Settlement of Piles Founded in Spatially Variable Soils

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ABSTRACT

This paper investigates the probabilistic settlement of a single pile founded in a spatially variable soil from a theoretical viewpoint. The goals of the paper are to a) derive a theoretical expression for the failure probability, i.e. that the pile design will result in an actual settlement which exceeds the design maximum tolerable settlement, and b) use this expression to derive the resistance factor required in the design process to achieve a maximum tolerable failure probability. The results of the paper are plots showing failure probability as a function of the soil's correlation length, elastic modulus coefficient of variation, and design resistance factor. These plots can be used to determine the required resistance factor to use in the design process.

1 INTRODUCTION

Deep foundations, or piles, are structural members made of steel, concrete, and/or timber to transmit some, or all of the, applied load to soils below the ground surface. Piles may be costly, yet necessary to ensure structural safety in situations where the upper soil layer is highly compressible and/or too weak to support the applied load. Soil usually improves with depth and piles are used to transfer the load to underlying bedrock or a stronger soil layer. Piles resting on a stiffer stratum are called end-bearing. If not end-bearing, they are called floating in which most of the resistance is derived from skin friction and/or cohesion. In this paper, only floating piles are considered and end-bearing is ignored.

As load is applied to the pile, the pile settles due to both deformation of the pile itself and deformation of the surrounding soil. Assuming that there is no relative movement between the pile and surrounding soil due to friction and/or adhesion, any displacement of the pile is associated with an equivalent displacement of the surrounding soil. Due to the elastic nature of the soil, this displacement is resisted by a force which is proportional to the soil's elastic modulus and the magnitude of the displacement. Thus, the support provided by the soil to the pile depends on elastic properties of the surrounding soil. As stated by Vesic (1977), the fraction of pile settlement due to deformation of the soil, is a constant (dependent on Poisson's ratio and pile geometry, as discussed shortly) times F/E_{eff}, where F is the applied load and E_{eff} is the effective soil elastic modulus. The effective soil elastic modulus, E_{eff}, is the uniform, spatially constant, value of the elastic modulus which would produce a settlement identical to the actual pile settlement (Fenton and Griffiths, 2007).

The paper first investigates the probabilistic settlement behavior of a pile subjected to a random vertical load and supported by a spatially random soil. The results are used to determine the geotechnical resistance factor required to achieve a target design reliability against excessive settlement. The pile is designed against excessive settlement using the Load and Resistance Factor Design (LRFD) methodology which specifies that the factored resistance must exceed the sum of the factored load effects which are trying to cause the failure. At the serviceability limit state (SLS) corresponding to excessive pile settlement, the LRFD requirement is

$$\varphi_{gs}\hat{R}_s \geq \sum_i \alpha_i \hat{F}_i \qquad [1]$$

where $\hat{F}_i$ is the i^{th} characteristic load effect, α_i is its corresponding load factor, $\hat{R}_s$ is the characteristic (design) serviceability geotechnical resistance determined using characteristic geotechnical parameters, and φ_{gs} is the serviceability geotechnical resistance factor. The characteristic serviceability geotechnical resistance, $\hat{R}_s$, is a function of the soil's characteristic elastic modulus, the maximum allowable pile settlement and the pile geometry. The geotechnical resistance factor, φ_{gs}, is typically less than 1.0 and accounts for uncertainties in geotechnical parameters (Allen, 2005). The load factor, α_i, is typically greater than 1.0 for ultimate limit states but usually assumed equal to 1.0 for serviceability limit states and accounts for uncertainty in loads. This paper will assume $\alpha_i = 1.0$ and only dead and live loads will be considered, so that the LRFD requirement becomes

$$\varphi_{gs}\hat{R}_s \geq \hat{F} = \hat{F}_L + \hat{F}_D \qquad [2]$$

in which $\hat{F}_L$ and $\hat{F}_D$ are characteristic live and dead loads, respectively, and $\hat{F}$ is the total characteristic load. In this work, we assume $\hat{F}_L = 1.41\mu_L$ (Becker, 1996) and $\hat{F}_D = 1.18\mu_D$ (Allen, 1975), where μ_L and μ_D are the means of the maximum lifetime live and dead loads, respectively, so that the total characteristic load is $\hat{F} = 1.41\mu_L + 1.18\mu_D$.

