Probabilistic Analysis of Foundation Settlement on Multilayered Soil with a Complex Layer-Boundary

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ABSTRACT

This paper describes the probabilistic analysis of settlement of shallow foundations on a multilayered soil profile with a complex layer-boundary. Each layer is modelled as a separate random field of soil elastic modulus properties in order to represent spatial variability. The soil layer boundaries incorporate complex features such as inclination, undulation and a transitory region, which attempts to model realistic geological features. The analysis investigates the failure probability due to excessive differential settlement of two shallow footings founded at the surface of a two layer soil profile.

Keywords: Shallow foundations, settlement, random field, probabilistic methods, multilayered soil profile

1. INTRODUCTION

The problem of uncertainty regarding the properties of soil beneath a construction site arises from natural variability, unavoidable limitation of the site investigation and mathematical model error. Natural, or spatial, variability exists as the result of the soil formation process, while statistical and measurement errors arise from site investigations limitation (Vanmarcke, 1977). The transformation model error arises from inaccuracy of a mathematical model which seeks to mimic reality (Baecher & Christian, 2003).

The soil formation process involves diverse factors and takes thousands to millions of years to create today's soil profiles. As a result, spatial variability and heterogeneity are common features of soil profiles, as are geological features such as multiple layers, which often exhibit inclination, undulation and transitory boundaries.

As the properties are incorporated in geotechnical design, the site investigation process introduces another source of uncertainty. The investigation is intended to measure the desired soil properties at a certain site, and ideally the measurement would accurately reflect these properties. However, insufficient sampling and testing, and measurement error are unavoidable during site investigation, resulting in uncertainty associated with the properties. Application of the measured data for geotechnical design through mathematical formulae is then subject to model error, resulting in further uncertainty.

Random field theory, introduced by Vanmarcke (1977), attempts to model the natural variability and the broader uncertainty of soil properties. The method uses the statistical properties such as the mean, standard deviation and correlation length to describe the variation of soil properties at a site. However, random field modelling is simplified when the values are stationary, which requires temporary removal of any trends associated with the soil properties. However, if the random field model consists of several layers, in order to model stratification, the removal of the deterministic trend is unnecessary, as the trend can be accounted for by adopting separate layers, as shown in Figure 1.

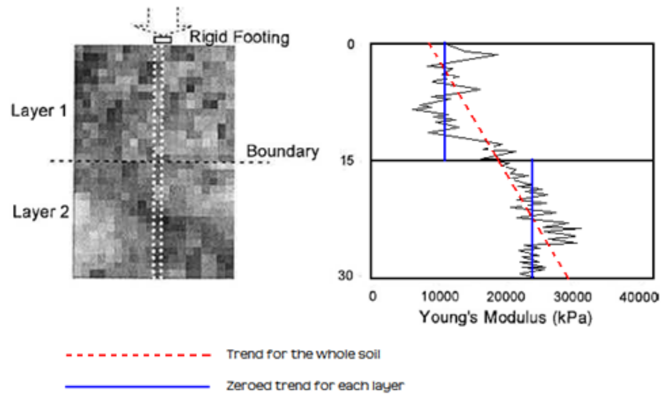

Figure 1. Removal of deterministic trend in multilayered random field model (adapted from Kuo et al. 2004)

The probabilistic method is used to examine the probability of failure of a foundation design. Monte Carlo simulation is employed by generating thousands of random fields which represent possible realisations of the site. The design failure probability is determined by dividing the number of realisations which fall short of the specified failure criterion by the total number of realisations.

A study by Kuo et al. (2004) used a probabilistic approach to examine the bearing capacity of a single, shallow foundation situated on a multilayered soil. However, this study was limited to horizontal layer boundaries. The study outlined in the present paper seeks to model more complex geologies and to determine their relationship to failure probabilities.

