The Role of Correlation Length on Probability of Failure for Soil Liner Systems

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ABSTRACT
The objective of this paper is to examine the influence of correlation length and hydraulic conductivity distribution on the probability of failure of a clay liner system. To achieve this objective, a two-dimensional (in plan) clayey liner system is simulated with a probabilistic modeling program, mrflow2d. The correlation length, variance, and mean of hydraulic conductivity of the clayey liner are varied in the simulations. In this study, the worst case correlation length for probability of failure was found to be approximately equal to 10% - 20% of the liner size in any direction when the mean hydraulic conductivity of the soil liner is the same as the regulatory specified hydraulic conductivity used in the simulation. This implies that there is a worst-case correlation length with results in a maximum probability of failure which is ideally what one would use when modelling such a system.

RÉSUMÉ
L'objectif de ce papier est d'examiner l'influence de longueur de corrélation et la distribution de conductivité hydraulique sur la probabilité d'échec d'un système de paquebot d'argile. Pour atteindre cet objectif, un à deux dimensions (dans le projet) le système de paquebot argileux est simulé avec un programme de modelage de probabilistic, mrflow2d. La longueur de corrélation, la variance, et les moyens de conductivité hydraulique du paquebot argileux est varié dans les simulations. Dans cette étude, la pire longueur de corrélation de cas pour la probabilité d'échec a été trouvée pour être approximativement d'égaler à 10% - 20% de la taille de paquebot dans n'importe quelle direction quand la conductivité hydraulique moyenne du paquebot de sol comme le même de la conductivité hydraulique, spécifiée et régulatrice utilisée dans la simulation. Ceci implique qu'il y a une longueur le pire des cas de corrélation avec a pour résultat une probabilité maximum d'échec qui est idéalement que l'un utiliserait en modelant un système.

1 INTRODUCTION

Containment facilities often rely on clayey liner systems to minimize the flow of liquid and/or contaminants from the facility. Clayey liner systems may consist of a naturally occurring clayey deposit or perhaps an engineered compacted clay liner system (Rowe et al, 2004). Whether the containment facility is a water retention pond situated on a natural clay deposit or a municipal solid waste landfill situated over a compacted clay liner system, there exists a probability (ideally very small) that the flow through the clayey liner will exceed some desired value. The common way to limit this flow is to specify the liner's hydraulic conductivity in a regulatory document (e.g., 10^{-9} m/s).

Hydraulic conductivity of a soil is generally measured over some selected volume, producing a local average hydraulic conductivity of the soil. The main effects of local averaging are to reduce the variance and to dampen contributions made by high frequency components of the random field in question. In a probabilistic simulation, the effects of local averaging and correlation length need to be considered when investigating the behaviour of a random field (Fenton and Griffiths, 2008). The correlation length provides a convenient measure of the "roughness" of a random field. The correlation length is essentially a measure of the distance over which points are significantly correlated. Points which are separated by a distance greater than the correlation length are largely uncorrelated (Fenton and Griffiths, 2008). A detailed discussion of local averaging and the correlation length can be found in Vanmarcke (1983) and Fenton and Griffiths (2008).

Benson et al. (1994) and Benson and Daniel (1994) investigated the influence of hydraulic conductivity mean and variance, and liner thickness, on the probability of failure of compacted soil liners through the use of stochastic models. However, due to the considerable computer-processing time required to conduct simulations of the flow through three dimensional soil liners and the lack of information on correlation structure, these studies assumed independence between elements of the soil model (i.e. each element was assigned random properties but there was no correlation between neighbouring elements). In Menzies et al (2009), it was indicated that the correlation length relative to the plan extent of the liner can have a significant impact on the probability of failure of compacted soil liners (CSLs) when used in combination with geosynthetic clay liners (GCLs).

The correlation length of a clay liner material is admittedly difficult to determine in practice. It is believed that further study will provide some insight into the general effects of correlation length on the probability of failure. The objective of this paper is to examine the influence of correlation length and hydraulic conductivity distributions on the probability of failure for a clay liner system. The correlation length, hydraulic conductivity variance and mean of the clayey liner are varied in the simulations to examine influences on probability of failure.

