Three-dimensional stability analysis of highly variable slopes by finite elements

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ABSTRACT: The paper will review the use of finite element (FE) methods as a powerful alternative to classical limit equilibrium method for tackling slope stability problems. The benefits of FE are particularly obvious when dealing with non-typical geometries such as 3-D and soils with variable properties. The paper will show several examples of finite element methods applied to slope stability including some recent work on 3-D random slopes

1. INTRODUCTION

The finite element method offers a powerful alternative to classical limit equilibrium methods of slope stability that have remained essentially unchanged for decades. The method offers the following main advantages:

- No assumption needs to be made in advance about the shape or location of the failure surface. Failure occurs "naturally" through the zones within the soil mass in which the soil shear strength is unable to sustain the applied shear stresses.

- Since there is no concept of slices in the finite element approach there is no need for assumptions about slice side forces. The finite element method preserves global equilibrium until "failure" is reached.

- If realistic soil compressibility data is available, the finite element solutions will give information about deformations at working stress levels.

- The finite element method is able to monitor progressive failure up to and including overall shear failure.

It is certainly not the case that the finite element method of slope stability analysis is a new technique. The first paper to tackle the subject by Smith & Hobbs (1974) is over 35 years old and this was followed by an important paper on the topic by Zienkiewicz *et al.* (1975). The Zienkiewicz paper had a very significant influence on the author's finite element slope stability software developments over the years. Early publications date back to Griffiths (1980) and the first ever published source code for finite element slope stability appeared in the second edition of the text by Smith & Griffiths (1988, 2004). Readers are also referred to Griffiths & Lane (1999) for a thorough review of how the methodology works.

2. 3-D SLOPES

The vast majority of slope stability analyses are performed in 2-D under the assumption of plane strain conditions. Even when 2-D conditions are not appropriate, 3-D analysis is rarely performed. There are a number of reasons for this. The vast majority of work on this subject has shown that the 2-D factor of safety is conservative (e.g. lower than the "true" 3-D factor of safety), and existing methods of 3-D slope stability analysis are not well established in practice. A further disadvantage of some 3-D methods, is that being based on extrapolations of 2-D "methods of slices" to 3-D "methods of columns", they are complex, and not readily modified to account for realistic boundary conditions in the third dimension. The advantages of FE slope stability methods become even more attractive in 3-D. The paper demonstrates some 3-D slope stability analyses by finite elements and shows that great care must be taken in subscribing to the received wisdom that "2-D is always conservative".

2.1 When is plane strain a reasonable approximation?

The first issue addressed here for a homogeneous slope, is to consider the question "how long does a slope need to be in the third dimension for a 2-D analysis to be justified?"

Figure 1 shows a simple mesh that might be used for a 3-D analysis of an undrained slope. A "rough-smooth" boundary condition implies a symmetric analysis about the plane $z = L/2$, thus only half of the actual depth L of the slope is analyzed. The bottom $(y = D)$ and far-side $(z = 0)$ of the slope are fully fixed, while the back $(x = 0)$ and front-side $(z = L/2)$ of the slope are constrained by vertical rollers.

