.

Table 1. Spatial statistics of soil profiles investigated

| Soil designation | Mean (kPa) | COV (%) | SOF (m) | | Poisson's ratio |
|------------------|------------|---------|---------|-------|-----------------|
| | | | Horiz. | Vert. | |
| 20{1,1} | 20,000 | 20 | 1 | 1 | 0.3 |
| 20{4,4} | 20,000 | 20 | 4 | 4 | 0.3 |
| 20{16,16} | 20,000 | 20 | 16 | 16 | 0.3 |
| 100{1,1} | 20,000 | 100 | 1 | 1 | 0.3 |
| 100{4,4} | 20,000 | 100 | 4 | 4 | 0.3 |
| 100{16,16} | 20,000 | 100 | 16 | 16 | 0.3 |

The sampling programme consists of between 1 and 9 samples. Sample locations are randomly selected within a 20 m × 20 m area surrounding the centre of the footing. Two different sampling methods in the vertical direction are also investigated. A process, where elastic moduli are recorded at each depth interval (0.5 m) of the random field, is considered continuous vertical sampling (CVS). Alternatively, discrete vertical sampling (DVS) records elastic moduli at every 3rd depth interval (1.5 m). DVS sampling is similar to the sampling in a standard penetration test, while CVS is closer to cone penetration testing, where the sampling frequency is much higher.

When a sampling programme consists of two or more samples, one of seven methods is used to reduce the results to yield a single set of values. A standard arithmetic (SA) average, as shown in Eq. 1, is the simplest and most intuitive method, where x_i is the individual test result, x_{SA} is the reduced result and n is the number of individual results to reduce. The geometric average (GA) and harmonic average (HA) are shown in Eqs. 2 and 3, respectively.

$$x_{SA} = \frac{1}{n} \sum_{i=1}^n x_i \quad (1)$$

$$x_{GA} = \left(\prod_{i=1}^n x_i \right)^{1/n} \quad (2)$$

$$\frac{1}{x_{HA}} = \frac{1}{n} \sum_{i=1}^n \frac{1}{x_i} \quad (3)$$

Unlike the SA, HA and GA methods, the inverse distance (ID) and inverse distance squared (I2) methods weight the results with respect to the distance between the footing and sample location. Eqs. 4 and 5 show the ID and I2 methods respectively, where d_i is the distance between the footing and sample location and d_{tot} is the sum of the distances between each sample and footing location. Threshold limits are also investigated in this paper where the minimum value (MN) of elastic modulus from all tests at each depth interval is chosen, as well as the value at which 25% of the values are lower (1st quartile – 1Q).

$$x_{ID} = \sum_{i=1}^n \frac{d_i}{d_{tot}} x_i \quad (4)$$

$$x_{I2} = \sum_{i=1}^n \frac{d_i^2}{d_{tot}^2} x_i \quad (5)$$

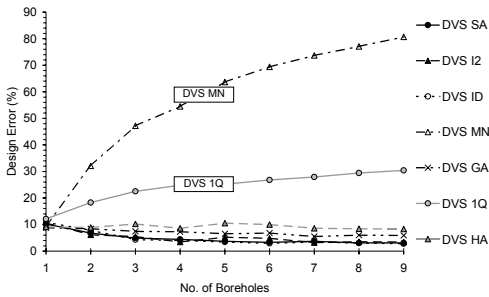
Two designs based on the complete knowledge of the soil profile are determined for each soil profile and compared to a design based on the results of the sampling programme using one of the reduction methods (SA, ID, I2, GA, HA, MN, 1Q). Results are displayed as a design error, which is a measure of the reliability of the design using the sampling programme and reduction method (d_s) compared to the design using complete and exact knowledge (d_c) of the underlying elastic moduli. The design error, D_e , is a normalized percentage (Eq. 6) calculated over 1000 realisations (nr) of a Monte Carlo simulation.

$$D_e = \frac{1}{nr} \sum_{i=1}^{nr} \frac{d_s - d_c}{d_c} \quad (6)$$

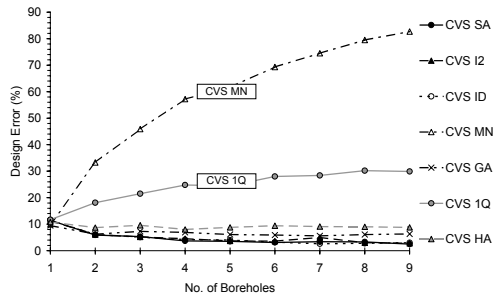
A positive design error indicates the design from the sampling programme is consistently over designed compared to the complete knowledge design. The design error may also be thought of as a normalised margin of safety, compared to the optimal design. In this framework, the reliability of the sampling programme and reduction method is improved as the design error is reduced. The most reliable and economical method will provide the smallest positive design error. However, as the design error has been averaged over 1000 realisations, a positive error does not explicitly signify an over design will occur for each realisation and a probability of under design or failure still exists.

