Influence of anisotropy and rotation on probabilistic slope stability analysis by RFEM

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ABSTRACT

This paper describes random finite element analyses of anisotropic 2-D undrained clay slopes with randomly distributed shear strength. The analyses take into account the rotational orientation of the principle axes of anisotropy. Particular focus is placed on the special case in which the spatial correlation in one of the orthogonal directions is set to infinity. It is found that rotated anisotropy can have a very significant effect on the probability of slope failure. Insight into the reasons for this sensitivity is provided by examining the nature of the failure mechanisms generated in the graphical output displays.

RÉSUMÉ

Cet article décrit des analyses par éléments finis aléatoires des 2-D pentes undrained anisotropes d'argile avec la résistance au cisaillement aléatoirement distribuée. Les analyses prennent en considération l'orientation de rotation des haches de principe de l'anisotropie. L'accent particulier est placé sur le cas spécial dans lequel la corrélation spatiale dans une des directions orthogonales est placée à l'infini. On le constate que l'anisotropie tournée peut exercer un effet très significatif sur la probabilité de l'échec de pente. L'aperçu des raisons de cette sensibilité est fourni en examinant la nature des mécanismes d'échec produits dans les affichages graphiques de rendement.

1 INTRODUCTION

Over the past few years there has been an increasing interest in applying probabilistic methods to geotechnical engineering applications. Combining random field generation with the finite element method (FEM) enables engineers to more accurately model the spatial variability of soils in computer simulations. The random finite element method (RFEM) in combination with the Monte Carlo Method allows for the calculation of a probability of failure, which can be more meaningful than the conventional factor of safety. Conventional methods account for variability and uncertainty with an all-encompassing factor of safety, but probabilistic methods carry the variability of soil properties and loadings through the calculations.

The Fenton and Griffiths RFEM program for slope stability analysis (Griffiths and Fenton (1993) and Fenton and Griffiths (1993)) has been modified to perform parametric slope stability analyses in this study. The original program allows for the user to enter anisotropic spatial correlation lengths using the x and y principal axes. This has been modified to allow the user to rotate the spatial correlation principal axes. This is ideal for modelling uplifted soils.

This study examines the effects of varying correlation lengths and principal axes orientations in a 1:1 slope comprised of undrained clay with random cohesion. It is found that correlation lengths and principal axes orientation can have up to an order of magnitude effect on the probability of failure calculation.

2 PAST RESEARCH

Recent publication by Shogaki and Kumagiai (2008), Attom and Al-Akhras (2008), and Comegna and Picarelli (2008) discuss the anisotropy of soils and its effect on shear strength. Unfortunately, there is little published research on soil anisotropy modelled by RFEM. There has, however, been published research on the general application of RFEM in geotechnical engineering. RFEM has been investigated by Fenton (1990), Fenton and Vanmarcke (1990), El Ramley etc al (2002), and Fenton and Griffiths (2008). In particular, investigations of probabilistic slope stability analysis by RFEM has been detailed by Szynakiewicz (2002), Tveten (2002), Griffiths and Fenton (2004), and Ziemann (2005). A study into the effects of spatial correlation anisotropy in slope stability analysis has been investigated by Schiermeyer (2009), and anisotropy in bearing capacity analysis has been investigated by Cho and Park (2009).

3 PROGRAM INPUTS

This study analyzes 1:1 undrained clay slopes with random cohesion and anisotropic spatial correlation lengths using a modified version of the Fenton and Griffiths RFEM program, rslope2d. The program uses the local average subdivision (LAS) algorithm for random field generation. The finite element analysis is performed using a stress redistribution algorithm that iteratively redistributes self weight stresses within the slope without exceeding the Mohr-Coulomb failure criterion (e.g. Griffiths and Lane 1999). For each simulation, if the global equilibrium cannot be achieved within a user-specified maximum number of iterations, then it is assumed that the slope failed. 1,000 Monte Carlo simulations are used for each case run to produce a probability of failure.

3.1 Slope and Mesh Properties

An example 1:1 slope with a finite element mesh and random field mapping is shown in Figure 1.