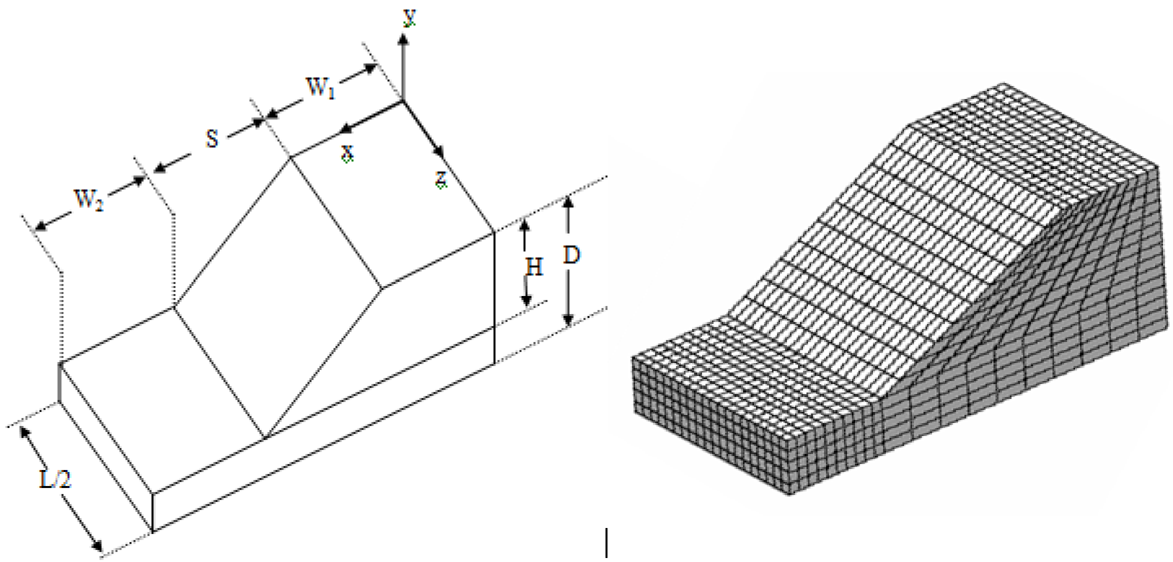

Figure 1. 3-D layout and typical mesh $(L/H = 2)$. FE analyses used 20-node hexahedral elements.

The results from a series of FE analyses with different depth ratios (L/H) while keeping all other parameters constant are shown in Figure 2. It can be seen that the factor of safety in 3-D was always higher than in 2-D but tended to the plane strain solution for depth ratios of the order of $L/H > 10$. It is shown that results of the same analysis with a coarser mesh gave slightly higher values of FS.

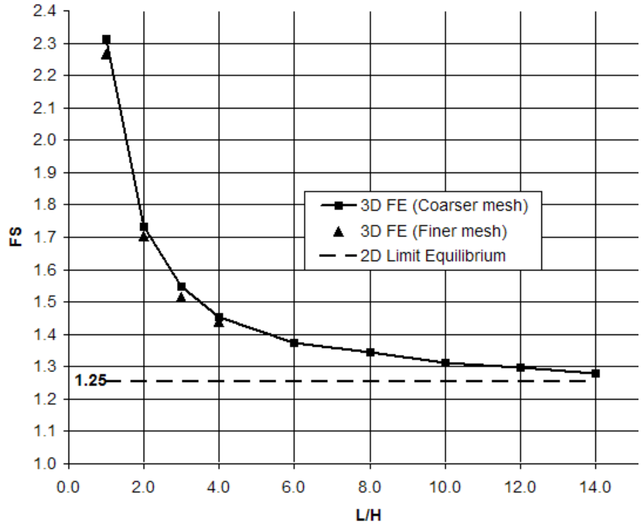

Figure 2: Comparison of 3-D FE and 2-D limit analyses for a $\phi_u = 0^o$ slope with $c_u / (\gamma \mathrm{H}) = 0.20$.

2.2 Is plane strain always conservative?

The assumption that 2-D analyses lead to conservative factors of safety needs some qualification. Firstly, a conservative result will only be obtained if the most pessimistic section in the 3-D problem is selected for 2-D analysis (see e.g., Duncan 1996). In a slope that contains layering and strength variability in the third dimension, this "most pessimistic" 2-D section may not be intuitively obvious. Secondly, the corollary of a conservative 2-D slope stability analysis is that back analysis of a failed slope will lead to an unconservative overestimation of the soil shear strength (e.g. Arellano & Stark 2000). Although some investigators (e.g. Hutchinson *et al.* 1985, Hungr 1988) have asserted that the factor of safety in 3-D is always greater than in 2-D, it cannot be ruled out that an unusual combination of soil properties and geometry could lead to a 3-D mechanism that is more critical. Bromhead & Martin (2004) argued that some landslide configurations with highly variable cross-sections could lead to failure modes in which the 3-D mechanism was the most critical. Other investigators have indicated more critical 3-D factors of safety (e.g., Chen & Chameau 1982 and Seed *et al*. 1990) although this remains a controversial topic.