2. METHODOLOGY

The complex multilayered soil profile model was developed by the introduction of several parameters used to describe the irregularities of the layer boundary, as shown in Figure 2. The parameter i describes the inclination of the layer boundary from the horizontal; σ_i is its standard deviation representing the inclination uncertainty; σ_{hb} is the standard deviation of the layer boundary depth, which can be thought of as the 'amplitude' of the layer-boundary undulation; $\theta_{\ln hb}$ is the correlation length of the layer boundary depth, which corresponds to the 'wavelength' of the boundary undulation; and t represents the thickness of layer boundary, which specifies the transition area between the two, adjacent layers. Figure 3 to **Error! Reference source not found.** depict how these parameters affect the shape of the layer boundary. The different shades of grey indicate soil elements with various values of Young's modulus, E, with darker shades representing stiffer soil.

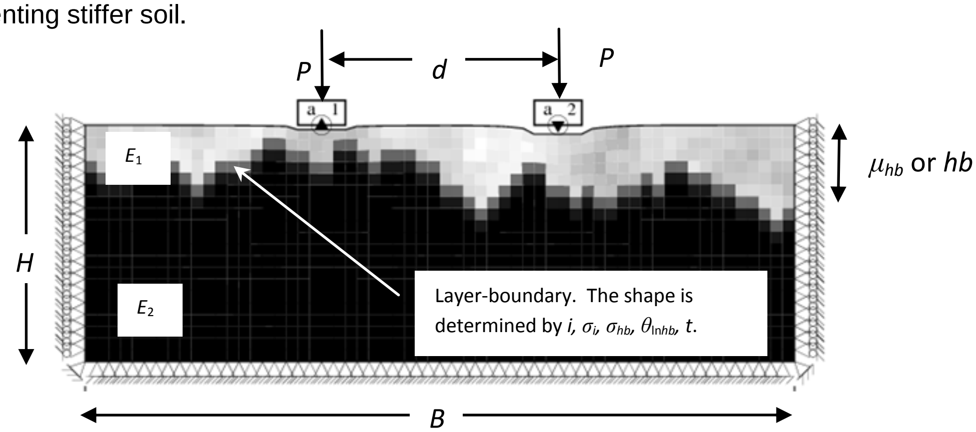

Figure 2. Random field soil model with complex layer boundary

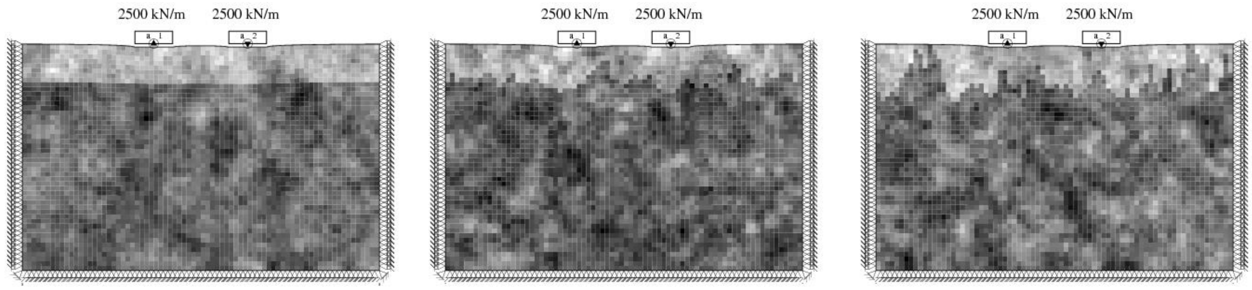

Figure 3. Layer boundaries using σ_{hb} = 0 m, 0.2 m and 0.5 m, respectively

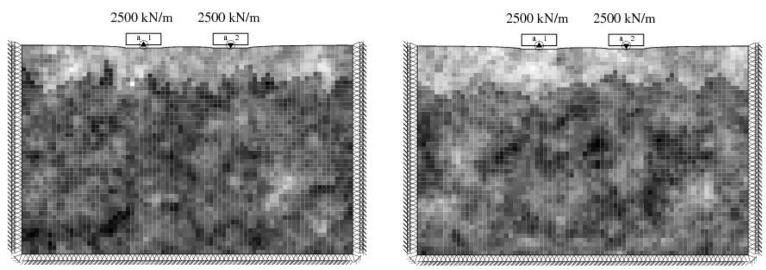

Figure 4. Layer-boundaries using $\theta_{\ln hb}$ = 1 m and 3 m, respectively

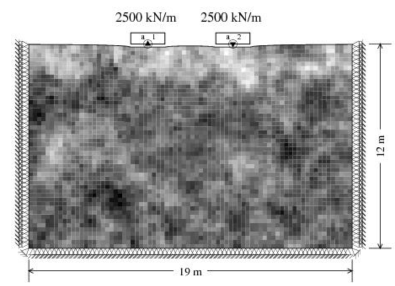

Figure 5. Gradational boundary, using $t > 0$

In actual soil profiles the properties of adjacent points will be roughly similar to one another, in other word correlated. The greater the distance between the two points, the smaller is the correlation. The correlation length, θ, describes the distance between two points within a soil profile where the properties are highly correlated, and this distance is used by the random field generator to specify properties at adjacent points. The same can also be said about the layer boundary depth, adjacent points tend to have similar depth and distant ones may have very different depths. Hence, when simulating layer boundaries the correlation length is also employed. In this paper the Markovian correlation function such as described in Fenton & Griffiths (2008) is used.

The analysis is conducted using a modified version of the shallow footing program RSETL2D, which is part of the random finite element model (RFEM) suite of Fenton & Griffiths (1990), which models the soil profile in 2D and hence assumes plane strain conditions. In the present paper, two soil types are modelled: (1) loose sand with an average elastic modulus, μ_E, of 17,000 kPa as the upper layer; and (2) medium-dense sand with μ_E of 22,000 kPa as the lower layer. Here, the number of layers is limited to two because the focus of the analysis is the effect of the layer-boundary uncertainty.

