Effect of spatial variability on reliability of soil slopes

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ABSTRACT

It is well known that the reliability of a soil slope is greatly influenced by the spatial variability of soil properties. Conventional slope stability analyses that are based on factor of safety cannot explicitly assess the effect of soil variability. Probabilistic methods are more rational means to incorporate uncertainty due to inherent spatial variability of soil properties. In the context of probabilistic slope stability analysis, the term 'probability of failure' is used to describe the safety or reliability of a slope. Inherent soil variability is commonly modelled by a random field, which can be described concisely by the coefficient of variation (COV) and the scale of fluctuation (SOF). This paper investigates the effect of spatial variability of soil properties on the reliability of a soil slope. This is achieved by using the Random Finite Element Method (RFEM), which combines the random field theory with non-linear elasto-plastic finite element slope stability analysis method. A cohesive slope problem is used herein to study the effect of soil variability. The undrained shear strength is treated as a random variable. The probabilistic study is carried out via the Monte Carlo Simulation method. The values of COV and SOF used in this study are selected based on the reported values from the literature. The influence of anisotropy of SOF is also considered in this study. Results from a series of parametric studies indicate that both the COV and the SOF have a significant effect on the probability of failure of a slope.

1 INTRODUCTION

The properties of natural soils are inherently variable from one location to another, even within a relatively homogenous deposit (Vanmarcke 1977a). In conventional slope stability analysis, however, the soil profiles are commonly assumed to be uniform and homogenous where characteristic soil parameters, based on laboratory or field tests, are used in the analysis. The uncertainty and variability in soil properties are traditionally accounted for by adopting a higher factor of safety (FOS) or a reduction factor for soil parameters. Engineering judgments based on local experience play an important role in the conventional analysis. However, it is not uncommon that engineering judgment leads to poor predictions of geotechnical performance. The factor of safety (FOS) has been proved as an inadequate tool in quantifying uncertainty and variability in soil properties (Duncan 2000). A more realistic approach is probabilistic analysis, which takes uncertainty and variability in soil properties into consideration. Probabilistic slope stability analysis has received considerable attention in the literature for the past three decades (e.g. Alonso 1976; Tang et al. 1976; Vanmarcke 1977b; Li and Lumb 1987; Mostyn and Soo 1992; Christian et al. 1994; Duncan 2000; El-Ramly et al. 2002). The measure of safety is expressed in terms of probability of failure (P_f) or reliability index, and the uncertainty and variability of the input soil parameters are incorporated systematically.

More recently, Griffiths and Fenton (2000; 2004) introduced a more advanced approach, called the random finite element method (RFEM), into slope stability analysis. This method combines random field theory (Vanmarcke 1977a; 1983) with non-linear elasto-plastic finite element analysis to explicitly account for the effect of the spatial variability of soil properties. The RFEM is adopted in this paper to investigate the effect of spatial variability on the reliability of a soil slope. A 45°

cohesive slope with a height H = 10 m is considered in this study. The typical finite element mesh of the slope in shown in Figure 1.

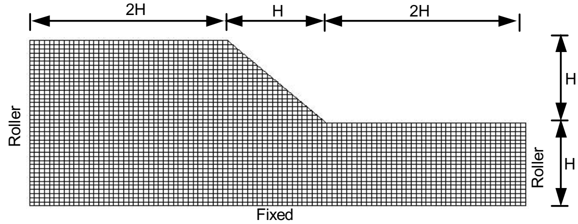


Figure 1: Mesh and slope geometry used for stability analyses.

2 PROBABILISTIC ANALYSIS OF A COHESIVE SLOPE

The slope is assumed to consist entirely of undrained clay with ϕ_u = 0, in which the soil strength is described by the undrained shear strength (c_u). In the context of probabilistic analysis, c_u is assumed to be lognormally distributed, which is characterised by a mean (μ) and a standard deviation (σ). The mean and standard deviation can be expressed in terms of the dimensionless coefficient of variation, (COV), defined as:

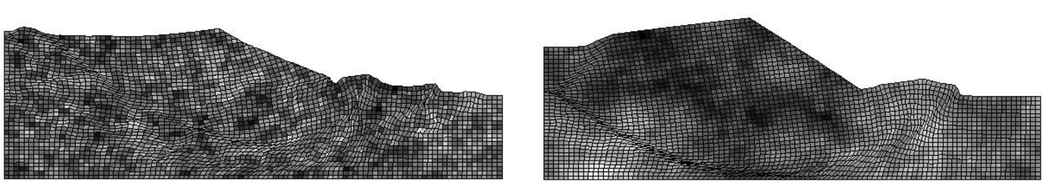
$$\mathrm{COV} = \frac{\sigma}{\mu} \tag{1}$$

The spatial variability of soil properties is modelled by the scale of fluctuation (θ). A large value of θ implies a more smoothly varying field, while a small value of θ indicates one that varies more randomly. In this study, the correlation structure of soil properties is defined by an exponentially decaying (Markovian) correlation function:

$$\rho(\tau) = \exp\left(-\frac{2|\tau|}{\theta}\right) \tag{2}$$

where τ is the distance between two points in the field.

