METHODS TO ASSESS RISK REDUCTION WHEN UTILIZING GCLS WITH COMPACTED SOIL LINERS

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ABSTRACT
This paper compares the effect that a GCL will have on the reduction in risk associated with advective flux through a spatially varying composite GCL/compacted soil liner relative to a regulatory CCL based soil liner. An analytical solution is proposed for the assessment and is verified numerically using a random finite element code (RFEM). The sensitivity of results is also examined in the context of the study.

RÉSUMÉ
Cet article compare l'effet qu'un géosynthétique bentonitique (GSB) aura sur la réduction du risque lié au transfert advectif créé par un système d'étancheité composé d'un GSB avec une couche d'argile compactée variant spacialement, comparativement à une couche d'argile compactée, telle que prescrite par les réglements. Une solution analytique est proposée pour l'évaluation et est vérifiée numériquement en utilisant un code d'éléments finis aléatoire. La sensibilité des résultats est également examinée dans le contexte de l'étude.

1 INTRODUCTION

Numerous factors can influence the hydraulic conductivity, k, of field constructed compacted clay liners (CCLs). This may include natural variability in soil composition (i.e. grainsize, atterberg limits, moisture) or variation in compaction energy (Benson et al 1994). Although many of these factors can be mitigated with proper quality control and quality assurance programs through inspection and sampling, the inherent variability in the hydraulic conductivity of a CCL still often remains. Attempting to ensure a minimum specification of 10^{-9} m/s for a compacted clay liner can also result in “wasting” clayey soil that may be above a 10^{-9} m/s specification for a containment site. For long term planning of landfills, transporting acceptable clay to the site from further distances can result in higher transportation costs. It would be an economic advantage to utilize this potential waste soil (hereafter referred to as a compacted soil liner (CSL)), perhaps in combination with a geosynthetic clay liner (GCL).

GCLs are commonly used in practice to reduce the hydraulic flux through barrier systems or provide a cost effective equivalent hydraulic barrier compared to CCLs. GCL variability (in terms of k) is most likely less than that of a constructed CCL. GCLs are a manufactured product, with plant manufacturing quality control and hence are subject to fewer construction irregularities. In figures 1a and 1b, the distribution of the hydraulic conductivity of a compacted soil and composite soil liner are shown. In this paper the distribution of hydraulic conductivity of soil is assumed to be lognormal (Benson et al 1994; Bogardi et al 1989, 1990).

The effective hydraulic conductivity of a composite liner system is closely approximated (assuming flow lines which remain perpendicular to the layers) by the harmonic average of the individual layer conductivities. Within each layer, however, the flow is free to avoid low conductivity zones and the effective conductivity is best captured by a geometric average of the layer conductivity field (see, e.g., Fenton and Griffiths, 1993). The geometric average k is simply the arithmetic average of ln k. If one considers the probability distribution of k for the CSL in Figure 1a based on its probability density function, the geometric average hydraulic conductivity of the CSL may be at or below the regulated value of 10^{-9} m/s, but there will be a probability that some parts of the CSL system are above the regulated value, as shown by the hatched area of Figure 1a. For situations where there is a possibility that the hydraulic conductivity may be exceeded because of variability in the constructed CSL, it is possible that a GCL placed over the CSL (see Figure 1b), will reduce the probability that flow will exceed the specified regulatory value. However, there is currently no known procedure to assess this reduction in risk. Benson et al (1994 and 1999) have published work related to the influence of hydraulic conductivity variability on the field performance of clayey soil liners. However, there is a lack of research pertaining to the role of GCLs in reducing the risk of flow due to the uncertainties mentioned above.