3 RESULTS EXCLUDING SETTLEMENT MODEL ERROR

The results presented in this section of the paper compare the use of Schmertmann's (1970) settlement model using the results of sampled data and the same settlement model using the actual elastic modulus data at the location of the proposed footing. This allows the measurement of design error with respect to the number of samples, reduction method and the vertical sampling frequency (CVS or DVS), independent of the model errors associated with the Schmertmann model. The reliability of the design with respect to the underlying spatial statistics of the elastic modulus is also investigated.

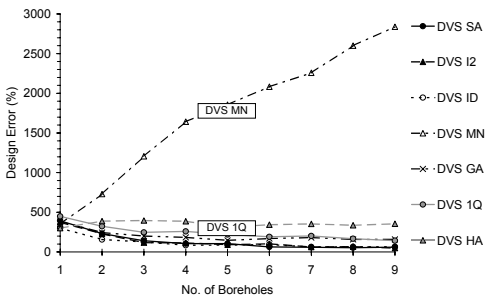


(a)

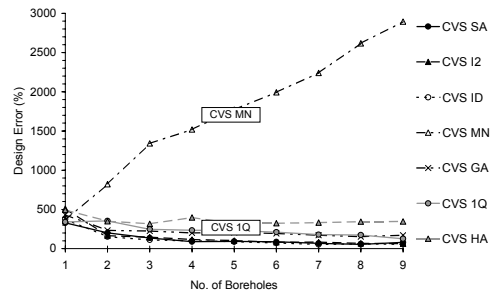


(b)

Figure 1. Design error versus sampling frequency for different combination methods on a soil profile with COV = 20% and SOF = 4 m using (a) DVS and (b) CVS



(a)



(b)

Figure 2. Design error versus sampling frequency for different combination methods on a soil profile with COV = 100% and SOF = 4 m using (a) DVS and (b) CVS

Figs. 1 and 2 show the design error due to the seven different test reduction methods for a relatively uniform (COV = 20%) and highly variable (COV = 100%) soil profile for the same SOF = 4 m. Figs. 1 and 2 also show the difference between (a) DVS and (b) CVS.

It is evident from Figs. 1 and 2, that the minimum value (MN) reduction method shows considerable decrease in reliability (increase in design error) for increasing number of samples. As the sampling frequency increases, there is a greater likelihood of obtaining low elastic moduli. This in turn results in a gross overdesign, leading to a large design error, as shown in Figs. 1 and 2. This phenomenon is also observed, but to a lesser extent, with the 1st quartile (1Q) method in the less variable or more uniform field (COV = 20%). However, the 1Q method in the highly variable soil (COV = 100%) shows an increase in reliability as the sampling frequency increases.

The standard arithmetic (SA), inverse distance (ID) and inverse distance squared (I2) methods appear to be the most reliable methods, showing low design errors. The geometric average (GA) and

harmonic average (HA) appear only slightly less reliable than the SA, ID and I2 for both soil types. Figs. 1 and 2 show little difference between DVS and CVS. Therefore, rather than investigating both methods for the remainder of the paper, the CVS has been abandoned in favour of DVS. The slight difference between CVS and DVS is most likely due to the small difference in sampling frequency. The CVS only records three times the samples of the DVS, leading to slight differences in estimated elastic modulus and therefore footing design.

Figs. 3 and 4 illustrate the sensitivity of the five reduction methods, observed to show an increasing reliability for increasing sample frequency in all soils (SA, GA, HA, ID, I2), with respect to the spatial variability of the profile. Fig. 3 shows the results for soil profiles with COV = 20%, while Fig. 4 presents the results on a profile with COV = 100%.