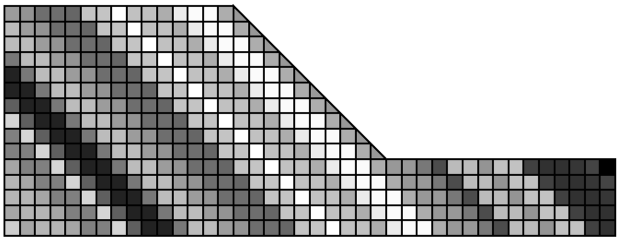

Figure 1. Example 1:1 slope

The lighter regions represent weaker soils while the darker regions represent stronger soils. The layers are sloped because the principal axes for spatial correlation lengths are rotated clockwise 45 degrees from horizontal line.

The same slope and mesh dimensions are used for all case runs in this study. The slope and finite element mesh dimensions are shown in Table 1.

Table 1. Slope and mesh dimensions.

Property	Value
x-elements to left of embankment	15
x-elements to right of embankment	15
y-elements in foundation	10
y-elements in embankment	5
Slope gradient	1
Element dimensions	0.5 x 0.5

3.2 Soil Properties

This study looks at an undrained clay slope with random cohesion. For this study, all distances are in meters and all forces in kN. The same soil properties are used for all case runs and are shown in Table 2.

Table 2. Soil properties.

Property	Mean	St Deviation	Distribution
Cohesion	20	4	Lognormal
Friction Angle	0	0	Deterministic
Dilation Angle	0	0	Deterministic
Unit Weight	20	0	Deterministic

3.3 Spatial Correlation Lengths

Spatial correlation lengths govern how rapidly random properties vary over space. A high spatial correlation length leads to properties that vary more gradually over space while a low spatial correlation length leads to properties that vary more rapidly over space. Figure 2 illustrates the difference between low and high spatial correlation lengths.

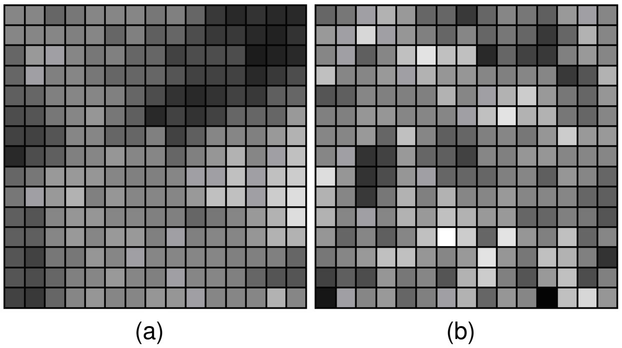

Figure 2. (a) High correlation (b) Low correlation

Spatial correlation lengths are specified using two orthogonal principal axes. These are customarily the x and y principal axes. This study examines the special case where one correlation length is set to infinity while the other is varied. In addition, this study rotates the principal axes in parametric studies at 15 degree increments from 0 to 180 degrees. Table 3 summarizes the spatial correlation lengths analyzed in this study.

Table 3. Spatial correlation lengths.

Case	First Spatial Correlation Length	Second Spatial Correlation Length
1	Infinity	0.5
2	Infinity	1.0
3	Infinity	2.5
4	Infinity	5.0
5	Infinity	7.5
6	Infinity	10.0
7	Infinity	12.5
8	Infinity	15.0

The spatial correlation lengths are entered as absolute lengths. Therefore, a spatial correlation length of 0.5 is equivalent to the size of an element in the finite element mesh, and a spatial correlation length of 5.0 is equivalent to the height of the embankment. It should be noted that a correlation length of infinity was not physically entered into the program. Rather, the program was modified to create the “layered” effect shown in Figure 3. A 1-D strip of random values was chosen then copied for all the elements in a row.

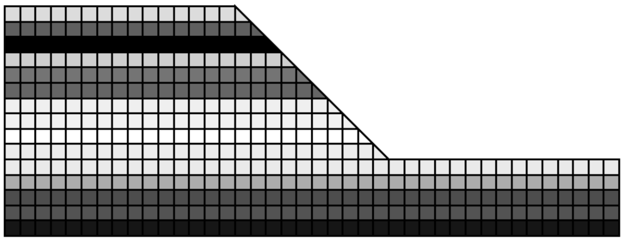

Figure 3. Slope with layered soil

4 RESULTS

The results for this study are shown in Figures 4 and 5. The results are plotted as probability of failure versus principal axes rotation angle and probability of failure versus spatial correlation length. The two plots are of the same results, but the plot axes are varied to better demonstrate the effects of principal axes rotation angle and spatial correlation lengths. In the plots, ly is the spatial correlation length perpendicular to the soil layering, and rfrot is the angle of random field rotation (spatial correlation principal axes rotation angle).