The uncertainty of E is modelled using a lognormal distribution function and the coefficient of variation ($COV_E = \sigma_E / \mu_E$). For both layers, COV_E is set at 10%, which is within the range suggested by Lee et al. (1983), and not too high to divert the focus of the present analysis from the layer boundary influence. The spatial correlation structure of the E field is again described by a Markovian function using correlation length ($\theta_{\ln E}$) values suggested by Phoon (1995), i.e. from 1 m to 2.2 m (in this paper from 1 m to 3 m). Poisson's ratio values are assumed to be constant, as suggested by Fenton and Griffiths (2002).

To simulate a zero average differential settlement between the two footings with equal load, the average layer-boundary inclination angle is set to horizontal (i = 0º). The footing load, P, corresponds to the limit of individual footing settlement design for footing on sands (Meyerhoff, 1965). As the model uses medium dense sand, 2500 kN/m is therefore adopted for each footing.

Differential settlement between the strip footings is selected as the failure criterion. Bjerrum (mentioned by Das, 1999) suggested 1/500 of the spacing between footings, d, as the safe differential settlement limit to prevent cracking of buildings. The spacing between footings is fixed at 5 m, thus making the differential settlement limit of 10 mm.

3. RESULTS AND ANALYSIS

Figure 6 shows the probability of failure of the footing design against the standard deviation of the layer boundary inclination σ_i, describing the degree of uncertainty of the inclination of a horizontal layer boundary. The figure depicts several curves showing the effect for different combinations of layer-boundary shape, as described by σ_{hb}, $\theta_{\ln hb}$ and t. All curves demonstrate that the probability of failure increases proportionately with the increase in σ_i.

A more undulating layer boundary also corresponds to a greater footing failure probability, where for the same σ_i, a layer-boundary with larger undulation 'amplitude' (σ_{hb}) results in an increased probability of failure. The contribution of the layer boundary spatial correlation distance ($\theta_{\ln hb}$), describing its undulation 'wavelength', is significantly smaller than the other factors.

