Estimation of Failure Probability by Limit State Sampling

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ABSTRACT

This paper presents a novel approach, referred to as Limit State Sampling, for estimating failure probabilities of engineering structures. The majority of methods used to evaluate failure probabilities involve a large number of simulations of the structural model. In situations with low failure probability and numerically complex structural models this can become a computationally unpractical task. The Limit State Sampling approach is developed here with the intention of reducing the number of simulations of the structural model in the process of evaluation of the failure probability. This is performed by introducing a pseudo probabilistic density function with the purpose of sampling around the failure limit state. Samples from the pseudo probability density function are then used to construct a surrogate model of the structural behavior at the failure limit state. Finally, the failure probability is estimated by utilizing the efficiency of the surrogate model, with reduced computational expense. The novelty of the approach comes from the formulation of the pseudo probability density function and the application to the probabilistic analysis of structures.

INTRODUCTION

Structural failure is commonly characterized as an event of an unsafe or undesired state of the structure. In reliability analysis of structures, the state of the structure is expressed by a performance function $g(\mathbf{X}, \lambda)$, where $\mathbf{X} = [X_1, ..., X_n]^T \in \Omega \subset \mathbb{R}^n$ is an *n*-dimensional vector of random structural variables in the variable space Ω, and λ is a parameter defining the failure criterion (e.g., displacement limit of the structure). The random vector $\mathbf{X}$ is associated with the *joint* probability density function (*pdf*) $\mathbf{X} \sim f_{\mathbf{X}}(\mathbf{x})$, where $\mathbf{x}$ is a value of $\mathbf{X}$. The performance function, $g(\mathbf{X}, \lambda)$, plays a central role in the reliability analysis of structures, because it separates the *n*-dimensional variable space Ω into a safe domain $g(\mathbf{X}, \lambda) > 0$, and an unsafe domain $g(\mathbf{X}, \lambda) \leq 0$ by the hypersurface denoted as the limit state $g(\mathbf{X}, \lambda) = 0$ (e.g., Hurtado & Alvarez, 2003).

One of primary interests in reliability analysis of structures is the estimation of failure probability. The probability of failure P_F, can mathematically be expressed as:

$$P_F = P(\mathbf{X} \in F) = \int_{g(\mathbf{x},\lambda)\leq 0} f_{\mathbf{X}}(\mathbf{x})d\mathbf{x} = \int_{\mathbb{R}^n} I_F(\mathbf{x}) f_{\mathbf{X}}(\mathbf{x})d\mathbf{x} \tag{1}$$

where F is the failure domain, I_F is an indicator function such that $I_F(\mathbf{X}) = 1$ if $\mathbf{X} \in F$ and $I_F(\mathbf{X}) = 0$ otherwise. The problem of calculating the P_F is *n*-dimensional as a result of the *n*-dimensionality of the integral in Equation 1.

In many practical applications numerical models are used to simulate the behavior of the structure, resulting in an implicit formulation of the performance function. As a result of the implicit formulation of the performance function, only pointwise evaluations of the performance function are obtainable, $g(\mathbf{x}^{(i)}, \lambda)$ for samples $\mathbf{x}^{(i)} \sim f_{\mathbf{X}}(\mathbf{x})$ from the variable space Ω.

In the majority of applications, the integral in Equation 1 is evaluated numerically using one of the optimization (e.g., First Order Reliability Method) or sampling methods (e.g., Monte Carlo, Importance Sampling) (e.g. Schuller et al., 2004). Among various numerical methods, the Monte Carlo (MC) method (Nicholas & Ulam, 1949) is extensively used as a result of its robustness and straightforward implementation (e.g. Schuller et al., 2004). The MC method is based on drawing N independent identically distributed (i.i.d.) samples $\mathbf{x}^{(i)} \sim f_{\mathbf{X}}(\mathbf{x}); i = 1, ..., N$ and evaluating the performance function $g(\mathbf{x}^{(i)}, \lambda)$ at these samples. The unbiased estimate of the failure probability is calculated as the ratio of the number of failed samples over the total number of samples:

$$\hat{P}_F = \frac{1}{N}\sum_{i=1}^{N} I_F(\mathbf{x}^{(i)}) \tag{2}$$

The MC method provides the estimate of the failure probability with high accuracy for an extensive range of problems in both low and high dimensional problems, provided that $f_{\mathbf{X}}(\mathbf{x})$ can be sampled (e.g. Koutsourelakis et al., 2004; Schuller et al., 2004). After observing that the failure event is a Bernoulli random variable: $I_F(\mathbf{x}^{(i)}) = 1$ if $g(\mathbf{x}^{(i)}, \lambda) \leq 0$ and $I_F(\mathbf{x}^{(i)}) = 0$ otherwise, the coefficient of variation $\hat{P}_F$ can be calculated as $CoV(\hat{P}_F) = \sqrt{(1 - \hat{P}_F)/(\hat{P}_F \cdot N)}$. After analyzing the $CoV(\hat{P}_F)$, one can detect that the accuracy of $\hat{P}_F$ is independent of the dimensionality of the problem (Schuller et al., 2004) and that the coefficient of variation reduces as the number of samples N increases. If $\hat{P}_F$ is small, a large number of samples N is necessary for $\hat{P}_F$ to converge. In computational terms that equates to N simulations of the structural model, required to evaluate the performance function. It is often unpractical to perform large number of simulations of the structural model, as the models can be computationally intensive. This observation characterizes the MC method as inefficient when applied in problems with small $\hat{P}_F$ and/or when complex structural models are used to evaluate the performance function. This is often the case in probabilistic analysis of structures.

This paper presents a novel approach in evaluating the P_F called Limit State

Sampling (LSS). The LSS method is developed with a purpose of concentrating the computational efforts on studying the behavior of the structure at and around the failure limit state. This is achieved by introducing a *pseudo pdf* as a function of the performance function with a mode at the failure limit state. The novelty of the approach comes from the formulation and implementation of the *pseudo pdf*. Detailed presentation of the LSS approach is presented in the following section.

LIMIT STATE SAMPLING

As discussed in the previous section, estimation of P_F in engineering structures is frequently conducted by sampling methods. Among a wide range of sampling methods, the MC method is predominantly used and it will be compared to the proposed LSS approach. One of the main disadvantages of the MC method in evaluating P_F is that the failure states are generally observed and sampled in small numbers, proportionally to P_F. In practical terms this translates to the requirement of a large number of samples, $\mathbf{x}^{(i)} \sim f_{\mathbf{X}}(\mathbf{x}); i = 1, ..., N$, and consequently a large number of simulations of the structural model for the evaluation of the performance function $g(\mathbf{x}^{(i)}, \lambda)$. In contrast, the idea behind the LSS approach is to concentrate the computational efforts on studying the performance of the structure at and around the failure limit state. This is achieved by introducing a *pseudo pdf*, which is a *pdf* formulated on, usually the only information related to the failure of the structure known prior to the simulation, the value of the performance function at the failure limit state $g(\mathbf{x}, \lambda) = 0$. Based on this information a *pseudo pdf* $\varphi(g(\mathbf{x}, \lambda))$ is formulated with a mode located at the failure limit state, as presented in Figure 1.

After sampling the *pseudo pdf*, a "cloud" of pointwise evaluations of the performance function around the failure hypersurface is obtained, $g^{(k)} \sim \varphi(g(\mathbf{x}, \lambda)); k = 1, ..., S$. Each sample from the *pseudo pdf*, $g^{(k)}$, corresponds to a certain combination of random parameters, $\mathbf{x}^{(k)}$. A set of pointwise evaluations of the performance function surrounding the failure hypersurface, $\left\{\left(g^{(k)}, \mathbf{x}^{(k)}\right) : \mathbf{x}^{(k)} \in \Omega, g^{(k)} = g(\mathbf{x}^{(k)}, \lambda), k = 1, ..., S\right\}$, can be used to formulate an approximation of the functional relationship between the random parameters and performance function, referred to as surrogate model.

The surrogate model is built for the purpose of accurate and efficient evaluation of the

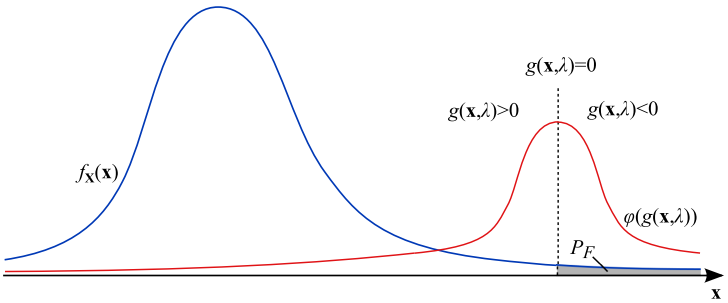


Figure 1: Limit State Sampling - one dimensional representation.

performance function for a large number of samples, providing significant reduction in computational time when compared to the actual structural model. The P_F is then estimated by the MC method or some of its derivatives, by sampling the *joint pdf* and evaluating the peformance function by the surrogate model. Details on the implementation of the LSS approach are presented in the following sections.
The approach is named Limit State Sampling because the method is not exclusively related to the failure limit state, any limit state can be investigated by the proposed approach as long as the *pseudo pdf* can be formulated and efficiently sampled.