2 SIMULATION METHOD

In this paper, "failure" of the clayey liner system is defined as the condition that total flow through the liner, Q, is greater than the total flow through a regulatory liner, Q_R, of similar area (i.e. through a 1 hectare, 1 m thick liner with hydraulic conductivity of 1×10^{-9} m/s). Both Q and Q_R would be calculated for the same gradients. A Monte Carlo simulation is performed in this research to examine the role of correlation length on probability of failure of a clay liner system. This simulation enables numerous parameters of the random field to be varied; including the mean, variance, correlation length, size of field, and variance function. The Monte Carlo simulation is carried out in this study using a modified version of the computer program mrflow2d described by Griffiths and Fenton (1993) and Fenton and Griffiths (1993). The original program was developed to compute the statistics of flow through a two dimensional soil mass. The program generates a two-dimensional (in plan), log-normally distributed random field of hydraulic conductivity (see Benson et al. 1994) with user prescribed mean, variance, and correlation function. Individual realizations were performed using the Random Finite Element Method (RFEM) and the conductivity realizations themselves are generated using the Local Average Subdivision (LAS) method (Fenton, 1990). The program was modified slightly to specify flow in the z-direction as shown in Figure 1.

As shown in Figure 1, each element in the x-y direction has random properties in terms of hydraulic conductivity, based on the correlation structure specified as well as the mean and variance specified for the soil liner. The total flow through the simulated liner can then be calculated as the sum of flow through each cell, and the effective hydraulic conductivity is thus the arithmetic average of all conductivity cells:

$$k_{eff} = \frac{Q}{A} = \frac{1}{m}\sum_{i=1}^{m} k_i \qquad [1]$$

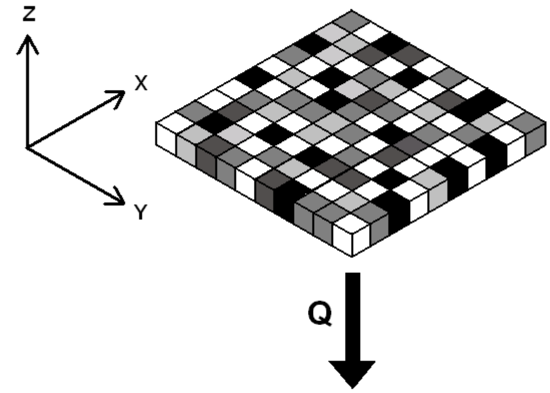

Figure 1. Diagram of hydraulic conductivity field, shades represent varying hydraulic conductivity of random field elements.

Modeling the hydraulic conductivity field in two dimensions, instead of three, imposes a restriction on the flow field. Flow is strictly one-dimensional, in the z-direction. This is deemed to be a reasonable approximation when the liner thickness is negligible compared to its area. As the goal of this investigation is to determine the influence of correlation length on probability of failure, it is felt that using a two-dimensional field will save computing time while still giving a reasonable approximation to the fully three-dimensional model.

In this paper, the mean hydraulic conductivity, μ_K, the hydraulic conductivity of individual elements, k, and the effective hydraulic conductivity, k_{eff}, were normalized with respect to the regulatory hydraulic conductivity, k_{crit}.

$$\mu_k' = \frac{\mu_k}{k_{crit}} \qquad (2)$$

$$k_i' = \frac{k_i}{k_{crit}} \qquad (3)$$

$$k_{eff}' = \frac{1}{m}\sum_{i=1}^{m} k_i' = \frac{k_{eff}}{k_{crit}} \qquad (4)$$

where:

μ_k' = normalized mean hydraulic conductivity

k' = normalized hydraulic conductivity of an individual element

k_{eff}' = normalized effective hydraulic conductivity

Having the hydraulic conductivity normalized to the regulatory hydraulic conductivity, k_{crit}, allows the results to be scaled to any desired regulatory conductivity.