The random field of shear strength values is simulated using the local average subdivision (LAS) method (Fenton and Vanmarcke 1990). The LAS algorithm generates correlated random variables in which their correlation is governed by the correlation function in Equation 2. Figure 2 shows two typical random field realisations of undrained shear strength with different scale of fluctuation. The darker elements indicate stronger soils.



(a) (b)

Figure 2: Typical random field realisations with scale of fluctuation of (a) 1 m and (b) 10 m.

The finite element analysis is based on an elastic-perfectly plastic stress-strain law with a Tresca failure criterion. It uses 8-noded quadrilateral elements and reduced integration in both the stiffness and stress distribution parts of the algorithm. The plastic stress distribution is accomplished by using a visco-plastic algorithm. The theoretical basis of the method is described more fully in the text by Smith and Griffiths (1998; 2004) and the paper by Griffiths and Lane (1999).

Based on a given set of statistics (μ, σ, θ) for a soil property, multiple possible random fields can be generated. These generated random fields are then mapped directly to the finite elements to set up the input soil properties for slope stability analysis. For each generated random field, a single finite element analysis is performed. Hence, in this context, probabilistic analysis involves the repeated finite element analysis for each realisation or repetition of soil properties as part of the Monte Carlo simulation process. The probability of failure (P_f) of a slope is therefore estimated by:

$$P_f = \frac{n_f}{n_{sim}} \tag{3}$$

where n_{sim} is the total number of realisations in the simulation process, and n_f is the number of realisations reaching failure. Slope failure is indicated by non-convergence of the finite element algorithm after 500 iterations (Griffiths and Fenton 2004). In this study, it was found that 2,000 realisations of the Monte Carlo simulation were adequate to give a reproducible estimate of the probability of failure.

Parametric studies were carried out to investigate the effect of spatial variability, by varying COV and SOF, on the probability of failure of the slope. The values of COV and SOF used in the analyses are as follow:

COV = 0.1, 0.3, 0.5, 1.0.
SOF = 1 m, 5 m, 10 m, 50 m, 100 m.

The effect of the anisotropic nature of SOF is also considered in this study by assuming different SOF in vertical and horizontal directions. This is achieved by varying the ratio between horizontal SOF, θ_H, and vertical SOF, θ_V (i.e. θ_H / θ_V = 1, 5, 10, 15, 100). The values of COV and SOF used in this study are within the typical range suggested by various researchers (Lee et al. 1983; Phoon and Kulhawy 1999; Baecher and Christian 2003)

3 RESULTS AND DISCUSSION

Figure 3 shows the influence of scale of fluctuation (SOF) on the probability of failure (P_f) for various coefficient of variation (COV) of two different cases of mean undrained shear strength (C_u):(a) 40 kPa and (b) 60 kPa. In this case, the SOF is assumed to be isotropic, i.e. vertical and horizontal SOF are equivalent. The corresponding deterministic factors of safety (FOS) were found to be 1.1 and 1.65 respectively. It is noted from Figure 3(a) that, for the case of small COV (i.e. COV = 0.1), P_f increases from zero to 21.5% as SOF increases from 1m to 100m, i.e. C_u changes from spatially random to perfectly correlated. However, when COV = 0.3, 0.5 and 1.0, P_f decrease as SOF increases. Figure 3(b) shows the results for the case of a higher value of mean C_u. In this case, when COV = 0.1, P_f remains as zero when SOF increases. P_f increases as SOF increases when COV = 0.3 and 0.5.