Figs. 3(a) and 4(a) display the results using the data obtained from 2 samples, while Figs. 3(b) and 4(b) show the results using 9 samples. It appears from the results in Figs. 3 and 4 that the SA, ID and I2 methods typically provide the most reliable designs showing low design error. However, the

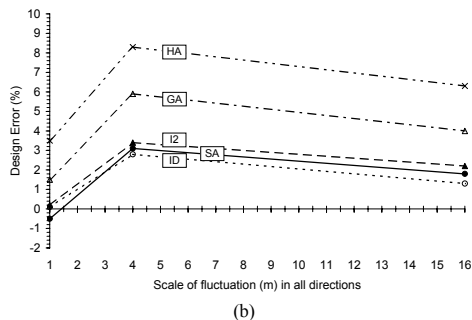
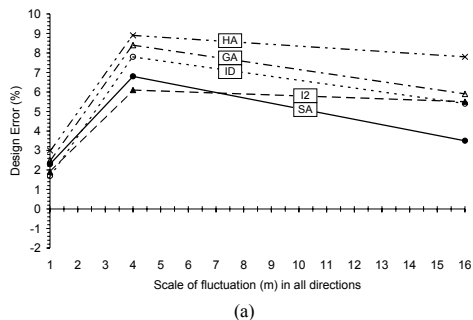


Figure 3. Design error for five reduction methods with respect to scale of fluctuation on a soil with COV = 20% and using the results of (a) 2 and (b) 9 samples

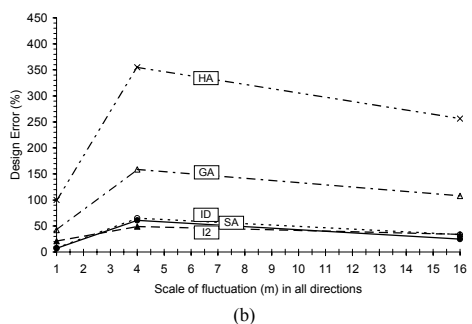
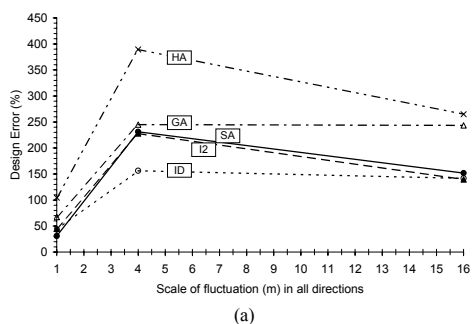


Figure 4. Design error for five reduction methods with respect to scale of fluctuation on a soil with COV = 100% and using the results of (a) 2 and (b) 9 samples

difference between the methods is not as clear as when only two samples have been used (Figs. 3(a) and 4(a)). In fact, the results of using two samples in the relatively uniform (COV = 20%) soil, suggests that the I2 is the most reliable method in an erratically and moderately continuous soil (SOF = 1 m and 4 m), but the SA method is the most reliable in a highly continuous soil (SOF = 16 m). These results are not in agreement with the conclusions that can be drawn from the more highly variable field or the low variability field, using the result of 9 samples. This suggests, it is not clear which reduction method explicitly provides the most reliable design using 2 samples in a low variability field (COV = 20%).

It is also evident from Figs. 3 and 4 that a local maximum occurs in design error for all reduction methods, when the variability of the soil properties is moderately continuous (SOF = 4 m). This maximum has also been observed in results of similar studies (Fenton et al., 2003) and has been suggested a function of the sampling and/or footing spacing. However, it appears that this phenomenon may be a

manifestation of the footing size, as it has been well recognised that a loaded area effects the soil surrounding the footing as well as the soil directly beneath. The mean footing size in terms of area for a soil profile with COV = 100% and using the SA reduction method of 2 samples is 6.69 m², 7.64 m² and 11.91 m², for profiles with SOF of 1 m, 4 m and 16 m, respectively. Based on a square footing, this corresponds to footing widths of 2.58 m, 2.76 m and 3.45 m. It is clear that, for this design situation, the footing width (2.76m) more closely resembles the correlation length (SOF = 4m).

Figs. 3 and 4 show that the HA method provides the least reliable design in all cases. However, Fenton et al. (2003) suggested that the GA provides a suitable elastic modulus in comparison to 2DFEA settlement. They determined the GA was effective in accurately characterising a soil profile to estimate the mean and variance of footing settlement on a spatially variable soil profile. As a consequence the HA method has been abandoned for the remainder of the analyses, in favour of investigating the SA, ID, I2 and GA methods.

4 RESULTS INCLUDING SETTLEMENT MODEL ERROR

Most commonly used settlement models are based on empirical correlations or simplified soil reaction models. Accordingly, they suffer from inherent model errors. On the other hand, 3DFEA allows the incorporation of spatial variability by assigning the soil properties from the random field to the elements of the finite element mesh. In this research, the design using 3DFEA and complete knowledge of the soil profile is considered the benchmark design. This allows the reliability of the other settlement models to be measured in comparison to 3DFEA.