The random field consisted of 102,400 (320 by 320) equal sized cells. Each cell was assumed to be 0.31 m by 0.31

m, yielding a plan area of 10,000 m^2 (1 hectare). For each set of soil statistics, 25,000 realizations were performed. The probability of failure is defined as the probability that the effective hydraulic conductivity is greater than the regulatory hydraulic conductivity:

$$\mathrm{P}\left[k_{eff} > k_{crit}\right] = \mathrm{P}\left[k'_{eff} > 1\right] \qquad (5)$$

For this study the correlation length was varied from 1 m to 100 m (1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 15, 20, 30, 40, 50, 100) and the variance was taken as 0.5, 1.0, and 2.0. The normalized mean, μ'_k, was varied from 0.1 to 2.0.

The correlation length was assumed to be isotropic for this study (i.e. equal in the x and y directions). In this analysis the correlation function is assumed to be Markovian with exponentially decaying correlation:

$$\rho\left(\tau_1,\tau_2\right) = \exp\left\{-2\sqrt{\frac{\tau_1^2}{\theta_{\ln k_1}^2} + \frac{\tau_2^2}{\theta_{\ln k_2}^2}}\right\} \qquad (6)$$

where:

$\theta_{\ln k_i}$ = correlation length in the i^{th} direction for a lognormally distributed random field.

τ_i = distance between points in the ith direction, for which the correlation coefficient is desired.

The correlation function has the corresponding variance function:

$$\gamma\left(T_1,T_2\right) = \frac{4}{T_1^2 T_2^2}\int_0^{T_1}\int_0^{T_2}\left(T_1-\tau_1\right)\left(T_2-\tau_2\right)\rho\left(\tau_1,\tau_2\right)d\tau_2 d\tau_1 \qquad (7)$$

which has no closed form solution, and is calculated in this paper using 20-point Gauss quadrature.

3 RESULTS

Figure 2 shows a typical histogram of the distribution of normalized effective hydraulic conductivity, k'_{eff}, of the clayey liner for 25000 simulations. The probability of failure of the clayey liner system is simply calculated as the hatched area under the curve greater than 1.0, the standardized regulatory value. If one considers the probability distribution of hydraulic conductivity, there is a probability that the effective hydraulic conductivity of the clay liner may be at or below the regulated value for any given realization, but there is also some probability that the effective hydraulic conductivity is above the regulated value. As shown in Figure 2, for μ'_K =1.0, υ^2_K=2.0 and correlation length, θ_{lnk}=10.0 for a 100 m by 100 m liner, the probability of failure can be estimated as 0.436 by counting the number of times k'_{eff} is greater than 1.0 during simulation and dividing by the total number of realizations. This high probability of failure would most likely be deemed unacceptable for most practical situations.

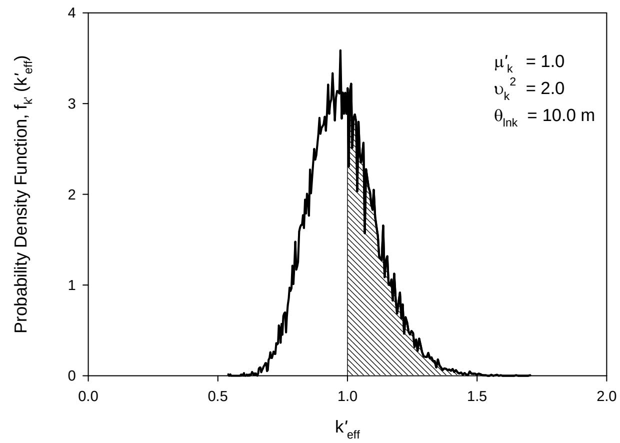

Figure 2. Typical histogram of effective hydraulic conductivity, for a 100 m by 100 m clayey liner system.