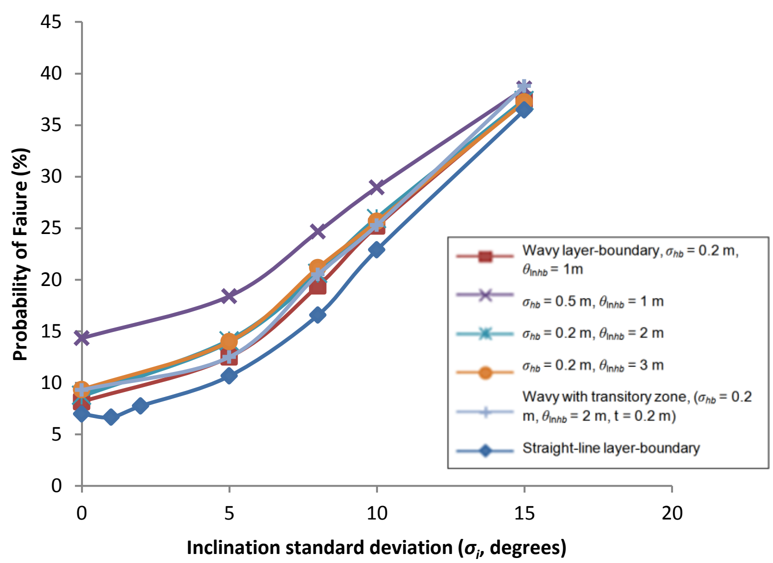

Figure 6. Effect of layer boundary inclination to the probability of failure (for hb = 2 m)

Figure 7 shows the probability of failure plotted against the σ_{hb}, showing the effect described above. For the same undulation amplitude (σ_{hb}), the increase in wavelength ($\theta_{\ln hb}$) initially results in a greater probability of failure, but subsequently this effect diminishes. This suggests that, as soon as the 'wavelength' approaches the spacing between the two footings, it ceases to affect the differential settlement, causing the footings to behave as if they are placed above a multilayered soil with a linear layer boundary. By plotting the probability of failure against $\theta_{\ln hb}$, Figure 8 shows this effect more clearly.

Figure 9 depicts the probability of failure against different thicknesses of the transitory region between the two layers. It shows that, for an undulating layer boundary, the presence of transitory region initially results in a small increase in the probability of failure. However, if the thickness of layer boundary is increased further the failure probability reverts back to its original value. This pattern is consistent for any layer boundary inclination uncertainty (σ_i), which means it is valid for any layer boundary inclination. For a horizontal layer boundary the transition zone, or blurriness, does not influence the failure probability.

Figure 10 depicts the probability of failure against the standard deviation of inclination, similar to Figure 6 but with curves representing soil profiles with different layer boundary depths. As shown, a deeper layer boundary decreases the failure probability. Moreover, the effect of undulation is also smaller for deeper layer boundaries as the footings are less influenced by any uncertainty in the layer boundary shape.

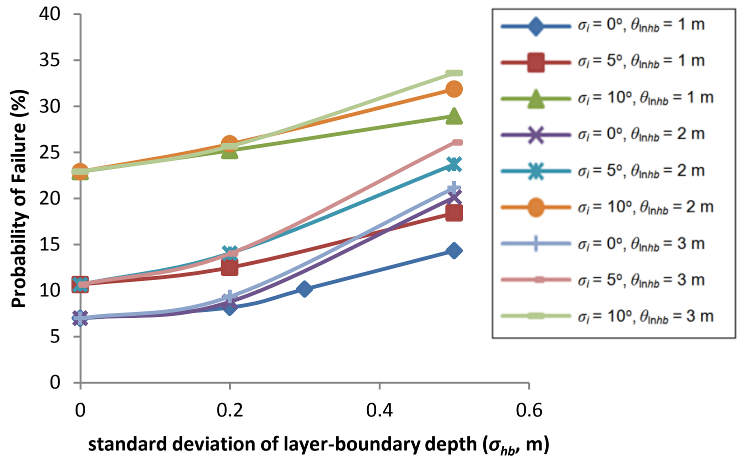

Figure 7. Effect of layer boundary undulation to the probability of failure (for hb = 2 m)