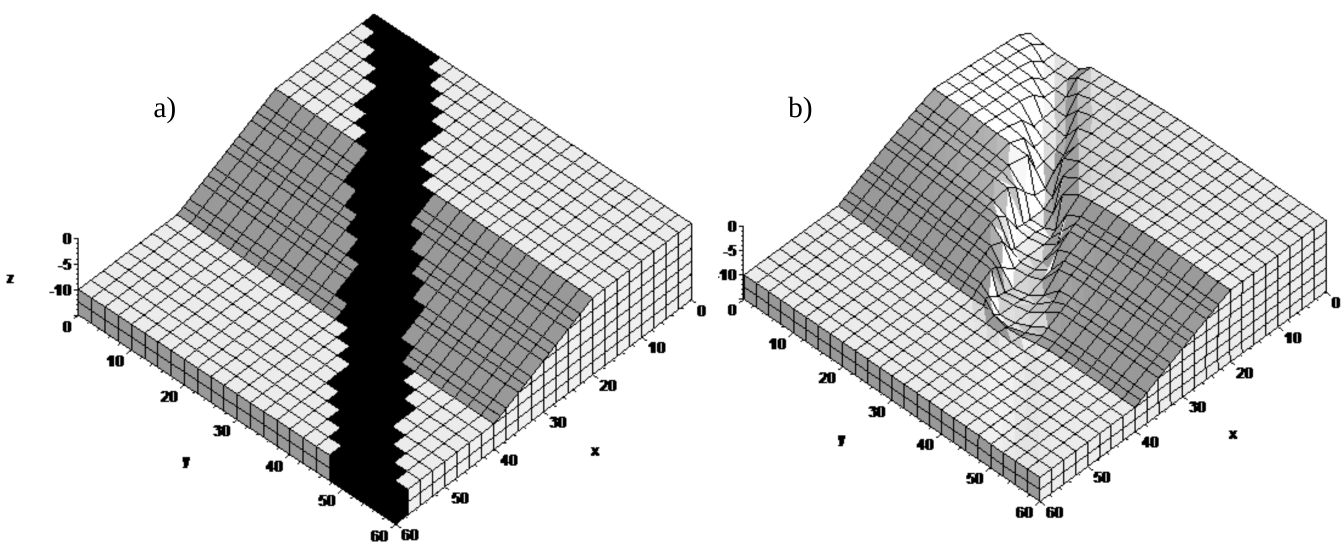

Figure 3: a) Three-dimensional slope at failure including an oblique layer of weak soil and b) failure mechanism by 3-D finite elements $\left(FS \approx 1.5\right)$

Finite element slope stability analysis offers us the opportunity to perform objective comparisons in which 2-D and 3-D factors of safety are compared for variable soil conditions. This point is highlighted in the 3-D example shown in Figure 3a) which represents a 2v:1h slope of height 10 m, foundation depth 5 m and a length in the out-of-plane direction of 60 m with smooth boundary conditions. An oblique zone of weak soil (shaded black) with undrained strength $c_u = 20\text{ kN/m}^2$ has been introduced into the slope with the surrounding soil four times stronger at $c_u = 80\text{ kN/m}^2$. The 3-D factor of safety is found to be approximately 1.5 and the mechanism clearly follows the weak zone as also shown in Figure 3b).

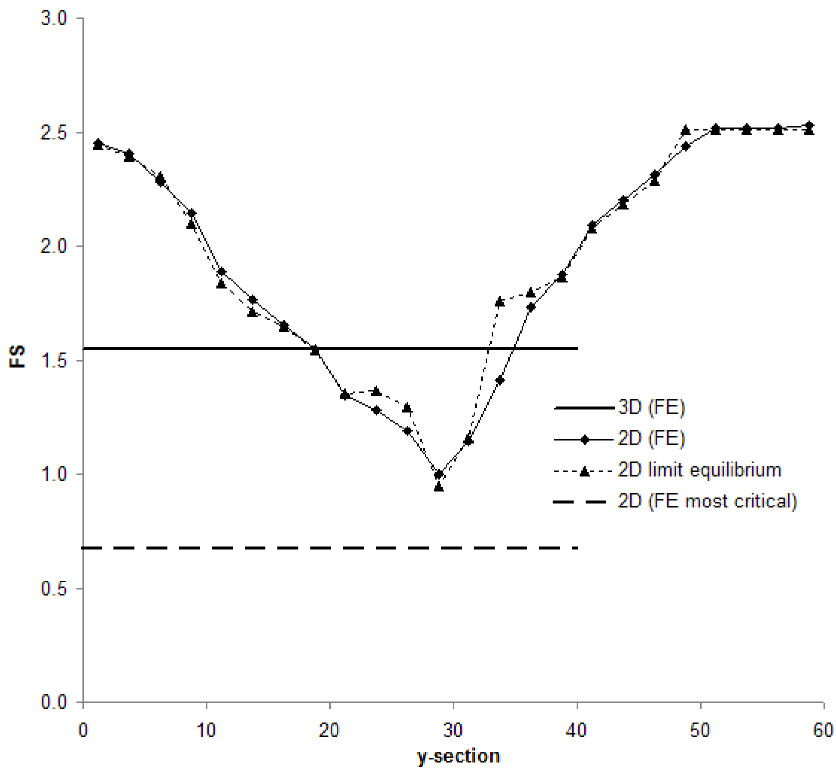

Figure 4: Factors of safety from 3-D analysis and various 2-D sections.