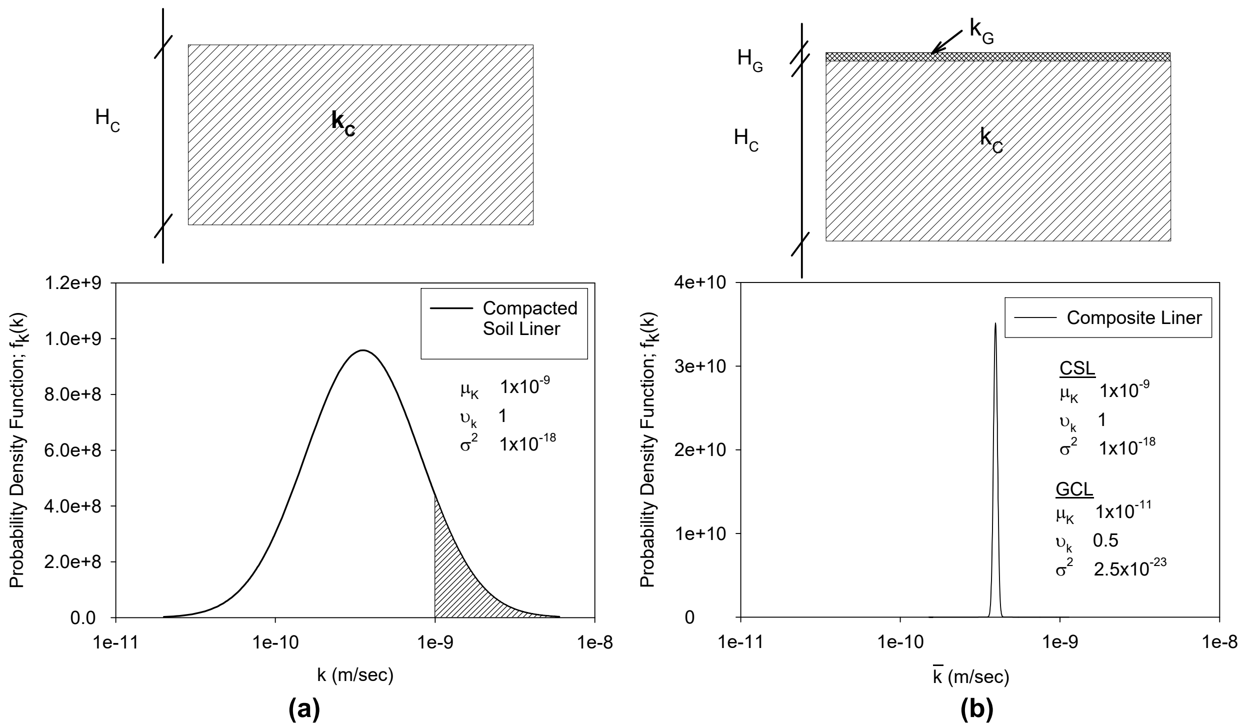

Figure 1. Probability Density Function of (a) CSL k and (b) composite soil liner $\bar{k}$.

The main purpose of this paper is to develop a set of relatively simple analytical solutions that can be implemented into spreadsheets to calculate the "probability of failure" of a GCL/compacted soil composite liner. The paper outlines the approaches taken to perform these analyses and verifies the analytical solutions with probabilistic modelling using the program mrflow3d developed by G.A. Fenton and D.V. Griffiths. An attempt is made to provide some insight on how different practical factors such as the mean k of the CCL and GCL, coefficient of variation of the k of the CCL and GCL, and how the area under consideration affects the "probability of failure". Results are presented in the context of potential benefits of utilizing a GCL to reduce uncertainty of the k of the compacted soil liner.

2 THEORY

2.1 Analytical Solutions

Most compacted soil liners for containment applications are constructed to a hydraulic conductivity specification of 1 x10^{-9} m/s, as well as to some specified thickness. The flux of water through a saturated compacted soil medium is given by Darcy's Law (in the z direction only):

$$v_{az} = k_C i_z = k_C \frac{\Delta h_z}{H_C} \quad [1]$$

where:
V_{az} = the water flux through the soil, in the z direction
k_C = the saturated hydraulic conductivity of the compacted soil liner
i_z = gradient in the z-direction
Δh_z = difference in total head across barrier
H_C = thickness of compacted soil liner in z-direction

Equation 1 essentially describes the flux of water through the compacted soil barrier in Figure 1a.