A variety of soils exist in nature which may be used in clayey liner systems. This study focuses on hydraulic conductivity means in the range (same order of magnitude) as the regulatory effective hydraulic conductivity, k'_{crit}. Means considered in this paper range from μ'_K = 0.1 to 2.0. Using the two-dimensional probabilistic simulation soil model described earlier, the specified mean of the random field can be changed to represent clayey liner systems of higher or lower effective hydraulic conductivity. As shown in Figure 3, changing the prescribed mean of the random field has the expected impact on the probability of failure of clayey liner systems in that increasing the mean hydraulic conductivity increases the probability of failure. Figure 3 shows the cumulative probability distribution of the effective hydraulic conductivity (i.e. the relationship between the probability of failure and the hydraulic conductivity mean), for υ^2_K =2.0 and θ_{lnk} =1 m, 10 m, and 100 m. For any given correlation length, the probability of failure approaches zero as the mean moves further below the regulated value of hydraulic conductivity. As the mean moves above the regulatory value, the probability of failure approaches 1.0. For lower correlation lengths, the probability of failure changes rapidly as the mean changes. For high correlation lengths, the probabilities of failure change much more gradually. This happens because the distribution of effective hydraulic conductivity is much wider for larger correlation lengths. It should be noted in Figure 3 that the probability of failure is not necessarily 0.5 when μ'_K =1.0. This is because the geometric average of the hydraulic conductivity tends toward the median, not the mean, for the lognormal distribution.

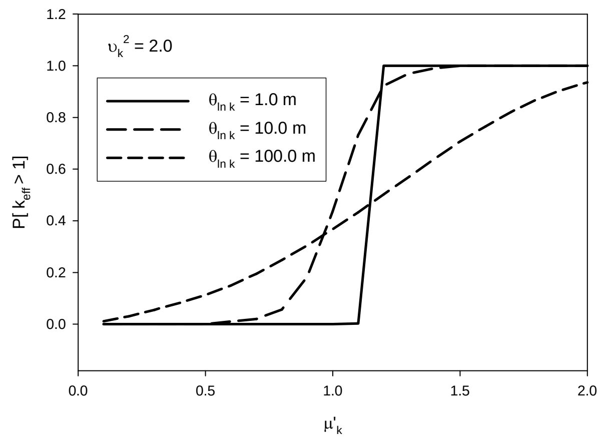

Figure 3. Probability of failure versus mean, μ'_k , for a 100 m by 100 m clayey liner system.

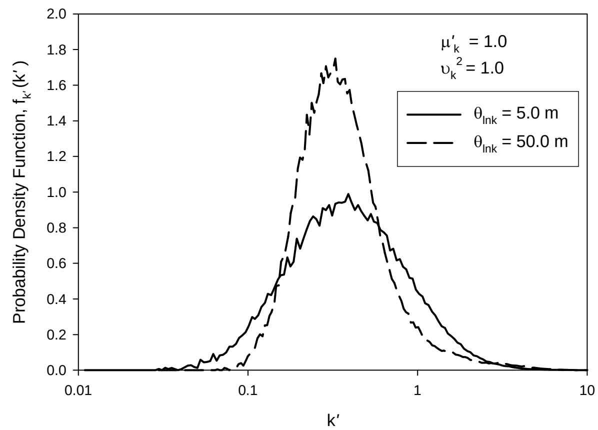

Figure 4. Histograms of single realizations of random field hydraulic conductivity with correlation lengths of 5.0 m and 50.0 m; for a 100 m by 100 m clayey liner system.

The results of Figure 3 implies that if the correlation length is small for compacted clay liners, as suggested by Benson and Daniel (1994), then the probability of failure should be low if the mean is below regulatory limits. Benson and Daniel (1994), Benson et al. (1994) and the results presented by Menzies et al. (2009) offer limited discussion on the influence of correlation length on probability of failure of compacted soil liners. Although Benson et al. (1994) recommend a correlation length of 1m to 3 m for compacted clay liners; the correlation length is varied from 0.01 m to 100 m in this study to fully investigate the influence of correlation length on probability of failure. Encompassing such a wide range of correlation lengths may allow this study to be useful not only for compacted clay liners (where the correlation length may be small as suggested by Benson and Daniel (1994)) but also natural clay deposits that are used as containment facilities.