When 2-D stability analyses are then performed on successive slices in the $x - z$ plane moving from $y = 0\text{ m}$ to $y = 60\text{ m}$, the result shown in Figure 4 is obtained. As a check, the 2-D analyses were performed both by finite elements and by a standard limit equilibrium program. It can be seen that towards the boundaries of the 3-D slope ($y < 21\text{ m}$ and $y > 34\text{ m}$) where the majority of soil in the sections is strong, the 2-D results led to higher and therefore unconservative estimates of the factor of safety. On the other hand, at sections towards the middle of the slope ($21\text{ m} < y < 34\text{ m}$) where there is a greater volume of weak soil, the 2-D results led to lower, and therefore conservative estimates of the factors of safety. An even more critical 2-D plane however, is the one that runs right down the middle of the weak soil. This 2-D plane has a 2.5v:1h slope which is flatter than the $x - z$ planes considered previously, however it is homogeneous and consists entirely of the weaker soil. A 2-D slope stability analysis on this plane gives an even lower factor of safety of about 0.7. This result, also shown on Figure 4, is less than half of the factor of safety given by the 3-D analysis and would be considered excessively conservative, even by geotechnical design standards.

Even in the rather simple problem considered here, the results have shown a quite complex relationship between 2-D and 3-D factors of safety. The results confirm that 2-D analysis will deliver conservative results if a pessimistic plane in the 3-D problem is selected, however this may lie well below the "true" 3-D factor of safety. It has also been shown however, that selection of the "wrong" 2-D plane could lead to an unconservative result.

3. RISK ASSESSMENT OF SLOPES

Risk assessment and probabilistic analysis in geotechnical engineering is a rapidly growing area of interest and activity for practitioners and academics. It fair to say that slope stability analysis has received greater attention from probabilistic tools than any other application of conventional geotechnical engineering (see e.g. Li and Lumb 1987, Mostyn and Lee 1993, Griffiths and Fenton 2000, Duncan 2000, El Ramly *et al.* 2002, Huang *et al.* 2010).

Soils and rocks are the most variable of all engineering materials, so when an engineer chooses "characteristic values" of the soil shear strength for a limit analysis (say), it is very likely that some parts of slope consist of soil that is stronger than the characteristic values, and other parts are weaker. How do the stronger and weaker soils interact and which of them have the greater influence in determining the factor of safety?

3.1 Checkerboard slope stability analysis.

In this section we take a simple 2-D slope and assign the slope two different properties arranged in a checkerboard pattern (Zhou & Griffiths 2009) as shown in Figure 5.

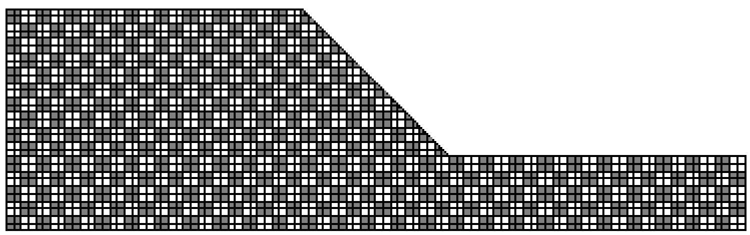

Figure 5: Slope stability analysis with checkerboard strength pattern. The darker zones are stronger.

The 1h:1v undrained clay slope has a height of $H = 10$ m and a foundation depth ratio of $D = 1.5$. The mean strength of $c_u = 50$ kPa was held constant, while the stronger soil was made stronger and the weaker soil was made weaker. The results of the factor of safety analysis by strength reduction are shown in Table 1. Clearly the weaker soil "wins"!