When a GCL is placed on top of a compacted soil barrier and flow is predominately in the z-direction (as shown in Figure 1b), the flow through the two-layered system can be calculated using the harmonic average of the two barriers (Rowe et al 2004):

$$v_{az} = \bar{k}\bar{i}_z \quad [2]$$

Where:
$\bar{i}_z$ = the hydraulic gradient across the entire composite system, and;
The harmonic average hydraulic conductivity of the GCL and CSL, $\bar{k}$, is calculated as:

$$\bar{k} = \frac{H_G + H_C}{\dfrac{H_G}{k_G} + \dfrac{H_C}{k_C}} \quad [3]$$

Where:
k_G = the saturated hydraulic conductivity of the GCL
H_G = thickness of GCL in z-direction

Equation 2 presented above can be used to calculate the flux of water through the GCL/CSL composite system shown in Figure 1b. As presented, both equations 1 and 2 are limited to one value of hydraulic conductivity for each of the barrier systems. As previously discussed, these equations do not allow us to assess risk associated with the barrier system presented in Figure 1a, nor the potential reduction in risk by using the barrier system in Figure 1b.

Although "probability of failure" can be defined in many ways, in terms of flow though barrier systems it may be more appropriate to use the definition of "probability of exceedance", where the probability of exceedance for the barrier system in Figures 1a or 1b is defined as the probability that the flux will exceed some predetermined regulatory value, v_R value. In most practical cases, the thickness of the barrier system is written into the regulation, and it is the hydraulic conductivity value that is uncertain for the risk assessment.

For this paper, the "probability of exceedance", P(E), is defined as:

$$P(E) = P\left[v_{az} \rangle v_R\right] \qquad [4]$$

Substituting equation [2] into [4] we get:

$$P(E) = P\left[\bar{k} i_z \rangle k_R i_R\right] \qquad [5]$$

And further, substituting equation [3] into [5] we get:

$$P(E) = \left[\frac{H_G + H_C}{\frac{H_G}{k_G} + \frac{H_C}{k_C}} \frac{\Delta h_z}{H_C + H_G} \rangle k_R \frac{\Delta h_z}{H_R} \right] \qquad [6]$$

Simplifying equation [6] forms equation [7] describing the probability of exceedance.

$$P(E) = P\left[\frac{H_G}{k_G} + \frac{H_C}{k_C} \langle \frac{H_R}{k_R} \right] \qquad [7]$$

where:
H_R = regulated thickness of soil liner
k_R = regulated saturated hydraulic conductivity of the soil liner

Substituting the variable, w, to simply equation [7],

$$w = w_G + w_C = \frac{H_G}{k_G} + \frac{H_C}{k_C} \qquad [8]$$

A simplified form of equation [7] becomes:

$$P(E) = P\left[w_G + w_C \langle \frac{H_R}{k_R} \right] \qquad [9]$$

Assuming that the CSL's hydraulic conductivity is lognormal, we have the following relationships between statistical parameters:

$$\sigma_{k_C}^2 = \left(\mu_{k_C} \upsilon_{k_C}\right)^2 \qquad [10a]$$

$$\mu_{\ln k_C} = \ln \mu_{k_C} - \frac{1}{2}\sigma_{\ln k_C}^2 \qquad [10b]$$

$$\sigma_{\ln k_C}^2 = \ln\left(1 + \upsilon_{k_C}^2\right) \qquad [10c]$$

where:
σ^2_{kC} = variance of k of CSL
μ_{kC} = mean k of CSL
v_{kC} = coefficient of variation of CSL k $= \sigma_{k_C} / \mu_{k_C}$
$\mu_{\ln kC}$ = log mean k of CSL
$\sigma^2_{\ln kC}$ = log variance of CSL k

Similar relationships hold for the GCL, assuming k_G is also lognormally distributed.

In previous probabilistic studies related to assessing risk of excessive flow through porous media, (Benson and Daniel, 1994, and Griffiths and Fenton, 1997) the mean k of the soil is generally normalized to mean one, and calculations are performed with a dimensionless k. However due to the complexities of computing the probability of exceedance of the harmonic average of two independent materials, normalization is difficult.

Before going any further in the development of the analytical solution a note needs to be made about the correlation structure of the random hydraulic conductivity field. In this analysis the correlation function is assumed to be Markovian with exponentially decaying correlation:

$$\rho(\tau) = \exp\left\{-2\frac{|\tau|}{\theta}\right\} \qquad [11]$$

where:
θ = scale of fluctuation
τ = averaging domain of the correlation function

This has the variance function,γ, (in one dimension):

$$\gamma(T) = \frac{\theta^2}{2T^2}\left[\frac{2|T|}{\theta} + \exp\left\{-\frac{2|T|}{\theta}\right\} - 1\right] \qquad [12]$$

where:
T = averaging domain of the variance function

For all cases investigated in this paper the scale of fluctuation, θ, is assumed to be 1m in all directions. This presents a reasonable estimate as shown by Benson et al, 1994, who state a scale of fluctuation for compacted clays is likely between 1 and 3 m.