The pile is assumed to be placed in a three-dimensional spatially random soil. A random load is applied vertically to the pile and the settlement of the pile is calculated using a simple elastic formula (modified from Das, 2000). The pile itself is assumed to be square, for reasons to be discussed later, with fixed cross-sectional dimension $d \times d$. The pile length, H, is determined as follows; *i*) the random soil is sampled at some location over a column of length D (as would occur if a CPT sounding were taken) to obtain a series of observations of the soil's elastic modulus, *ii*) the characteristic elastic modulus used in design, $\hat{E}$, is determined from the soil sample, and *iii*) the required pile length H is obtained via the LRFD requirement of eq. [2]. The details are discussed in the following sections. Once the pile length has been determined, the 'failure' probability that the pile settlement exceeds the maximum tolerable settlement can be determined from the theory developed below and plots of failure probability can be developed as a function of the statistics of the soil's random elastic modulus field (mean, variance, and correlation length) and the resistance factor used in the design process. It is to be emphasized that the work presented here is preliminary, laying out the basic theory. However, the theory has not yet been validated by comparison with simulation – this is the subject of future research and will appear in a future publication.

The remainder of this paper is organized as follows: Random soil and load models are described in Section 2. A reliability-based design approach to the pile settlement problem is discussed in Section 3 and the theoretical results are presented in Section 4. Conclusions and recommendations for future work are then given in Section 5.

2 RANDOM SOIL AND LOAD MODELS

The actual spatially varying elastic modulus field is probabilistically characterized by two numbers; one is the effective soil elastic modulus, E_{eff}, which is that value which yields the same settlement in a uniform elastic modulus field as the pile experiences in the actual spatially varying soil (Fenton and Griffiths, 2007). The second is the characteristic soil elastic modulus, $\hat{E}$, which is an estimate of E_{eff} obtained from a soil sample. Both numbers are derived in the next section as geometric averages of the actual spatially varying elastic modulus field, E, which is assumed to be lognormally distributed with mean μ_E, standard deviation σ_E and spatial correlation length, $\theta_{\ln E}$. The lognormal distribution is commonly used to represent non-negative soil properties and means that ln$E$ is normally distributed with parameters $\mu_{\ln E}$ and $\sigma_{\ln E}$. The correlation coefficient between the log elastic modulus at two points x_1 and x_2 is defined by a correlation function, $\rho_{\ln E}(\tau)$ in which τ is the distance between the two points. In this study, a simple exponentially decaying (Markovian) correlation function will be employed, having the form

$$\rho_{\ln E}(\tau) = \exp\left\{\frac{-2|\tau|}{\theta}\right\} \qquad [3]$$

As mentioned in Section 1, only live and dead loads are considered in this paper, which is a typical assumption in code development. The load applied to the pile takes two forms. One is the characteristic total load used in the pile design, which comes from current code provisions and is assumed to be deterministic: $\hat{F} = \hat{F}_L + \hat{F}_D = 1.41\mu_L + 1.18\mu_D$. The other is the 'true', but random, total load applied to the pile, F. It is assumed that the total load is equal to the sum of the maximum lifetime live load, F_L, and the relatively static dead load, F_D, i.e,

$$F = F_L + F_D \qquad [4]$$

where F_L and F_D are assumed to be lognormally distributed. In this case the mean and variance of total load, F, assuming live and dead loads are independent, are given by,

$$\mu_F = \mu_L + \mu_D \ , \ \ \sigma_F^2 = \sigma_L^2 + \sigma_D^2 \qquad [5]$$

Fenton et al. (2008) found that F is approximately lognormally distributed even though, strictly speaking, the sum of two lognormally distributed random variables is *not* lognormally distributed. The distribution parameters of the total load, F, are thus

$$\mu_{\ln F} = \ln(\mu_F) - \frac{1}{2}\sigma_{\ln F}^2 \ , \ \ \sigma_{\ln F}^2 = \ln(1 + v_F^2) \qquad [6]$$

where v_F is the coefficient of variation of the total load,

$$v_F = \frac{\sigma_F}{\mu_F} \qquad [7]$$

3 RELIABILITY-BASED SETTLEMENT DESIGN

In this section, a reliability-based design methodology is proposed for the pile length and a mathematical theory is presented to theoretically estimate the failure probability of an individual pile placed in a spatially varying soil. Due to space limitations, only the basic theory is presented in this paper. A more complete discussion, along with simulation-based validation of the theory, will be published by the authors shortly.