It is noted that the results in Figure 3 are similar to those observed by Griffiths and Fenton (2004) for a 26.6° (2:1) slope with mean undrained shear strength of 50 kPa. As previously discussed by Griffiths and Fenton (2004), the two different trends observed with respect to the variation of P_f with SOF (i.e. P_f either increases or decreases as SOF increases) are governed by the *median* C_u of the simulated field. In deterministic analysis, a C_u of 36.4 kPa will cause the slope being considered in this study to have a FOS = 1.0. Hence, in probabilistic analysis, if median C_u > 36.4 kPa, which implies the output statistic has a median FOS > 1.0, P_f is expected to increase as the SOF increases. This is because, by increasing the SOF, the variation in the output statistic is also increased and this consequently increases the chances of a 'failed' slope (i.e. having a FOS < 1.0) to occur. This is observed in Figure 3(a) for the case with COV = 0.1 and in Figure 3(b) for the cases with COV = 0.1, 0.3 and 0.5. These specific cases of COV have caused the median C_u > 36.4 kPa. On the other hand, if the median C_u < 36.4 kPa, larger values of SOF will increase the chances of the occurrence of a 'safe' slope (i.e. having a FOS > 1.0). Hence, a decrease in P_f is expected in this case. This is observed in Figure 3(a) for the case with COV = 0.3, 0.5, and 1.0 and in Figure 3(b) for the cases with COV = 1.0. These results in Figure 3 suggest that assuming a perfectly correlated field (large SOF) and completely ignoring spatial correlation in probabilistic slope stability analysis could either overestimate or underestimate the probability of failure of a slope.

Figure 4 shows the plot of P_f against FOS for various isotropic SOFs when COV is fixed at (a) 0.3 and (b) 0.5. The mean values of C_u being considered are 36.4 kPa, 40.0 kPa, 43.7 kPa, 47.3 kPa, 51.0 kPa and 54.6 kPa, which corresponding to a FOS of 1.0, 1.1, 1.2, 1.3, 1.4 and 1.5 respectively. As expected, P_f decreases as FOS increases for all cases of SOF. It is noted that from Figure 4(a) that all the curves crossover approximately at FOS = 1.2. For FOS < 1.2, P_f decreases as SOF increases, while P_f decreases as SOF decreases when FOS > 1.2. The crossover occurred at FOS ≈ 1.45 when the COV is fixed at 0.5, as shown in Figure 4(b).

Figure 5 shows the influence of anisotropy of SOF on P_f. The horizontal SOF, θ_H, is assumed to be 5, 10, 15 and 100 times larger than the vertical SOF, θ_V. The vertical SOF is fixed at 1 m and 10 m, respectively. It can be noted from Figures 5 that P_f increases with increasing degree of anisotropy (i.e. θ_H/θ_V) for higher FOS with the crossover points approximately at 1.2 and 1.25 for the case of θ_V equal to 1 m and 10 m, respectively. The results indicate that assuming isotropic SOF could either overestimate or underestimate the P_f. However, a further increment in the degree of anisotropy or the horizontal SOF will result in small changes to P_f.

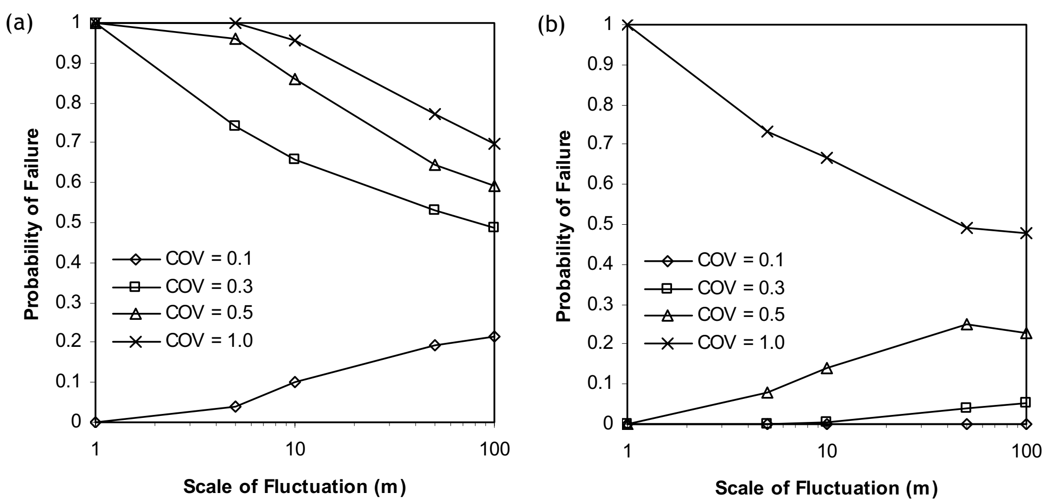


Figure 3: Probability of failure versus scale of fluctuation for various COV. (a) C_u = 40 kPa; FOS = 1.1, (b) C_u = 60 kPa; FOS = 1.65.