If k_C is lognormally distributed, then w_C is also, where:

$$w_C = \frac{H_C}{k_C} \quad [13a]$$

$$\ln w_C = \ln H_C - \ln k_C \quad [13b]$$

and its parameters are:

$$\mu_{\ln w_C} = \ln H_C - \mu_{\ln k_C} \quad [13c]$$

$$\sigma^2_{\ln w_C} = \sigma^2_{\ln k_C} \gamma_C(D) \quad [13d]$$

similarly for w_G.

Where:
$\gamma_C(D)$ is a three dimensional variance function and D is the averaging domain = H_C x Area (Vanmarcke 1983).

Most engineers are not familiar with the variance function and hence will not know how to calculate it for a given problem. Essentially, the variance function is an indication of how the size and scale of fluctuation (the distance beyond which points are essentially uncorrelated) of the problem influence the variance. Figure 2 plots values of the variance function as a function of the size of problem examined in this paper; that is problem areas with equal length and width (square) with θ_x equal to θ_y; (X: width or length) and scale of fluctuations (x- direction: θ_x and y- direction: θ_y). Due to the nature of the variance function used, thickness of the liner and scale of fluctuation used in the vertical direction have limited influence at large area. This essentially means the variance function changes little for CSLs and GCLs, and plots nearly on top of each other in Figure 2.

To obtain values for P(E), the lognormal parameters of w_C and w_G must be determined. Equations 14a, and b show the inverse transformations of Equations 10b, and c. These equations enable us to transform parameters back to normal state in order to sum them correctly.

$$\mu_{w_C} = \exp\left(\mu_{\ln w_C} + \frac{\sigma^2_{\ln w_C}}{2}\right) \quad [14a]$$

$$\sigma^2_{w_C} = \left[\exp\left(\sigma^2_{\ln w_C}\right) - 1\right] \exp\left(2\mu_{\ln w_C} + \sigma^2_{\ln w_C}\right) \quad [14b]$$

A similar transformation can be performed for the GCL to obtain μ_{wG} and σ^2_{wG}.

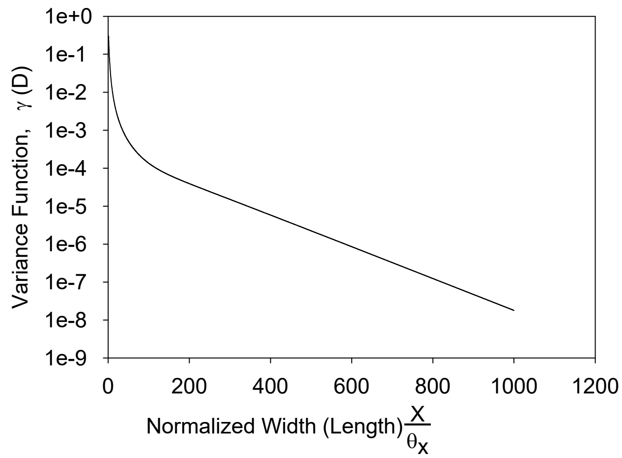

Figure 2. Plot of variance function corresponding to equation [11] (Markov correlation function).

Summing the normalized inverse mean hydraulic conductivity of the CSL and GCL, and the variance of the CSL and GCL in normal space we obtain:

$$\mu_w = \mu_{w_{GCL}} + \mu_{w_{CSL}} \quad [15a]$$

$$\sigma^2_w = \sigma^2_{w_{GCL}} + \sigma^2_{w_{CSL}} \quad [15b]$$

assuming independence.