The reliability-based design goal is to determine the required pile length, H, such that the probability, p_f, of exceeding a specified maximum tolerable settlement, δ_{max}, is acceptably small, i.e. to find H such that

$$\mathrm{P}[\delta > \delta_{max}] = p_f \leq p_{max} \qquad [8]$$

in which δ is the actual (random) pile settlement. Design failure occurs if the actual pile settlement, δ, exceeds the maximum tolerable settlement, δ_{max}, which is taken as 0.025 m in this study.

Various methods are available to calculate the settlement of a pile; the basis of design used in this paper is a modified Das (2000) relationship

$$\hat{\delta} = u_1 \frac{\hat{F} d}{p H \hat{E}} \qquad [9]$$

where $\hat{\delta}$ is the characteristic pile settlement, $\hat{F}$ is the characteristic load calculated using Eq. 2, d is the pile width, $p=4d$ is the pile perimeter for a square pile, H is the pile length, $\hat{E}$ is the estimated characteristic soil elastic modulus, and u_1 is an influence factor which includes the effect of Poisson's ratio ($\nu = 0.3$ in this research).

The calibration of u_1, which leads to a modification to Das's (2000) relationship, is done here by calculating the deterministic settlement of a pile of length H surrounded by a soil with uniform (spatially constant) elastic modulus $\hat{E}$, Poisson's ratio ν, and supporting load $\hat{F}$ using the finite element method (Smith and Griffiths, 2004). The pile is founded in a three-dimensional linearly elastic soil mass underlain by bedrock. The mesh selected is 32 elements by 32 elements in plan by 64 elements in depth. Eight-node brick elements are used with dimensions: 0.3 m by 0.3 m by 1 m in the X, Y (plan) and Z (vertical) directions. Within this mesh, the pile is modeled as a column of elements having depth H and elastic modulus 21 Gpa, which is several orders of magnitude higher than that of the surrounding soil. Thus, the pile is assumed here to be of square cross-section with dimension $d = 0.3$ m and depth ranging from 0 to 64 m, rounded to the nearest 1 m. Because the stiffness matrix of a 32 by 32 by 64 element mesh requires about 2 GBytes of memory, a conjugate gradient iterative solver is employed to avoid the need to assemble the entire stiffness matrix.

Repeating the finite element prediction of settlement over a range of pile lengths, H, and back-calculating u_1 using Eq. 9 results in the solid curve shown in Figure 1. The following curve was fitted by regression to the finite element results,

$$u_1 = 3.16 H^{0.809} \qquad [10]$$

and the excellent match is also shown in Figure 1. The predicted $u_1$value given by Eq. 11 is used in the remainder of this paper.

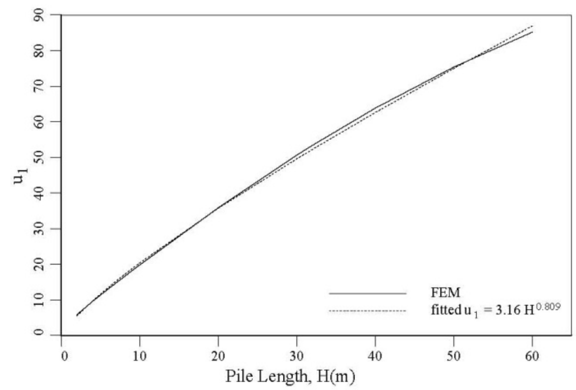

Figure 1. Calibration of u_1 using FE model

It is hypothesized now that the actual (random) pile settlement, δ, can be determined using eq. [9] by replacing the characteristic load $\hat{F}$ with the true (random) load F and the characteristic elastic modulus $\hat{E}$ with the (random) effective elastic modulus E_{eff},

$$\delta = u_1 \frac{Fd}{pHE_{eff}} \qquad [11]$$

Investigations by Fenton and Griffiths (2002) suggest that the effective elastic modulus as seen by a shallow foundation is a geometric average of the soil's elastic modulus under the foundation. It will be similarly assumed here that the effective elastic modulus as seen by the pile, E_{eff}, is a geometric average of the soil's elastic modulus over the pile depth,

$$E_{eff} = \exp\left\{\frac{1}{H}\int_0^H \ln E(z)dz\right\} \qquad [12]$$

where $E(z)$ is the elastic modulus of the soil at depth z.