The spatial correlation lengths, ly, in Figures 4 and 5 have been normalized by the height of the embankment. Thus, a ly of 0.1 is equivalent to an element size, and a ly of 1.0 is equivalent to the embankment height.

Figures 4 and 5 include some results in which the correlation length is approximately equal to the element size implying significant local averaging. It may be noted that local averaging of a lognormal distribution preserves the median, but causes both the mean and the standard deviation to be reduced. While the mesh discretization in these cases is not refined enough to accurately model low spatial correlation lengths, the qualitative nature of the results indicating the influence of rotation angle is striking.

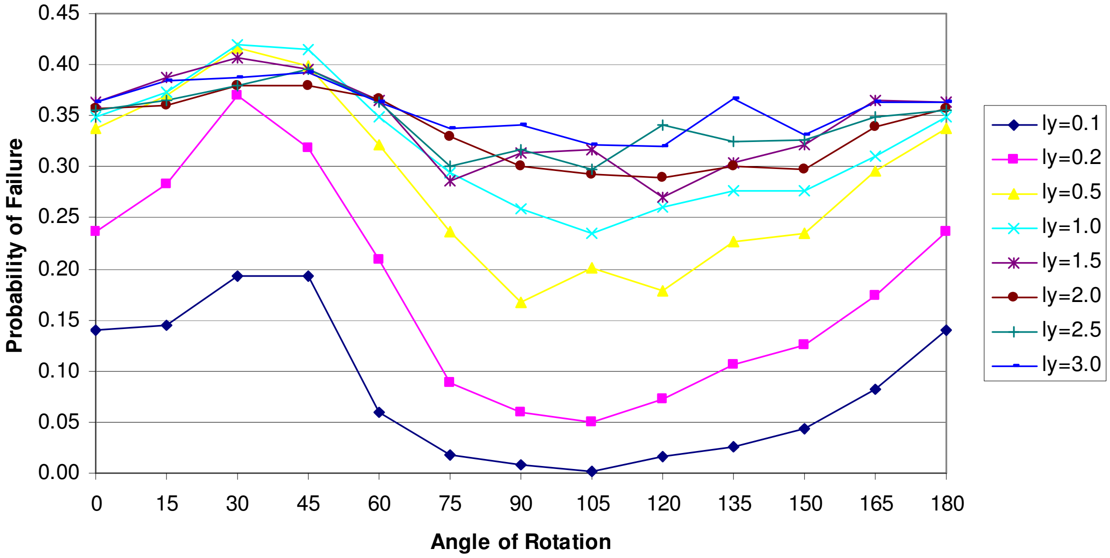

Figure 4. Probability of failure vs. angle of rotation

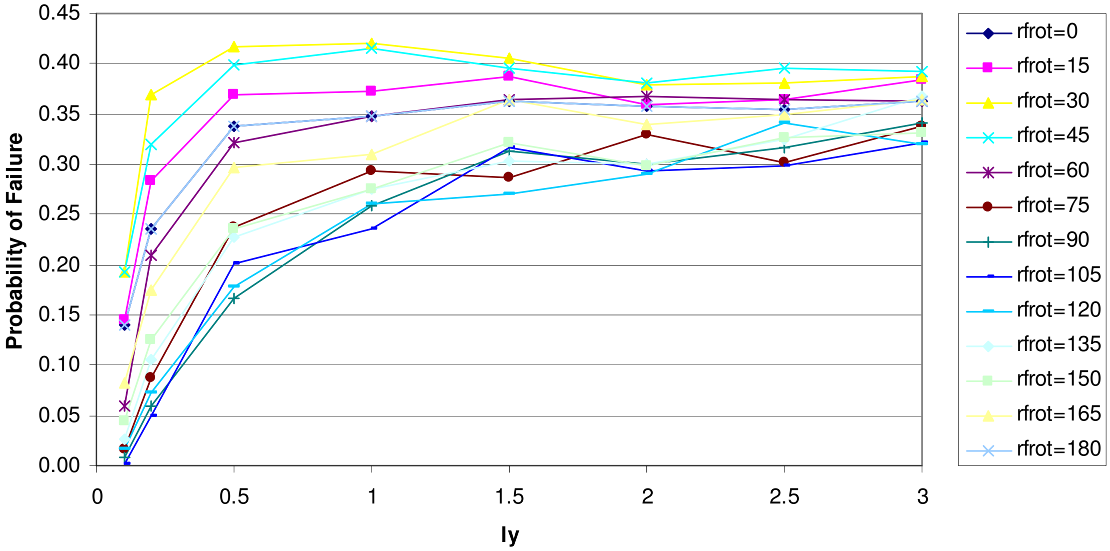

Figure 5. Probability of failure vs. spatial correlation

The results plotted in Figure 4 suggest that higher probabilities of failure occur when the soil layers are aligned parallel to the slope. Similarly, lower probabilities of failure are found when the soil layers are aligned perpendicular to the slope. These results can be explained by examining the failure mechanisms that lead to slope failure.

Basic slope stability theory suggests that slopes will fail along a circular failure path (figure 6).

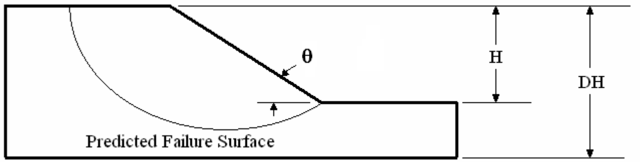

Figure 6. Predicted failure surface

The critical failure surface is the one with the lowest factor of safety. For uniform soils, this critical failure surface is a function of the slope geometry. However, for non-uniform (random) soils, the critical failure surface is dependent upon the location of the weaker soil regions.