Finally, assuming w can be approximated by the lognormal distribution, the mean and variance of w can be transformed back to the parameters of the lognormal distribution with equations 13a, and b; enabling the calculation of probability of exceedance, P(E):

$$\mu_{\ln w} = \ln \mu_w - \frac{1}{2}\sigma^2_{\ln w} \quad [16a]$$

$$\sigma^2_{\ln w} = \ln\left(1 + \frac{\sigma^2_w}{(\mu_w)^2}\right) \quad [16b]$$

so that

$$P(E) = \Phi\left(\frac{\ln\left(H_R / k_R\right) - \mu_{\ln w}}{\sigma_{\ln w}}\right) \quad [17]$$

Where, Φ is the cumulative distribution function of the standard normal variate.

Through the use of these equations, and interpolating values of the variance function from Figure 2, the probability of exceedance can be calculated simply with the use of a spreadsheet (as was done for this paper).

2.2 Simulation

To perform the calculations described for the analytical solution in section 2.1, a log normal distribution must be assumed for the hydraulic conductivity of individual layers. However, a harmonic average calculation alters the cumulative distribution of hydraulic conductivity to something other than the lognormal distribution and no common distribution is exact. Therefore, to provide some confidence in this approach, Monte Carlo simulation analysis can be performed to assess the validity of this log normal distribution for the analytical solution discussed in section 2.1.

Probabilistic simulation was performed with a random finite element code, mrflow3d, described by Griffiths and Fenton (1997), which was designed to analyze fluid flow through stochastic fields. This program generates a spatially-varying log normally distributed hydraulic conductivity in three-dimensions for a soil layer which is characterized by a given hydraulic conductivity mean, variance and correlation function. With mrflow3d, hydraulic conductivity realizations are created using the Local Average Subdivision (LAS) method and are subsequently analyzed using the Finite Element Method (FEM). Details of this approach can be found in Griffiths and Fenton (1997). For the present study mrflow3d is used to generate two independent hydraulic conductivity fields; one for the CSL and one for the GCL. For each realization performed, mrflow3d generates a correlated field of "local averages of" hydraulic conductivity values. The geometric average of these point scale hydraulic conductivity values is then calculated and recorded. This type of averaging retains the point scale lognormal distribution and physically represents some lateral flow around low hydraulic conductivity zones. In a post-processing stage, each realization of the geometric average hydraulic conductivity of the CSL is harmonically averaged with a corresponding realization of the geometric average hydraulic conductivity of the GCL. After this is completed for 5000 simulations, the mean and standard deviation of composite liner hydraulic conductivity is determined using the harmonic average of independent layers

3 VERIFICATION OF ANALYTICAL SOLUTION

Although this type of analysis can be performed with mrflow3d, significant expertise and knowledge is required to perform a probabilistic simulation. The purpose of developing an analytical solution for the problem described herein is to generate a set of equations which can be entered into a spreadsheet in order to quickly generate "probabilities of exceedance" for varying liner compositions.

Figure 3 shows the similarity of a typical histogram developed from Monte Carlo simulation performed by mrflow3d (solid line) and the analytical solution (dashed line). It can be seen that the assumption of a lognormal distribution fitted to the mean and standard deviation of the simulated data matches the histogram generated from the analytical solution. The "probability of exceedance" of the liner system is shown on the histogram as the area under the curve to the left of the regulatory value (hatched area). For example, in Figure 3, the area under the curve is 1.0 and the probability of exceedance of the composite CSL/GCL liner is calculated as 0.035. Histograms were generated for a range of assumed means and variances and found to match well to the simulation results. It is therefore concluded that the assumption of a lognormal distribution for w is reasonable and hence provides confidence in the derivation of the analytical solutions provided earlier.

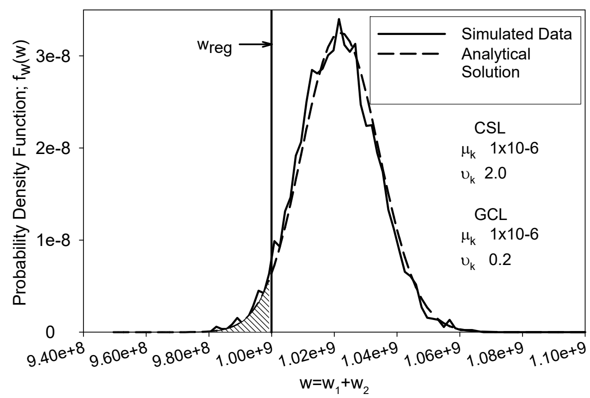

Figure 3. Typical Probability Density Function of Composite Liner w.