The characteristic elastic modulus, $\hat{E}$, is estimated using observed values of the soil's elastic modulus. To obtain the observed soil properties, the soil is assumed to be sampled over a single column somewhere in the vicinity of the pile, for example by a single CPT or SPT sounding test taken near the pile, which yields a sequence of m observed elastic modulus values, $E_1^o, E_2^o, \ldots, E_m^o$. If $\hat{E}$ is to be a good estimate of E_{eff}, which is assumed to be a geometric average, then it should be similarly determined as geometric average of the observed sample $E_1^o, E_2^o, \ldots, E_m^o$

$$\hat{E} = \left(\prod_{j=1}^{m} E_j^o\right)^{1/m} = \exp\left\{\frac{1}{m}\sum_{j=1}^{m} \ln E_j^o\right\} \qquad [13]$$

The design pile length can now be determined by returning to eq. [9] and replacing $\hat{\delta}$ with the maximum tolerable settlement, δ_{max}. In order to achieve the desired design reliability, a resistance factor, φ_{gs}, is introduced so that the pile design length H satisfies the following settlement prediction,

$$\delta_{max} = u_1 \frac{\hat{F}d}{pH\varphi_{gs}\hat{E}} \qquad [14]$$

Replacing u_1 with eq. [10] leads to

$$\delta_{max} = u_1 \frac{\hat{F}d}{pH\varphi_{gs}\hat{E}} = 3.16H^{0.809}\frac{\hat{F}d}{pH\varphi_{gs}\hat{E}} = \frac{3.16\hat{F}d}{pH^{0.191}\varphi_{gs}\hat{E}} \qquad [15]$$

so that the design pile length can be determined as

$$H = \left(\frac{3.16\hat{F}d}{\delta_{max}p\varphi_{gs}\hat{E}}\right)^{5.236} \qquad [16]$$

Now that the pile length has been designed, attention can be turned to evaluating the probability that the design fails (see eq. [8]) i.e., that the actual pile settlement δ exceeds the design maximum tolerable settlement δ_{max}. Using eq. [10] in eq. [11], and replacing H with eq. [16], the actual (random) pile settlement can be estimated to be,

$$\delta = u_1 \frac{Fd}{pHE_{eff}} = \frac{3.16Fd}{pH^{0.191}E_{eff}} = \frac{3.16Fd}{p\left(\frac{3.16\hat{F}d}{\delta_{max}p\varphi_{gs}\hat{E}}\right)E_{eff}} = \frac{\delta_{max}\varphi_{gs}\hat{E}}{E_{eff}}\left(\frac{F}{\hat{F}}\right) \qquad [17]$$

which means that the design requirement of eq. [8] becomes,

$$\mathrm{P}[\delta > \delta_{max}] = \mathrm{P}\left[\frac{\varphi_{gs}\hat{E}}{E_{eff}}\left(\frac{F}{\hat{F}}\right) > 1\right] = \mathrm{P}\left[F\left(\frac{\hat{E}}{E_{eff}}\right) > \frac{\hat{F}}{\varphi_{gs}}\right] \le p_{max} \qquad [18]$$

If the soil's elastic modulus, E, is lognormally distributed, as assumed, then both $\hat{E}$ and E_{eff} will also be lognormally distributed since geometric averages preserve the lognormal distribution. In addition, if F is at least approximately lognormally distributed, as assumed here (Fenton et al., 2008), the quantity W, defined as

$$W = F\left(\frac{\hat{E}}{E_{eff}}\right) \qquad [19]$$

will be lognormally distributed and its parameters can be determined by considering the individual distributions of F, $\hat{E}$ and E_{eff} as follows. Since W is (at least approximately) lognormally distributed, then

$$\ln W = \ln F + \ln \hat{E} - \ln E_{eff} \qquad [20]$$

is (at least approximately) normally distributed and p_f can be found from

$$p_f = \mathrm{P}[W > \hat{F}/\varphi_{gs}] = \mathrm{P}[\ln W > \ln(\hat{F}/\varphi_{gs})] = 1 - \Phi\left(\frac{\ln(\hat{F}/\varphi_{gs}) - \mu_{\ln W}}{\sigma_{\ln W}}\right) = 1 - \Phi(\beta) \qquad [21]$$

where Φ is the standard normal cumulative distribution function, and β is the desired reliability index.