The stress redistribution algorithm used in the RFEM program puts no restrictions on the geometry for the development of the failure surface. However, the results from the RFEM program tend to follow slope stability theory, which predicts a circular failure surface. It is found that the program seeks out the weaker soil regions to develop the failure surface. This ideally models real-world slope failures that seek out weaker soil regions.

It is easiest for the program find a failure mechanism when weaker soil regions are aligned along the predicted failure path. Figure 5 illustrates that higher correlation lengths generally lead to higher probabilities of failure because the properties vary more gradually and weak soil layers are aligned along similar weak soil layers.

Two primary failure mechanisms are found – series failure mechanism and parallel failure mechanism. When the principal axes are rotated 45 degrees as shown in Figures 7 and 8, the failure mechanism in the embankment is mainly series, in which failure mechanism is able to seek out and primarily pass through a single weak layer of soil. However, the failure mechanism must pass through several strong and weak soil regions in the foundation and therefore be parallel. In the figures, the lighter regions represent weaker soil, and the darker regions represent stronger soil.

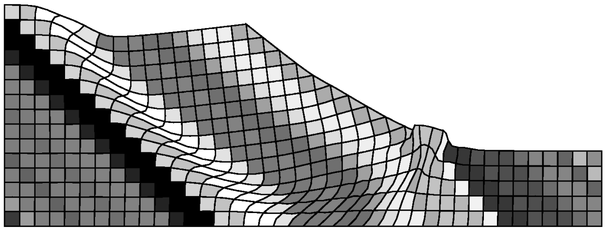

Figure 7. Series failure mechanism #1

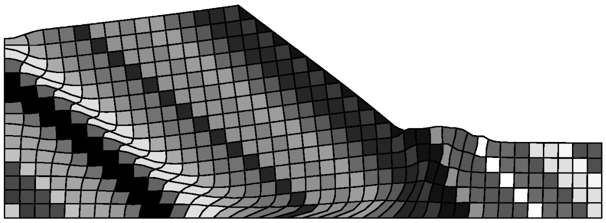

Figure 8. Series failure mechanism #2

When the principal axes are rotated 135 degrees as shown in Figures 9 and 10, the failure mechanism is mainly parallel in the embankment and series in the foundation.