Implementation of the LSS method. The LSS method is developed around the idea of studying the performance of structures at the failure limit state by sampling the *pseudo pdf*. In many practical situations, the performance function and consequently the *pseudo pdf* are implicit functions of the random model variables. As a result of the implicit formulation of the performance function, geometrical properties of the *pseudo pdf* in the variable space and the normalizing constant are unknown prior to the probabilistic analysis. These complexities in the formulation of the *pseudo pdf* are avoided by employing the Metropolis Hastings (MH) algorithm to sample the *pseudo pdf*. MH is one of the algorithms of the Markov Chain Monte Carlo (MCMC) method (Hastings, 1970; Metropolis et al.,1953), and it is selected for its flexibility in sampling a wide range of distributions and simplicity of implementation. MCMC is a group of methods used for sampling a distribution of interest by constructing a Markov Chain that has the distribution of interest as its limiting distribution. Markov Chain is a stochastic process which defines a sequence (chain) of states of the process, where the transition to the next state of the process is dependent only on the current state. The changes in the states of the process are controlled by the transition kernel, defining the particular transition probabilities and an initial state of the process. The transition kernel in the MH algorithm is defined by a proposal distribution, which is a *pdf* centered at the current state of the chain. A new state of the Markov chain is drawn from the proposal *pdf* and accepted with probability α. The acceptance probability α is a function of the proposal *pdf* and the distribution of interest at the current and proposed state of the Markov chain (e.g., Gamerman & Lopes, 2006).
In the LSS approach the distribution of interest is the *pseudo pdf*. Drawing samples from the *pseudo pdf*, $g^{(k)} \sim \varphi(g(\mathbf{x}, \lambda)); k = 1, ..., S$, will provide an insight into the behavior of the structure at the failure limit state as each sample, $g^{(k)}$, is associated with a certain combination of random variables, $\mathbf{x}^{(k)}$, in the variable space Ω. Sampling is performed by the MH algorithm where the transition kernel is defined by a proposal *pdf*, q. Proposal *pdf* is usually a multivariate normal or an uniform *pdf* centered at the current state of the chain $g^{(j-1)}$. The proposal state of the Markov Chain, $\hat{g}$, is generated from the *pdf* centered at the corresponding current state of the Markov Chain in the variable space $\hat{\mathbf{x}} \sim q(\cdot|\mathbf{x}^{(j-1)})$. After a corresponding proposal state in the variable space $\hat{\mathbf{x}}$ is sampled, the performance function is evaluated in order to generate $\hat{g} = g(\hat{\mathbf{x}}, \lambda)$. The move from the current state to the proposed state is accepted with the probability α, as presented in the following algorithm.
The *pseudo pdf* is formulated as a function of the performance function and various

formulations are feasible, as long as they are able to provide samples around the limit state. Some of the possible formulations are presented in Figure 2. Geometrical properties of the *pseudo pdf* are controlled by the shape parameter a as presented in Figure 2. The shape parameter controls the "width" of the *pseudo pdf*, which can affect the performance of the method. The optimal value of the shape parameter can be selected by conducting a few short trial simulations of the algorithm.

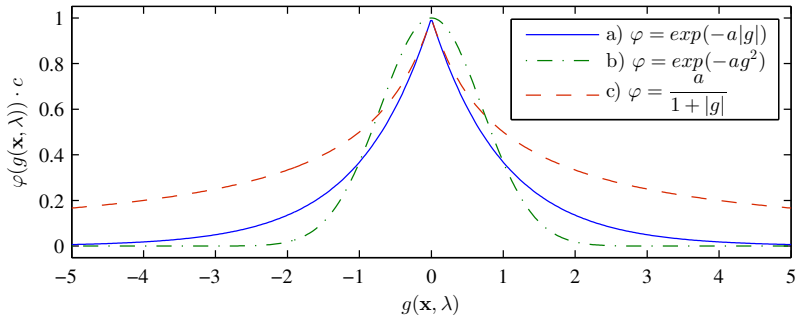


Figure 2: Several formulations of the *pseudo pdf*; a shape coefficient, c normalizing constant.

Implementation of the MH algorithm in the LSS method:
1) *Initialize the counter* $j = 1$ *and set the initial value* $\mathbf{x}^{(0)}$ and $g^{(0)}$.
2) *Obtain a proposal state of the Markov Chain by sampling* $\hat{\mathbf{x}} \sim q(\cdot|\mathbf{x}^{(j-1)})$, *and evaluating the performance function,* $\hat{g} = g(\hat{\mathbf{x}}, \lambda)$.
3) *Evaluate the acceptance probability of the move*

$$\alpha = min\left(1, \frac{\varphi(\hat{g})q(\mathbf{x}^{(j-1)}|\hat{\mathbf{x}})}{\varphi(g^{(j-1)})q(\hat{\mathbf{x}}|\mathbf{x}^{(j-1)})}\right) = min\left(1, \frac{\varphi(g(\hat{\mathbf{x}},\lambda))q(\mathbf{x}^{(j-1)}|\hat{\mathbf{x}})}{\varphi(g(\mathbf{x}^{(j-1)},\lambda))q(\hat{\mathbf{x}}|\mathbf{x}^{(j-1)})}\right)$$