Table 1: Influence of variable soil in a checkerboard pattern

$C_{u(\text{strong})}$ (kPa)	$C_{u(\text{weak})}$ (kPa)	$C_{u(\text{strong})} / C_{u(\text{weak})}$	FS
50	50	1.00	1.39
60	40	1.50	1.30
70	30	2.33	1.17
80	20	4.00	1.03
90	10	9.00	0.88

Failure mechanisms in the homogeneous and the most variable cases are shown in Figure 6a) and b) respectively. In the most variable case, it can be seen that multiple mechanisms are attracted to the "diagonals" of weak soil and show a more dramatic outcrop on the downhill side.

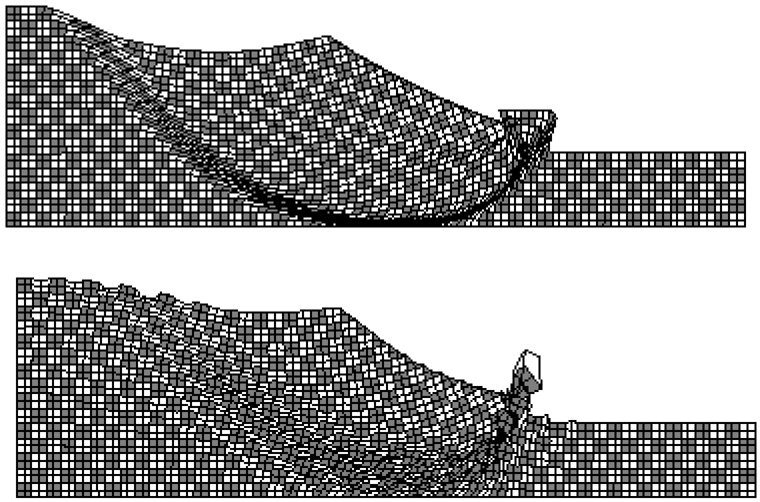

Figure 6: Failure mechanisms in "checkerboard" analysis. a) Homogeneous slope, b) Slope with strength ratio $C_{u(\text{strong})} / C_{u(\text{weak})} = 9$

3.2 The Random Finite Element Method (RFEM).

The goal of a probabilistic slope stability analysis is to estimate the probability of slope failure as opposed to the ubiquitous factor of safety used in conventional analysis. Several relatively simple tools exist for performing this calculation that include the First Order Second Moment (FOSM) methods and the First Order Reliability Methods (FORM).

A legitimate criticism of these first order methods is that they are unable to properly account for spatial correlation length in the random material. This parameter recognizes that two sites could have the same mean and standard deviation of strength parameters, but quite different spatial correlation lengths. The spatial correlation length is the distance in length units, over which soil properties tends to be correlated.

To overcome these deficiencies, the author and Gordon A. Fenton of Dalhousie University, have developed an advanced probabilistic analysis tool called the Random Finite Element Method (RFEM) that combines random field theory with elasto-plastic finite element analysis. Input to RFEM is provided in the form of the mean, standard deviation and spatial correlation length of the soil strength parameters. Spatial correlation length may be expressed in dimensionless form as Θ in which the spatial correlation length is normalized by diving by the slope height. Following generation of a locally averaged random field the properties are assigned to the mesh and gravity loads are applied. The slope either fails or not, and the process is repeated. Following a sufficient number of Monte-Carlo simulations, the probability of failure is simply the proportion of the total number of simulations that failed. The interested reader is directed to publications by Griffiths and Fenton (2000, 2004) and the textbook by Fenton and Griffiths (2008) for more detail. The method is becoming recognized as the state-of-the-art in probabilistic geotechnical analysis and is being used by several research groups worldwide. The RFEM codes developed by Griffiths and Fenton have now been applied to numerous areas of geotechnical engineering and are freely available in source code from the authors' web site at www.mines.edu/~vgriffit/rfem.

3.3 Analysis of 3-D random slopes.

With reference to the 2h:1v slope shown in Figure 7 with a height of 10m and no foundation layer, all the RFEM analyses that follow assume that the bottom of the mesh is fully fixed and the back of the mesh is allowed to move only in a vertical plane. It is noted that unlike the deterministic study shown previously, there is no symmetry in the RFEM analyses due to the spatial varying soil properties. In these analyses, both "rough" and "smooth" boundary conditions have been considered at the ends of the mesh in the out-of-plane direction $(z = 0 \text{ and } L)$. In the rough cases the ends are fully fixed and in the smooth case, they are allowed to move only in a vertical plane. In this study, it was determined that 2000 simulations of the Monte-Carlo process for each parametric group, was sufficient to give reliable and reproducible estimates of the probability of failure p_f. It can be noted that neither the rough nor the smooth vertical boundary conditions are particularly realistic. Real 3-D slopes tend to have rough sloping sides as might be observed at the abutments of an earth dam. In this paper however, we have considered only simple boundary conditions in order to focus on the influence of 3-D failure mechanisms.