4 UTILIZING ANALYTICAL SOLUTIONS TO EXAMINE PROBABILITY OF EXCEEDANCE

Given the suitability of the analytical solutions for calculating the "probability of exceedance", it becomes a fairly easy exercise to examine some practical factors which can influence risk reduction of using a GCL over a CSL. The results for a range of hydraulic conductivities of the CSL and GCL are presented below. Also presented below is the influence of the barrier area on the analysis.

4.1 Influence of CSL and GCL Mean Hydraulic Conductivity on Probability of Exceedance

A variety of soils exist in nature that can potentially be used for barrier systems, especially when used as some form of composite barrier system with GCLs. In this study, mean CSL hydraulic conductivities were varied from $1x10^{-10}$ to $1x10^{-4}$ m/s to represent soil types ranging from low permeability clays to coarse sands (Das, 2002). Three typical GCL conductivities ($1x10^{-12}$ m/s, $1x10^{-11}$m/s, and $5x10^{-11}$ m/s) were chosen to utilize in combination with the CSL to provide harmonic average hydraulic conductivities in the vicinity of 1 x 10^{-9} m/s. A summary of the ranges of values examined can be found in Figure 4.

The results of Figure 4 are presented as a series of points for each analysis. For example, if one examines Figure 4b for a GCL/CSL composite liner system in which the mean GCL hydraulic conductivity is 1 x 10^{-11} m/s, υ_c is 0.1 and scale of fluctuation is 1m in all directions, and the CSL mean hydraulic conductivity is 1x10^{-4} m/s with υ_c is 0.5 and scale of fluctuation is 1m in all directions, they will find a probability of exceedance of 0.2. In other words, given the statistics of the CSL and the GCL combined, there is 1 in 5 probability that the flow will be higher than that of 1 m thick compacted soil liner a uniform hydraulic conductivity of 1 x10^{-9} m/s. In a similar manner, it can be seen for the same GCL used in combination with a compacted soil liner of mean hydraulic conductivity of 1 x 10^{-8} m/s (CSL υ_c of 0.5), the probability of failure is close to zero. A similar exercise can be performed for any of the three GCL mean hydraulic conductivities and the CSL mean hydraulic conductivities chosen. If other GCL mean hydraulic conductivity values were required to be used in the analysis, one could generate similar plots using the analytical solution presented in section 2. It should be noted that the results shown in Figure 4 are for a plan area of 20 m x 20 m.

Based on the results shown in Figure 4, the following points can be noted:

- As the mean hydraulic conductivity of the CSL increases beyond the regulatory value of 1x10^{-9} m/s, the risk of exceedance also increases.
- Regardless of the mean k of the CSL, as the mean hydraulic conductivity of the GCL decreases to 10^{-12} m/s, the risk of exceedance approaches zero. This implies that for very low values of hydraulic conductivity of the GCL, the probability of exceedance posed by the risk of high k values of the CSL become minimal.
- For “marginal” k soils (near but slightly above the regulatory value of 1 x10^{-9} m/s), a GCL with k values of 1 x 10^{-11} m/s and below can significantly reduce the probability of exceedance.
- The influences of CSL variance are counterintuitive to what one would naturally assume. Increasing CSL variance reduces the probability of failure for any pair of CSL and GCL mean hydraulic conductivities. This is because with higher variability of the material, it is more likely that there will be lower values of k in the flow path for a given mean k of the CSL.