Figs. 5 and 6 illustrate the performance of seven different settlement models as a measure of design error. Each is compared to the optimal design found by using 3DFEA and complete knowledge of the soil profile. Fig. 5 shows the results for a foundation on a relatively uniform soil profile (COV = 20%), while Fig. 6 displays the results for a soil profile with greater variability (COV = 100%). Both Figs. 5 and 6 show the results for soil profiles with elastic moduli that exhibit moderately continuous fluctuations (SOF = 4 m). The traditional settlement models investigated were Timoshenko and Goodier (1951); Janbu et al. (1956); Schmertmann (1970); Perloff (1975) and the stress-strain models with stress distributions found by Newmark (1935), Westergaard (1938) and "2:1" (United States Army Corp of Engineers, 1990) integration. Each model has been used with data obtained from the same sampling programme discussed earlier and one of four reduction methods (SA, ID, I2, GA).

It is evident from both Figs. 5 and 6 that there are significant differences between the settlement models. Theoretically, these are a result of how each method treats the strain and/or stress distributions caused by the loaded area. Another limitation of the methods proposed by Timoshenko and Goodier (1951), Janbu et al. (1956) and Perloff (1975) is the use of a single elastic modulus. Irrespective of the reduction method used to determine this value, a single elastic modulus cannot represent variations in both the horizontal and vertical directions.

The soil variability has a large impact on the reliability of the settlement model, as shown in Figs. 5 and 6. Fig. 5 (COV = 20%) shows that there is only marginal benefit of increasing the sampling frequency for most settlement models and all reduction methods. However, when the foundation is underlain by a soil of greater variability (COV = 100%) there is great benefit in increasing the sampling frequency, as shown in Fig 6.

The relative reliability of the settlement models appears consistent with respect to the variability of

the elastic modulus, where the "2:1" method seems to be the most conservative on highly variable soil profiles (COV = 100%), while the Perloff model, which uses influence values for a rigid rectangular footing adopted from Harr (1966), appears to be the most conservative on more uniform soil profiles (COV = 20%). It is also evident from Figs. 5 and 6 that the settlement models become more conservative when the soil underlying the footing is more variable. It appears that the Westergaard and Newmark integration of the stress distributions are consistently the least conservative. The similarities in errors shown for these two models are a result of the inherent similarities between the models themselves. Fig. 5 indicates that the best-performing settlement model is the Schmertmann relationship. However, this method shows some overall under conservatism, which can lead to under designs and therefore possible failures. The Janbu settlement model on the other hand, generally appears to provide the most reliable designs, showing the smallest positive design error. It must be noted however, that the Janbu settlement model has been calibrated using 3DFEA on a soil profile with uniform elastic modulus and therefore is expected to provide designs similar to 3DFEA.

Fig. 6 also indicates there is typically a greater increase in reliability (reduction in design error) for increased sampling when reducing results with the SA when compared to the other methods. In fact, there appears little benefit using more than 2 samples with the GA, as shown in Fig. 6(d). This suggests the GA makes better use of limited information. Yet, in general, the GA provides a more conservative design than the SA. This is most likely due to the GA providing results that tend to the median elastic modulus rather than the mean, as estimated by the SA.

5 CONCLUSIONS

The reliability of the sampling programme, reduction method and settlement model has been measured against an optimal design accounting for all system uncertainties. This has enabled the effects of sampling, reduction method and model uncertainties to be evaluated for the design scenario presented. Results have indicated that the SA reduction method best accounts for reduction method uncertainty by providing the most reliable designs for all sampling frequencies and most settlement models. The GA, on the other hand, appears to be a more reliable reduction method when there is limited samples or the sampling uncertainty is high.