The failure probability p_f in eq.[21] can be estimated once the mean and variance of $\ln W$ are determined, where

$$\mu_{\ln W} = \mu_{\ln F} + \mu_{\ln \hat{E}} - \mu_{\ln E_{eff}} \qquad [22]$$

$$\sigma^2_{\ln W} = \sigma^2_{\ln F} + \sigma^2_{\ln \hat{E}} + \sigma^2_{\ln E_{eff}} - 2\mathrm{Cov}[\ln \hat{E}, \ln E_{eff}] \qquad [23]$$

under the assumption that the total load, F, and soil's elastic modulus, E, are independent. As discussed in section 2, the total load, F, is equal to the sum of the live load, F_L, and the dead load, F_D, i.e. $F = F_L + F_D$, and the mean and variance of $\ln F$ can be evaluated using eq. [6].

With reference to eq. [13],

$$\mu_{\ln \hat{E}} = \mathrm{E}[\ln \hat{E}] = \mathrm{E}\left[\frac{1}{m}\sum_{j=1}^{m} \ln E_j^o\right] = \frac{1}{m}\sum_{j=1}^{m} \mathrm{E}[\ln E_j^o] = \frac{1}{m}\sum_{j=1}^{m} \mu_{\ln E} = \mu_{\ln E} \qquad [24]$$

and

$$\sigma^2_{\ln \hat{E}} \cong \mathrm{E}\left[\left(\ln \hat{E} - \mu_{\ln \hat{E}}\right)^2\right] \cong \mathrm{E}\left[\left(\left(\frac{1}{m}\sum_{j=1}^{m} \ln E_j^o\right) - \mu_{\ln \hat{E}}\right)^2\right]$$

$$\cong \frac{1}{m^2}\sum_{i=1}^{m}\sum_{j=1}^{m} \mathrm{Cov}[\ln E_i, \ln E_j] \cong \frac{\sigma^2_{\ln E}}{m^2}\sum_{i=1}^{m}\sum_{j=1}^{m} \rho(z_i^o - z_j^o) \qquad [25]$$

in which z_i^o is the spatial location of the center of the i^{th} soil sample, for $i=1,...,m$, and ρ is the correlation function defined by eq.[3]. An approximation in the covariance occurs due to the fact that correlation coefficients between the local averages associated with observations are approximated by correlation coefficients between the local average centers. Assuming that $\ln \hat{E}$ represents a local average of $\ln E$ over the sample domain of depth, D, then $\sigma^2_{\ln \hat{E}}$ may be better computed as

$$\sigma^2_{\ln \hat{E}} \cong \sigma^2_{\ln E}\gamma(D) \qquad [26]$$

where $\gamma(D)$ is the variance reduction function, defined as,

$$\gamma(D) = \frac{1}{H^2}\int_0^H\int_0^H \rho(z_1 - z_2)dz_1dz_2 \qquad [27]$$

which measures the reduction in variance due to local averaging over the sample depth D. The sample depth, D, in this research, is assumed to be $D = \Delta z \times m$ where m is the number of observations and Δz is the vertical dimension of each soil sample.

Similarly, and with reference to eq. [12],

$$\mu_{\ln E_{eff}} = \mathrm{E}[\ln E_{eff}] = \mathrm{E}\left[\frac{1}{H}\int_0^H \ln E(z)dz\right] = \frac{1}{H}\int_0^H \mathrm{E}[\ln E(z)]dz = \frac{1}{H}\int_0^H \mu_{\ln E}\, dz = \mu_{\ln E} \qquad [28]$$

$$\sigma^2_{\ln E_{eff}} \cong \mathrm{E}\left[\left(\ln E_{eff} - \mu_{\ln E_{eff}}\right)^2\right] \cong \mathrm{E}\left[\left(\left(\frac{1}{H}\int_o^H \ln E(z)dz\right) - \mu_{\ln E_{eff}}\right)^2\right] \qquad [29]$$

$$\cong \frac{\sigma^2_{\ln E}}{H^2}\int_0^H\int_0^H \rho(z_1 - z_2)\, dz_1dz_2 \cong \sigma^2_{\ln E}\gamma(H)$$