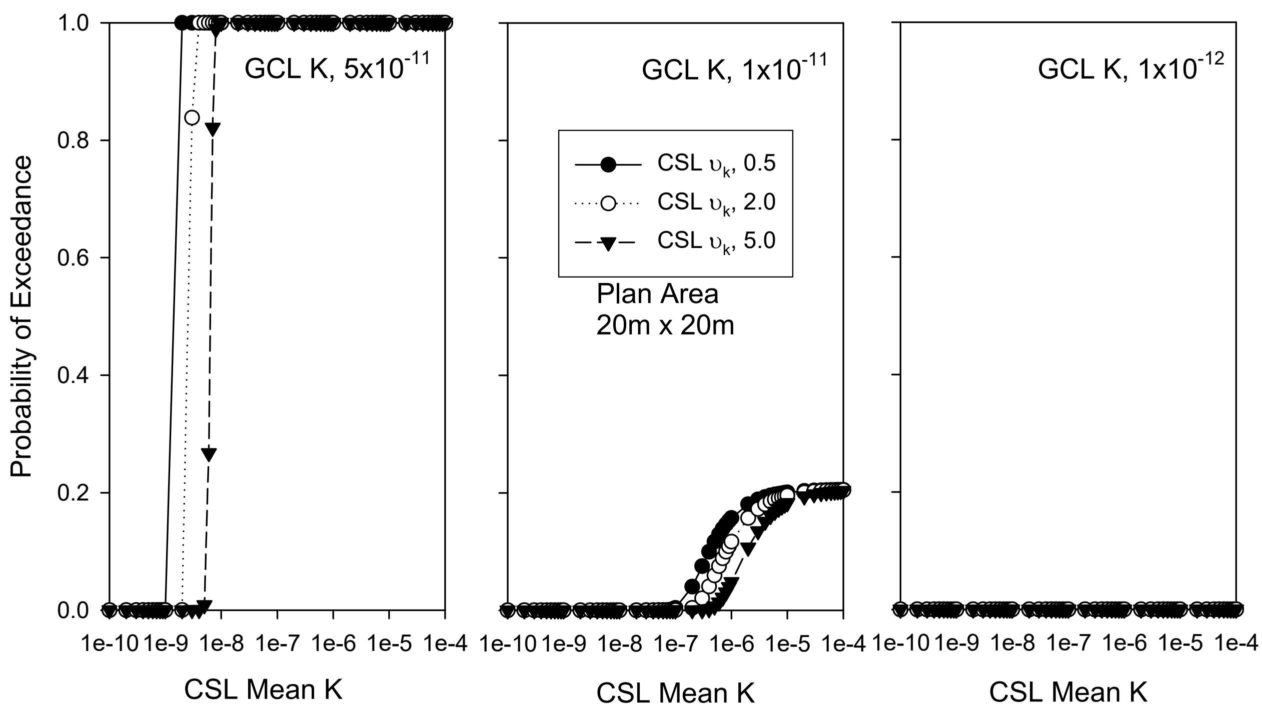

Figure 4. Probability of Exceedance with GCL Mean k; liner area of 20m x 20m.

4.2 Effect of Liner Area

In practice, liner systems are used in projects varying in size from hundreds of square meters (small lagoons) to thousands or even millions of square meters (large landfills). The size of the problem will inherently affect the probability of exceedance. For this reason it is prudent to investigate the effect, if any, of liner area.

For all cases considered (for a 1 m thickness), larger area liners have a lower probability of exceedance than smaller liners of the same composition (Figure 5). This emphasises the importance of material quality and construction techniques, especially for small, lagoon size, projects. Mathematically, increasing the area reduces the three dimensional variance function used in both simulation and the analytical solutions described in section 2 and shown in Figure 2. The decrease in variance produces a narrowing of the lognormal distribution, which results in a lower probability of advective flux exceeding the regulated value, shown in Figure 6. Hence the probability of failure becomes either zero or one at large areas, depending on the harmonic average hydraulic conductivity, as shown in Figure 5. What it also shows is that for areas larger than 20 m x 20m examined in Figure 4, the GCL can eliminate risk of exceedance for a wide variety of k of the CSL.