Figure 4 demonstrates the influence of correlation length on the distribution of element hydraulic conductivity, k'. Two histograms are presented; one of a single realization of the random conductivity field having μ'_K =1.0, υ^2_K =1.0, and θ_{lnk} =5.0 m; the other of a single realization of the random conductivity field having μ'_K =1.0, υ^2_K =1.0, and θ_{lnk} =50.0. For the 50 m correlation length, points within the liner are more correlated with each other resulting in a narrower distribution of hydraulic conductivity; at 5 m correlation length, points are less correlated resulting in a wider distribution. In effect, increasing the correlation length reduces the variance of a single realization of the random field. As the correlation length approaches infinity, (i.e. θ_{lnk} approaches infinity), all conductivities in the field become equal and the variability of each realization approaches zero. Note, however, that another realization of the field will have a different effective hydraulic conductivity (i.e. there is still variability over the ensemble of clayey liner system realizations).

For multiple realizations of a random field, at low correlation lengths the hydraulic conductivity of points within the field is highly variable, but all realizations of the random conductivity field have similar distributions, and hence there is less variation in effective hydraulic conductivity, k_{eff}, relative to random fields with higher correlation lengths. At higher correlation lengths the hydraulic conductivity is more correlated, points within the field are closely related to each other, but multiple realizations of the clayey liner have more variation in effective hydraulic conductivity. To demonstrate this, Figure 5 shows three histograms of the effective hydraulic conductivity, k'_{eff}, of a clayey liner system, each for 25000 realizations, for μ'_K =1.0, υ^2_K =2.0, and θ_{lnk} =5 m, 10 m, and 50 m. This figure shows narrowing of the effective hydraulic conductivity distribution with decreasing correlation length.

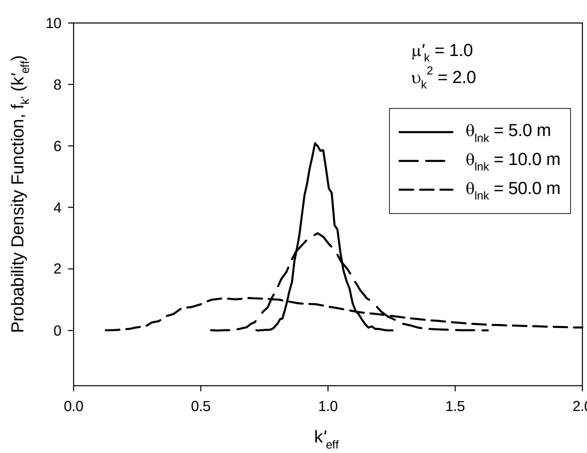

Figure 5. Histograms of clayey liner system effective hydraulic conductivity, for three correlation lengths; for a 100 m by 100 m clayey liner system.

Figure 6 summarizes the influence of correlation length on probability of failure. Three curves are shown for 100 m x 100 m clayey liner systems with μ'_K =1.0, υ^2_K =0.5, 1.0, and 2.0. All three curves reach a maximum probability of failure when the correlation length is approximately 15.0 m. Similar simulations were performed for liners with different areas; 50 m by 50 m and 200 m by 200 m. The 50 m by 50 m and 200 m by 200 m clayey liners reached their maximum probabilities of failure with correlation lengths approximately equal to 10 m and 30 m respectively. In other words, in terms of probability of failure, the worst case correlation length is approximately equal to 10% - 20% of the liner size in any direction. A theoretical explanation for this is that at both very small and very large correlation lengths, the field is the same everywhere. More specifically, as the correlation length decreases, the field actually becomes increasingly 'different', but at increasingly finer scales. As the correlation length approaches zero, the field is infinitely variable (i.e. k at one point might be completely different than k at 0.0000001 mm away). However, since k is measured on a local average (over some volume) this infinitesimal variability becomes infinitely damped. Therefore local averages all become similar when correlation length approaches zero. They also all become similar when correlation length approaches infinity. It is only at intermediate correlation lengths that the local averages show variability, and thus an increased probability of failure. The worst case correlation length is an interesting and useful observation as often the scale of fluctuation is unknown for a given site, especially in the design phase.