The covariance in eq. [23] between the geometric average of the observed elastic modulus values over sample domain, $D = \Delta z \times m$, and the effective elastic modulus along the pile length, H, is obtained as follows,

$$\mathrm{Cov}[\ln \hat{E}, \ln E_{eff}] \cong \mathrm{E}\left[\left(\ln \hat{E} - \mu_{\ln \hat{E}}\right)\left(\ln E_{eff} - \mu_{\ln E_{eff}}\right)\right]$$

$$\cong \mathrm{E}\left[\left(\left(\frac{1}{m}\sum_{j=1}^{m} \ln E_j^o\right) - \mu_{\ln \hat{E}}\right)\left(\left(\frac{1}{H}\int_o^H \ln E(z)dz\right) - \mu_{\ln E_{eff}}\right)\right] \qquad [30]$$

$$\cong \frac{\sigma^2_{\ln E}}{mH}\sum_{j=1}^{m}\int_0^H \rho\left(\sqrt{r^2 + (z - z_j^o)^2}\right) dz \cong \sigma^2_{\ln E}\gamma_{HD}$$

where γ_{HD} is the average correlation coefficient between the elastic modulus samples over domain D and the elastic modulus along the pile of length H, and ρ is the correlation function between $\ln E(z_j^o)$ and $\ln E(z)$. In detail, γ_{HD} is defined by,

$$\gamma_{HD} \cong \frac{1}{mH}\sum_{j=1}^{m}\int_0^H \rho\left(\sqrt{r^2 + (z - z_j^o)^2}\right) dz \qquad [31]$$

where r is the horizontal distance between the pile centerline and the centerline of the soil sample column. Substituting eq.'s [6], [24], [26], [28], [29], and [30] into eq.'s [22] and [23], leads to

$$\mu_{\ln W} = \mu_{\ln F} \qquad [32]$$

$$\sigma^2_{\ln W} \cong \sigma^2_{\ln F} + \sigma^2_{\ln E}[\gamma(D) + \gamma(H) - 2\gamma_{HD}] \qquad [33]$$

If the reliability index is specified by p_{max}, where p_{max}=1-$\Phi(\beta)$, then the geotechnical resistance factor is determined by

$$\varphi_{gs} = \exp\left(\ln \hat{F} - \mu_{\ln W} - \beta\sigma_{\ln W}\right) \qquad [34]$$

4 THEORETICAL RESULTS

In this section, the theoretical failure probability results will be discussed. The statistical parameters used in this study are listed in Table 1.

Table 1 – Input parameters used in this study

Parameter	Values Considered
μ_L	84.5 kN
μ_D	253.5 kN
μ_F	338.0 kN
$v_L = \sigma_L/\mu_L$	0.30
$v_D = \sigma_D/\mu_D$	0.15
$v_F = \sigma_F/\mu_F$	0.135
Poisson's ratio, ν	0.3
$v_E = \sigma_E/\mu_E$	0.1- 0.5
μ_E	30 Mpa
$\theta_{\ln E}$	0.0-100.0 m
φ_{gs}	0.5,0.6,0.7,0.8,0.9

The effect of correlation length on the probability of failure, computed analytically using eq. [21], is illustrated in Figure 2 for, v_E=0.4, and $\theta_{\ln E}$=4 m, when the soil is sampled at r=4.5 m from the pile location. It is observed from Figure 2 that the probability of failure, p_f, increases with resistance factor, φ_{gs}, as expected. Also, it is evident that the probability of failure reaches a maximum at an intermediate correlation length $\theta_{\ln E} \approx 4$ m. This is as expected, since for small and large correlation lengths the values of $\hat{E}$ and E_{Eff} will coincide for stationary random fields and so the largest difference between $\hat{E}$ and E_{Eff} will occur at intermediate correlation lengths. This worst case is important, since the correlation length is very hard to estimate and will be unknown for most sites. In other words, in the absence of knowledge about the correlation length, the lowest resistance factor in these plots, at the worst case correlation length, should probably be used.