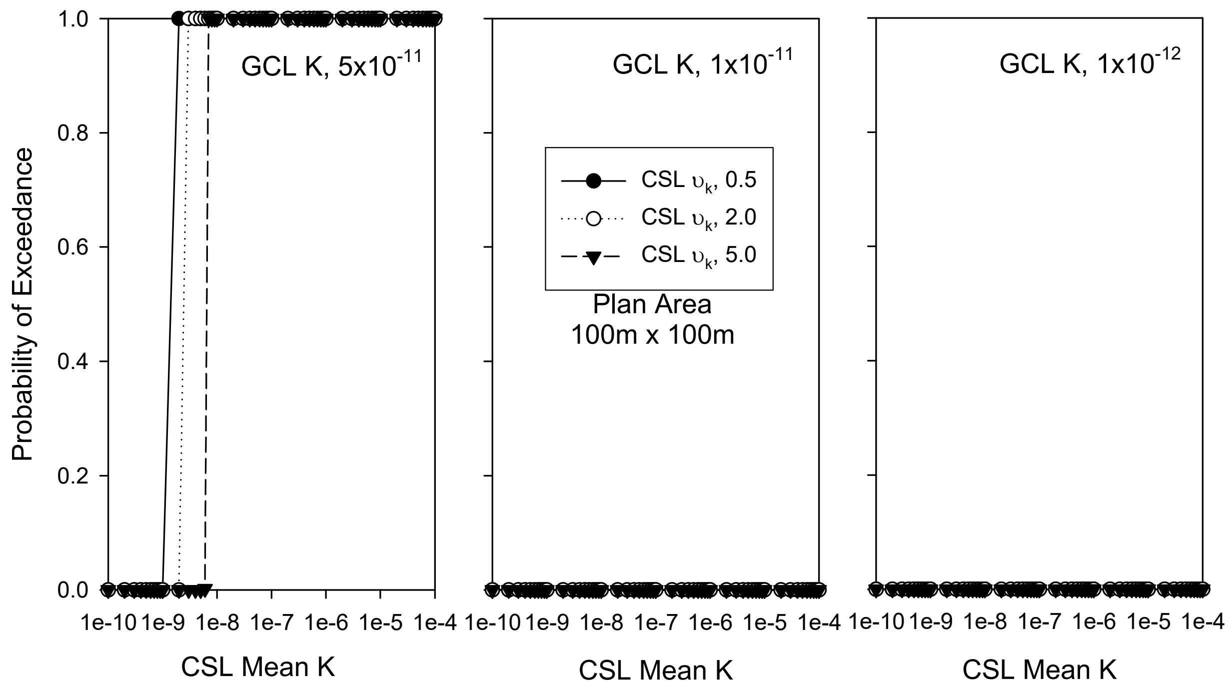

Figure 5 Probability of Exceedance with GCL Mean k; 100m x 100m

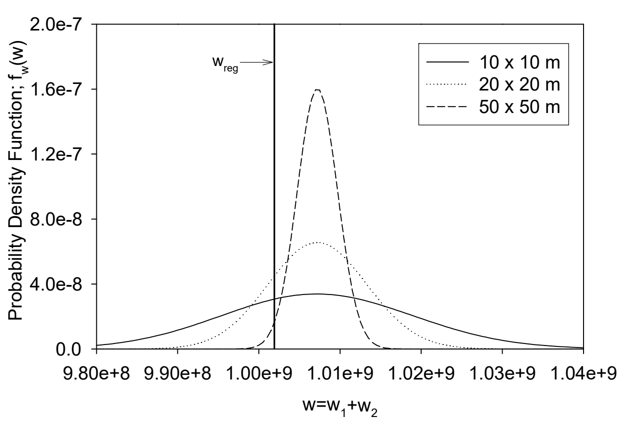

Figure 6. Probability Density Function of Composite Liner with Changing Area

5 CONCLUSIONS

A set of relatively simple analytical solutions were developed, and verified with simulation, in order to predict the influence adding a GCL to a CSL has on the reduction in risk of advective flux exceeding a regulated condition. It was shown for the assumed properties of CSL and GCL that decreases in a CSL's mean k resulted in lower probabilities of exceedance, as expected. Typically large changes in CSL k are required for noticeable changes in harmonic average k of the composite liner system. However in some instances with large areas, greater than 100m x 100m, slight increases in CSL k result in the probability of exceedance changing from zero to one.

In all situations considered throughout this study, GCL k has a large effect on the probability of exceedance of the composite liner. As expected, decreasing GCL k produces lower probabilities of advective flux exceeding a regulated value. To produce intermediate probabilities of exceedance (i.e. values other than zero or one), GCL k must be chosen to produce a harmonic average k sufficiently close to the regulated value.

Liner area does have an effect on the probability of advective flux exceeding a regulated value. Increased liner areas reduce the probability of achieving an intermediate probability of exceedance, probability other than zero or one. This is caused by a narrowing of the probability density function resulting from a decrease in variance due to the application of the variance function, which continually decays with constant scale of fluctuation and increasing area.

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