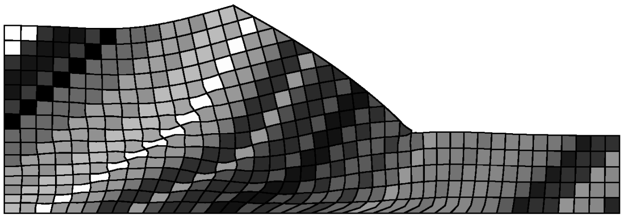

Figure 9. Parallel failure mechanism #1

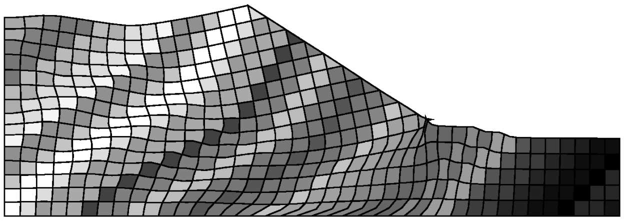

Figure 10. Parallel failure mechanism #2

The series and parallel failure mechanisms help to explain the results plotted in figure 4. As shown in Figures 7 to 10, the failure mechanism in the embankment involves larger volume of soil than the failure mechanism in the foundation do. When the soil layers are aligned parallel to the slope (rfrot=45), the failure mechanism is mainly series and leads to high probability of failure. However, when the soil layers are aligned perpendicular to the slope (rfrot=135), the failure mechanism is mainly parallel and leads to low probability of failure.

This study demonstrates that the orientation of the spatial correlation principal axes can have a significant effect on the probability of failure calculation. Thus, it is important to not only account for the degree of anisotropy in soil but also the orientation.

5 CONCLUSIONS

RFEM with the Monte Carlo Method is a powerful tool for modelling soils as spatially random materials. However, it is important to properly model the spatial variability of the soils. Spatial correlation lengths and their orientation can have a significant effect on the calculated probability of slope failure. Their effect can be explained by examining the program’s ability to seek out weaker soil regions when developing a failure surface.

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REFERENCES

Attom M.F., and Al-Akhras N.M. 2008. Investigating anisotropy in shear strength of clayey soils. *Proceedings of the Institution of Civil Engineers-Geotechnical Engineering,* 161(5): 269-273.

Comegna L., and Picarelli L. 2008. Anisotropy of a shear zone. *Geotechnique,* 58(9): 737-742.

El-Ramly, H., Morgenstern, N.R., and Cruden, D.M. 2002. Probabilistic slope stability analysis for practice. *Canadian Geotechnical Journal*, 39(3): 665-683.

Fenton, G.A. 1990. Simulation and analysis of random fields. Ph.D. thesis, Pinceton University, Pinceton, NJ.

Fenton, G.A., and Griffiths, D.V. 1993. Statistics of block conductivity through a simple bounded stochastic medium. *Water Resources Research*, 29(6): 1825-1830.

Fenton, G.A., and Griffiths, D.V. 2008. *Risk assessment in geotechnical engineering*, John Wiley & Sons Ltd, Hoboken, NJ.

Fenton, G.A., and Vanmarcke, E.H. 1990. Simulation of random-fields via local average subdivision. *Journal of Engineering Mechanics-ASCE*, 116(8): 1733-1749.

Griffiths, D.V., and Fenton, G.A. 1993. Seepage beneath water retaining structures founded on spatially random soil. *Geotechnique*, 43(6): 577-587.

Griffiths, D.V., and Fenton, G.A. 2004. Probabilistic slope stability analysis by finite elements. *Journal of Geotechnical and Geoenvironmental Engineering*, 130(5): 507-518.

Griffiths, D.V., and Lane, P.A. 1999. Slope stability analysis by finite elements. *Géotechnique,* 49, No.3, pp.387-403, (1999)

Schiermeyer R.P. 2009. Probabilistic methods applied to slopes and footings. Masters Thesis, Colorado School of Mines, Golden, CO.

Shogaki T., and Kumagai N. 2008. A slope stability analysis considering undrained strength anisotropy of natural clay deposits. *Soils and Foundations*, 48(6): 805-819.

Szynakiewicz, T. 2002. Probabilistic analysis of slope stability using random field theory and the finite element method. Masters Thesis, Colorado School of Mines, Golden, CO.

Tveten, D. 2002. Application of probabilistic methods to stability and earth pressure problems in geomechanics. Masters Thesis, Colorado School of Mines, Golden, CO.

Ziemann, H.R. 2005. The analysis of passive earth pressure and slope stability by the random finite element method. Masters Thesis, Colorado School of fMines, Golden, CO.