Figure 3 shows the effect of resistance factor on estimated probability of failure, computed analytically using eq. [21], for different values of v_E, and the worst case correlation length, $\theta_{\ln E}$=4 m, when the soil is sampled at r=4.5 m from the pile location. This figure can be used for design by drawing a horizontal line across at the target probability, p_{max}, and then reading off the required resistance factor for a given v_E. For p_{max}=0.05, it can be seen that φ_{gs} is almost 0.51 for the 'worst case' v_E=0.5. For all other v_E's considered, the required resistance factor is between 0.59 and 0.9. Apparently, the probability of failure is more sensitive to resistance factor changes, when the soil variability is higher. For lower target probabilities, say p_{max} = 0.01, the resistance factor for the 'worst case' v_E decreases to 0.4, and ranges between 0.44 and 0.81, for all other considered v_E's.

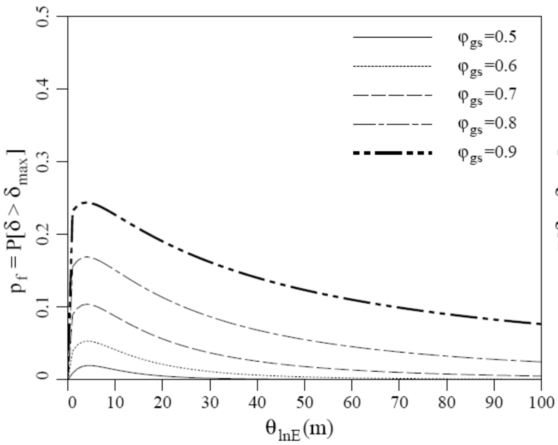

Figure 2. Effect of correlation length, θ_{lnE}, on probability of failure, p_f, for v_E=0.4 and μ_E=30Mpa.

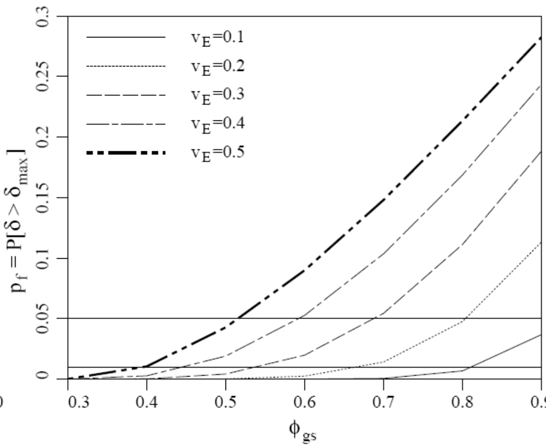

Figure 3. Effect of resistance factor, φ_{gs}, on probability of failure, $p_{f,}$, for θ_{lnE}=4 m.

5 CONCLUSIONS

In this work, a reliability-based design of deep foundations was studied and a mathematical theory was developed to analytically estimate the probability of pile settlement failure, which was then employed to derive the resistance factor required in the design process to achieve a maximum tolerable failure probability.

The effect of the soil's spatial variability and site understanding on the geotechnical resistance factor has been investigated by theory, using various soil statistics where the soil sample is taken at distance r=4.5 m from the pile location.

The computation of required geotechnical resistance factors for pile design involves the soil field's uncertainty level (e.g. coefficient of variation, v_E), correlation level (e.g. correlation length, θ) and sampling location. Since coefficient of variation, v_E, and correlation length, θ, are usually unknown for a given site, various v_E's are considered in this study for deep foundation limit state design, along with a worse case value of θ, i.e. the intermediate value of θ corresponding to the higher probabilities of failure.

The resistance factors recommended in this study for serviceability limit state design of deep foundations should be considered to be upper bounds because measurement and model errors are not considered. These additional error sources can be accommodated here by using a value of v_E greater than would actually be true at a site (e.g. if v_E = 0.35 at a site, the effects of measurement and model error might be accommodated by using v_E = 0.5 in the relationships presented here) or by assuming that the soil samples were taken further away from the pile centerline than they actually were (e.g. if low-quality soil samples are taken at the pile location, r = 0, the geotechnical resistance factor corresponding to a larger value of r, say r = 4.5 m should be used).

Although the analytical results presented in Figure 2 demonstrate that a ’worst case’ correlation length exist, the results should be validated by comparison with simulation in order to support the analytical results, which is the subject of future research.

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