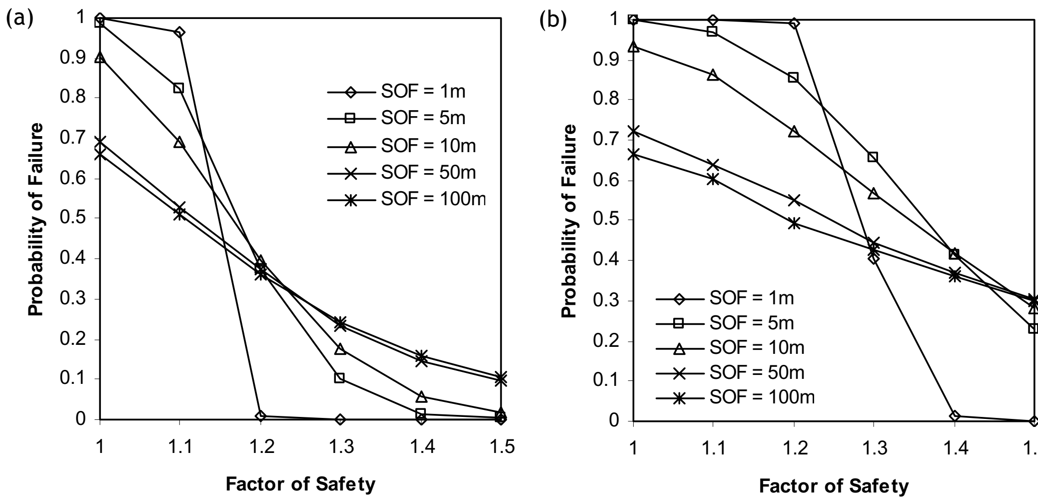


Figure 4: Probability of failure versus factor of safety for isotropic scale of fluctuation. (a) COV = 0.3, (b) COV = 0.5.

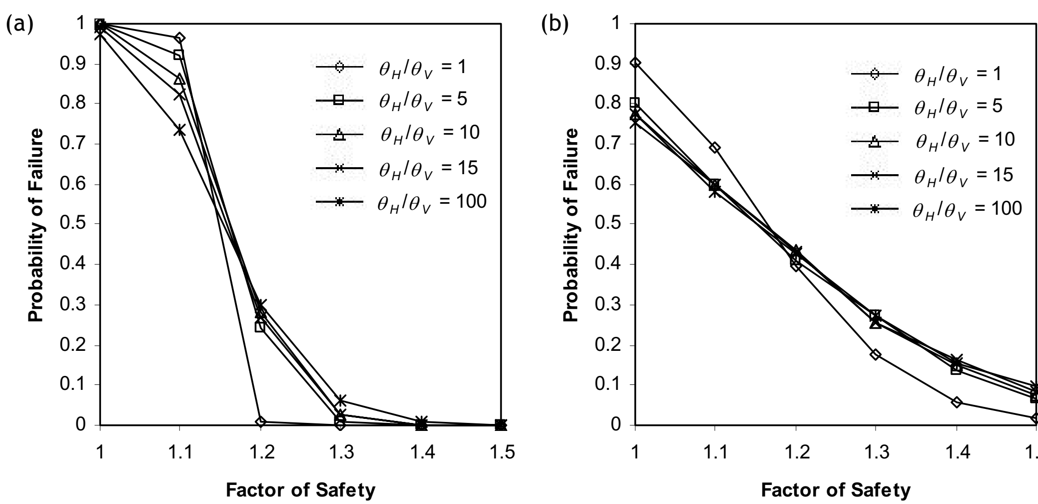


Figure 5: Probability of failure versus factor of safety for anisotropic scale of fluctuation with COV = 0.3. (a) θ_V = 1m, (b) θ_V = 10m.

4 CONCLUSIONS

In this paper, we have investigated the influence of the spatial variability of undrained shear strength (c_u) on the reliability of a spatially cohesive slope. The random finite element method (RFEM), which uses random field theory and elasto-plastic finite element analysis, is adopted in this study. The undrained shear strength is treated as a spatially random variable, which is lognomally distributed. The probability of failure (P_f) of a slope is computed via the Monte Carlo simulation process.

Results from the parametric studies indicated that spatial variability of undrained shear strength, which is measured by scale of fluctuation (SOF) and coefficient of variation (COV), has significant influence on the probability of failure. Ignoring spatial variability in slope stability analysis could either overestimate or underestimate the probability of failure. It is also found that the anisotropy of SOF has equally important effect on the probability of failure.

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