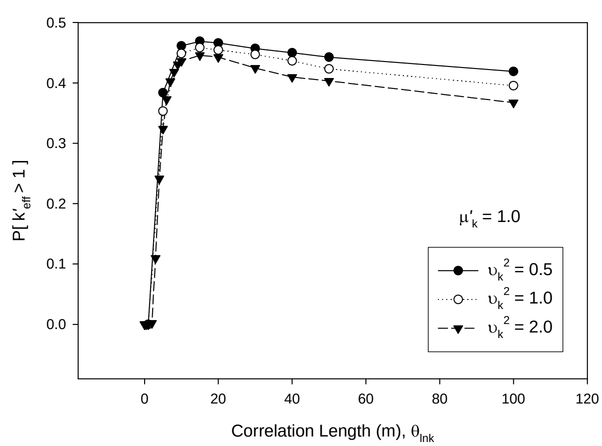

Figure 6. Probability of failure versus correlation length, for a 100 m by 100 m clayey liner system.

Increasing the variance of the clayey liner hydraulic conductivity distribution produces a decrease in the probability of failure when the mean is fixed. This counter-intuitive result arises because increasing the variance, for a fixed mean and correlation length produces a small increase in the probability of high effective hydraulic conductivity values, and an increase in the probability of low effective hydraulic conductivity values. Figure 7 illustrates the distribution shift; it presents histograms of the clayey liner effective hydraulic conductivity for two variances, where μ'_K =1.0, and θ_{lnk} =100m. It can be seen in Figure 6, for constant mean and correlation length, the probability of failure decreases with increasing hydraulic conductivity variance. For example, in both Figures 6 and 7; increasing the variance from υ^2_K =0.5 to υ^2_K = 2.0 decreases the probability of failure from 0.419 to 0.367 for μ'_K =1.0, and θ_{lnk} =100 m.

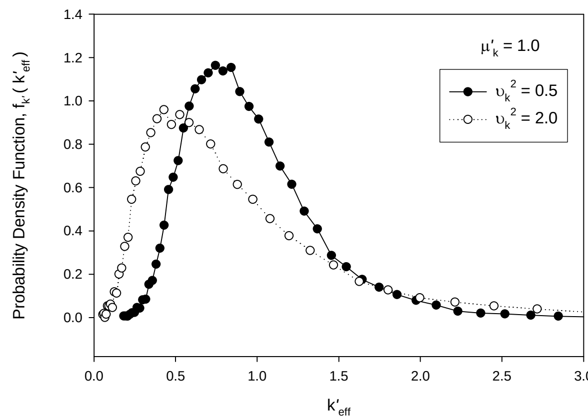

Figure 7. Histograms of clayey liner system effective hydraulic conductivity for two variances; for a 100 m by 100 m clayey liner system.

4 SUMMARY AND CONCLUSIONS

The model mrflow2d was used to generate and compute statistics of flow through a stochastic two dimensional, lognormally distributed soil mass. The model computes the probability of failure (the probability that the effective hydraulic conductivity of a clayey liner system, k_{eff} is greater than a regulatory hydraulic conductivity, k_{crit}; P[k_{eff}>k_{crit}]). Simulation was performed to determine the influence of the correlation length on the probability of failure. The influence of mean and variance of the clayey liner system hydraulic conductivity on the probability of failure was also investigated.

It was shown that the hydraulic conductivity mean, variance, correlation length, and size of the clayey liner system had the following influence on the probability of failure of a soil liner:

- Increasing the hydraulic conductivity mean produced an increase in probability of failure, as expected.
- Increasing the hydraulic conductivity variance resulted in a decrease in probability of failure for fixed mean.
- At low correlation lengths (less than 1% of the field dimension), the probability of failure approaches zero, provided the mean hydraulic conductivity is lower than the regulatory value. Increasing the correlation length results in an increase in probability of failure to some maximum value, further increasing the

correlation length of the random field results in a decrease in probability of failure to some intermediate limiting value at very large correlation lengths. In this study the worst case correlation length for probability of failure was found to be approximately equal to 10% - 20% of the liner size in any direction when μ'_k=1.0. This value could be used as a conservative estimate for probabilistic modelling.

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