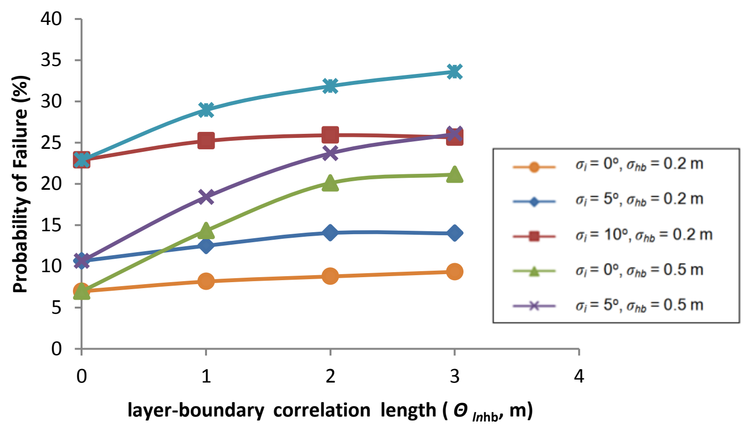

Figure 8. Effect of layer boundary correlation length on the probability of failure (for hb = 2 m)

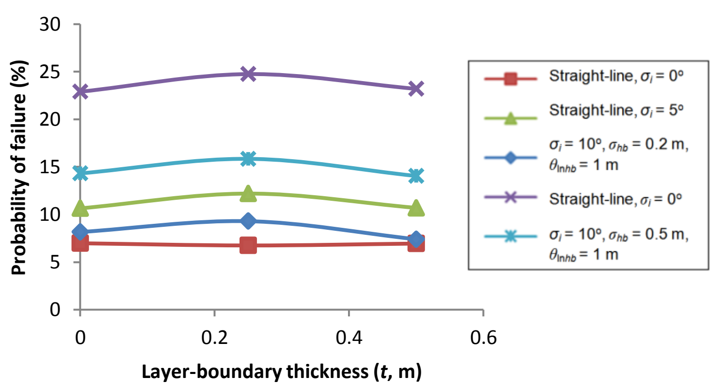

Figure 9. Effect of layer boundary thickness on the probability of failure (hb = 2 m, σ_i = 0º)

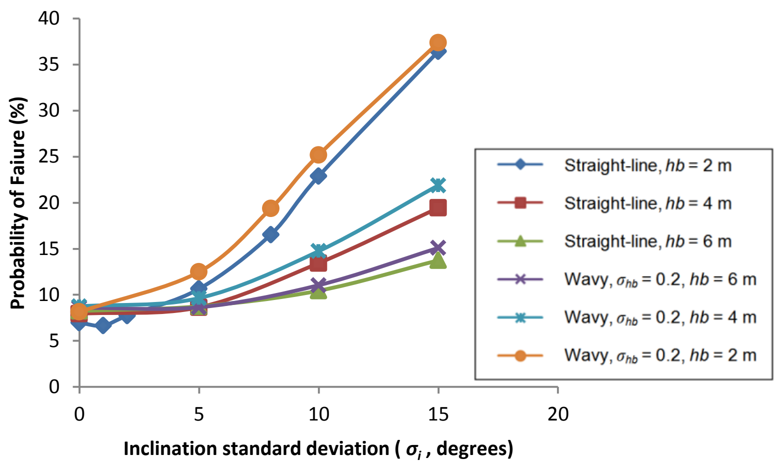

Figure 10. Effect of layer boundary undulation for different layer-boundary depths

4. CONCLUSION

The paper has examined the probability of failure associated with the differential settlement of two shallow footings founded on a two-layered soil profile incorporating geotechnical parameter uncertainty and realistic layer boundaries. Monte Carlo simulation was adopted in conjunction with the random finite element method. The probability of failure of the footings was found to be significantly affected by the uncertainty associated with the inclination and undulation of the layer boundary. The influence of inclination increases in proportion with its degree of uncertainty. The influence of undulation increases with its amplitude, while it decreases with its correlation length, or 'wavelength', as it approaches the spacing of the footings. The effect of both inclination and undulation of the layer boundary decreases as the depth of the layer boundary increases. Finally, a transitional layer boundary exhibited virtually no effect on the failure probability of the footings.

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