Figs. 7, 8 and 9 show typical failed slopes with different (isotropic) correlation lengths given by $\Theta = 0.2,\ 2.0$ and 200.0. The grey scale depicts the undrained strength, although it should be emphasized that each figure represents just one simulation sampled from a suite of 2000 Monte-Carlo repetitions. It can be seen that the failure zone, when it occurs, typically involves a greater volume of soil when the spatial correlation length is either much smaller or much larger than the slope height.

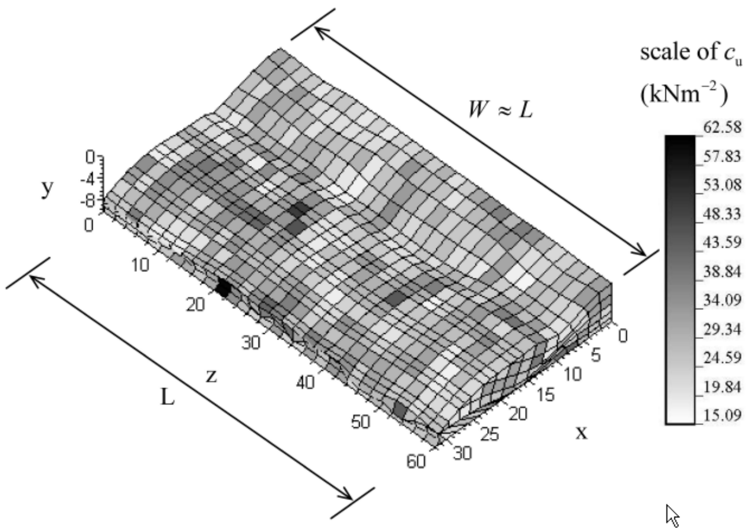

Fig. 7. Slope failure with $\Theta = 0.2$ and smooth boundary condition (all dimensions in metres)

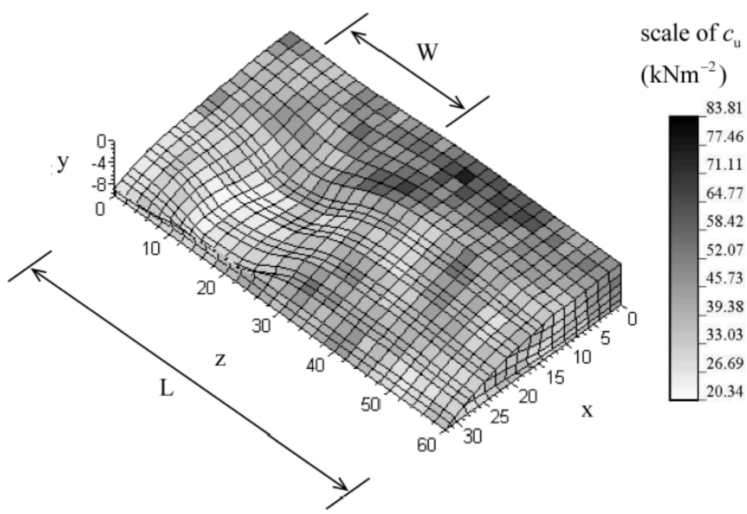

Fig. 8. Slope failure with $\Theta = 2.0$ and smooth boundary condition (all dimensions in metres)