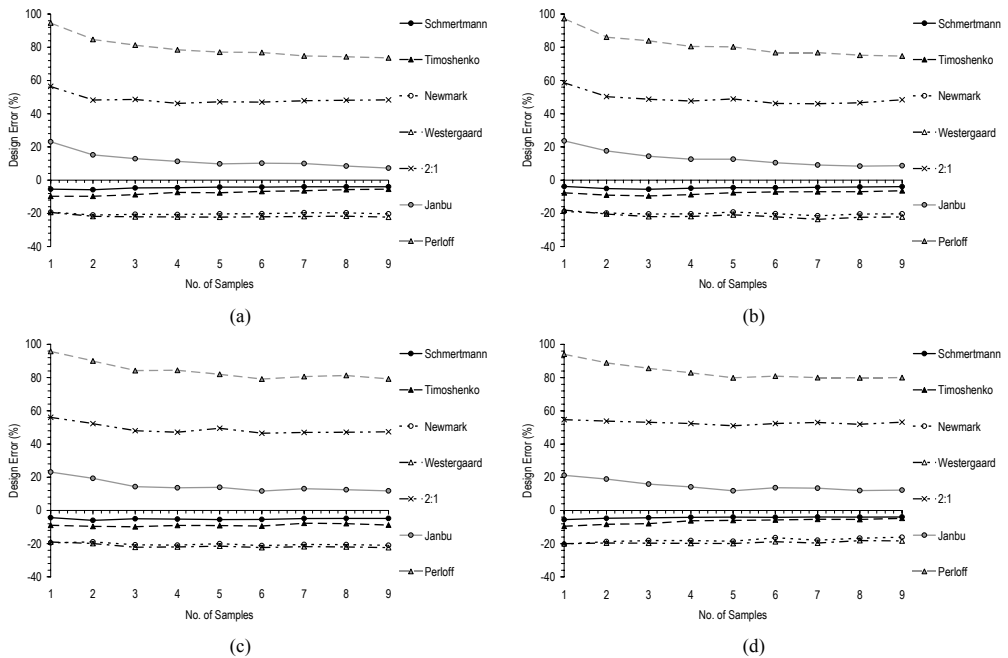


Figure 5. Design error of settlement models with respect to optimal design on a soil profile with COV = 20% and SOF = 4 m using reduction methods: (a) SA, (b) ID, (c) I2 and (d) GA

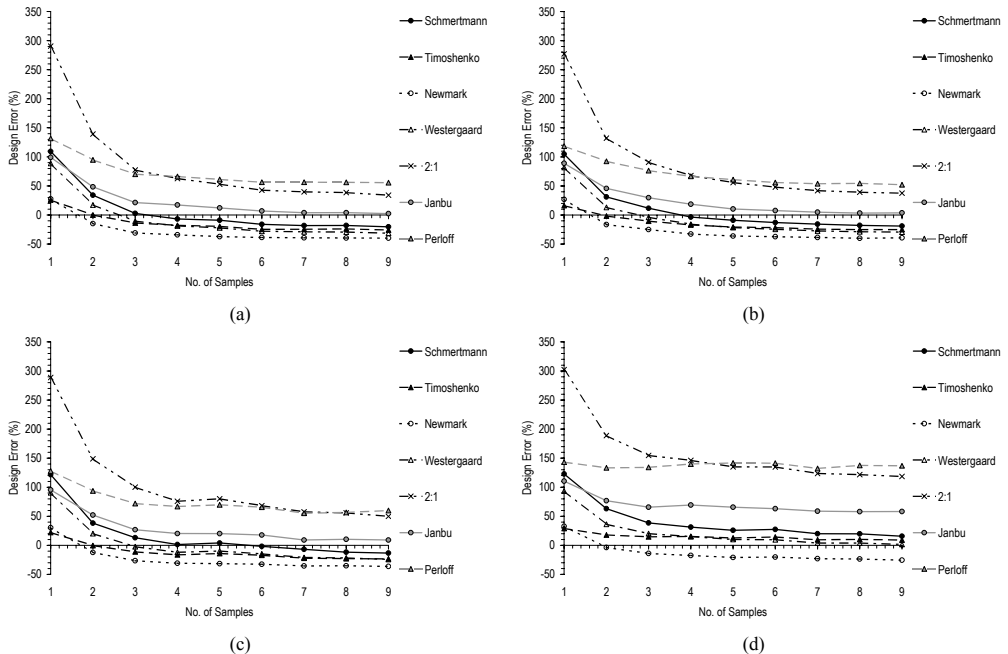


Figure 6. Design error of settlement models with respect to optimal design on a soil profile with COV = 100% and SOF = 4 m using reduction methods: (a) SA, (b) ID, (c) I2 and (d) GA

Significant differences in the reliability of commonly used settlement models have also been observed. The Janbu model appears to provide the most reliable design for most soil profiles, especially the highly variable profiles. The models proposed by Schmertmann, Timoshenko and Goodier, Newmark and Westergaard result in small absolute design errors, however, such errors were typically negative, suggesting under designs and therefore, potential failures.

The spatial variability of the elastic modulus has also been shown to have a significant effect on the reliability of the sampling programme, reduction method and settlement model. The occurrence of a “worst-case” SOF has been identified, similar to other research concerning settlement reliability.

The results presented in this paper will assist a geotechnical professional to adequately manage the uncertainties of spatial variability, limited site knowledge and model error through the use of increased sampling frequency, reduction methods and settlement models. It will also provide a relative measure of the reliability of these methods to attain an optimal design, which takes into account all system uncertainties.

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