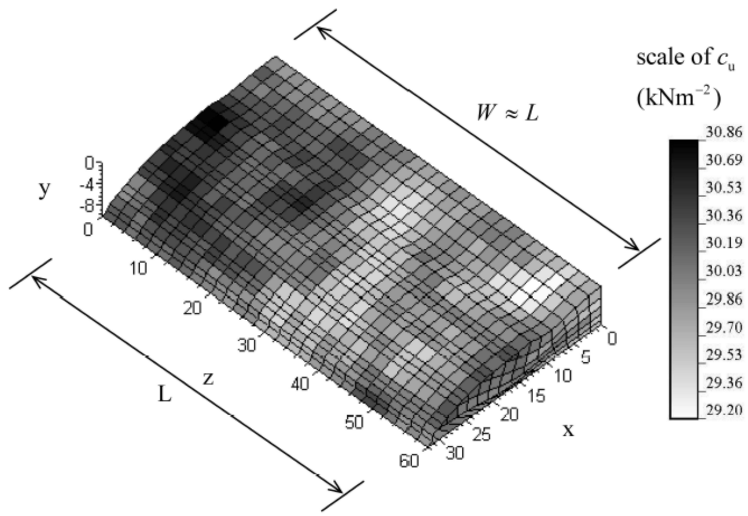

Fig. 9. Slope failure with $\Theta = 200.0$ and smooth boundary condition (all dimensions in metres)

Fig. 8 demonstrates an important characteristic in 3-D slope analysis called the "preferred" failure mechanism width W. This is the width of the failure mechanism in the $z-$direction that the finite element analysis "seeks out". Over a suite of Monte-Carlo simulations the average preferred failure mechanism width is called W_{crit}. It will be shown that this dimension has a significant influence on 3-D slope reliability depending on whether the length of the slope L is greater than or less than W_{crit}.

For given values of v_{C_u} (coefficient of variation) and Θ let us define the critical slope length L_{crit} and the critical slope length ratio $\left(L/H\right)_{crit}$ as being that value of L/H for which the slope is safest and its probability of failure p_f a minimum. It will be shown that this minimum probability of failure in the smooth case occurs when $L_{crit} \approx W_{crit}$.

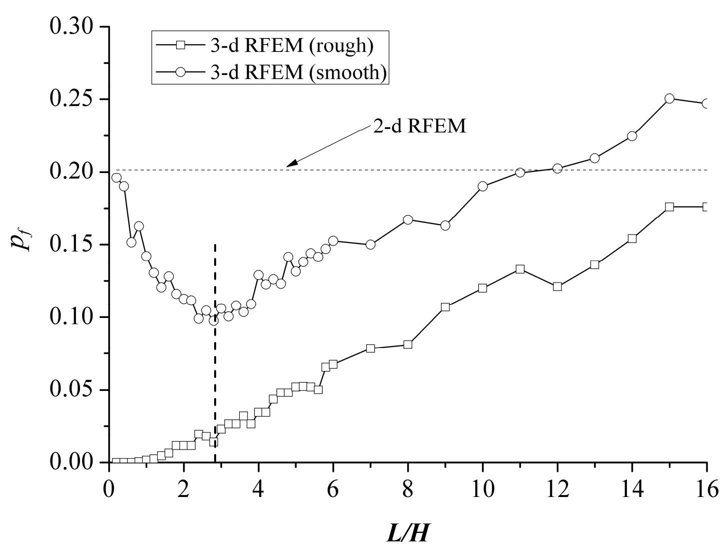

Fig. 10. Probability of failure versus slope length ratio $\left(v_{C_u} = 0.5,\ \Theta = 1.0\right)$

As shown in Figure 10 for the smooth case, if we reduce the slope length ratio below this critical value $\left(L < L_{crit}\right)$, the slope finds it easier to form a global mechanism spanning the entire width of the mesh with smooth end conditions, so the value of p_f increases, tending eventually to the plane strain value. However, if we increase the slope length ratio above this critical value $\left(L > L_{crit}\right)$, the slope finds it easier to form a local mechanism. Since $L > W_{crit}$ the mechanism has more opportunities to develop somewhere in the $z-\text{direction}$ hence p_f again increases.

4. CONCLUDING REMARKS

The paper has focused onthe use of finite element methods for slope stability analysis in variable soils. Observations were made on the depth of a 3-D slope in the out-of-plane direction needed to justify plane strain conditions respectively.

An investigation of the popular assumption that 2-D slope analysis is conservative compared to 3-D was found to rest entirely on the suitable selection of a “pessimistic” 2-D slice. A poorly selected 2-D slice could lead to unconservative predictions of the 3-D factor of safety.

Finally, the paper described some 3-D probabilistic slope stability methods using an important new method developed by the author and co-workers called the Random Finite Element Method (RFEM). These approaches target the *probability of failure* of a slope as opposed to the classical slope *factor of safety*. The influence of spatial correlation length was highlighted and the concept of a “preferred failure width” in 3-D slope analysis highlighted for the first time.

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