4) *Generate an independent uniform* $u \sim U(0,1)$ *and if* $u \leq \alpha$ *accept the move* $g^{(j)} = \hat{g}$ *;* $\mathbf{x}^{(j)} = \hat{\mathbf{x}}$, *otherwise reject the move* $g^{(j)} = g^{(j-1)}$ *;* $\mathbf{x}^{(j)} = \mathbf{x}^{(j-1)}$.
5) *Update the iterator* $j = j + 1$ *and return to step 2 until desired number of samples is drawn, or convergence is reached.*

Evaluation of failure probability. Sampling of the *pseudo pdf* results in a set of pointwise evaluations of the performance function surrounding the failure hypersurface: $\left\{\left(g^{(k)}, \mathbf{x}^{(k)}\right) : \mathbf{x}^{(k)} \in \Omega, g^{(k)} = g(\mathbf{x}^{(k)}, \lambda), k = 1, ..., S\right\}$. Samples from the *pseudo pdf* are used to generate a surrogate model of the performance function, $\tilde{g}(\mathbf{X}, \lambda)$, by implementing interpolation or some of the regression or classification approaches from the group of statistical learning methods (e.g., Hastie et al., 2001; Hurtado & Alvarez, 2003). The indicator function, based on the surrogate model, has the following formulation, $\tilde{I}_F(\mathbf{X}) = 1$ if $\tilde{g}(\mathbf{X}, \lambda) \leq 0$ and $\tilde{I}_F(\mathbf{X}) = 0$ otherwise. Once the approximation model is formulated, P_F can be estimated by implementing crude MC method or some of its derivaties (e.g, Latin Hypercube Sampling) to sample $\mathbf{x}^{(j)} \sim f_{\mathbf{X}}(\mathbf{x}); j = 1, ..., K$, while the performance function is efficiently evaluated

by $\tilde{g}(\mathbf{x}^{(j)}, \lambda)$. The failure probability is estimated by the following equation:

$$\hat{P}_F = \frac{1}{K} \sum_{j=1}^{K} \tilde{I}_F(\mathbf{x}^{(j)}) \tag{3}$$

The accuracy of the LSS method is controlled by the quality of the representation of the failure hypersurface, which is dependent on the number and position of the samples from the *pseudo pdf*, and the accuracy of the method approximating the performance function. In order to have a good representation of the failure hypersurface it is important to sample the *pseudo pdf* in the region of the variable space populated by samples $\mathbf{x}^{(j)} \sim f_{\mathbf{X}}(\mathbf{x}); j = 1, ..., K$. In low dimensional problems this can be ensured by focusing the MH algorithm to the region of the variable space defined by the following limits: $\mathbf{x}_{min} = \min(\mathbf{x}^{(j)}), \mathbf{x}_{max} = \max(\mathbf{x}^{(j)})$. The sampling limits should be determined prior to sampling the *pseudo pdf* for the proposal states in the MH algorithm to be generated from a subset of the variable space, $\{\mathbf{x} \in \Omega_{lim} \subset \Omega : \mathbf{x}_{min} \leq \mathbf{x} \leq \mathbf{x}_{max}\}$.

PROBABILISTIC ANALYSIS OF A LATERALLY LOADED PILE USING LSS

An investigation of a pile response to lateral loading is essential in the design of certain engineering structures (e.g., oil platforms, offshore wind turbine foundations). In the case of offshore wind turbines, a single large diameter pile (i.e., monopile), is commonly chosen as a foundation option (e.g., LORC, 2013).

Numerical pile-soil model. A numerical model, known as the p-y model (Matlock, 1970), is extensively used for the analysis of the pile-soil system. The p-y model was developed by backcalculating a series of field tests (Matlock, 1970; Reese et al., 1974) and it is currently in the recommended practice of several offshore wind turbine design codes (e.g., DNV, 2010). The model presents an extension of Winkler's beam on an elastic foundation (Winkler, 1867), where mechanical properties of springs are modeled by nonlinear p-y curves. Detailed presentation of the p-y formulation can be found in Matlock, (1970).

Laterally loaded pile with two soil layers. The performance of the LSS method is examined using an example of a monopile offshore wind turbine foundation. The monopile is L=30 m long, with a diameter of D=3.86 m and a pile wall thickness of t=0.04 m. Pile material is steel with yield strength $f_y = 235.0$ MPa, Young's modulus of E=$2.1 \cdot 10^5$ MPa, and a Poisson's ratio of ν=0.3. Material behavior of the pile is assumed to be linear elastic.

The soil domain is divided into two 15-m, medium stiff clay, layers and discretized by 30 springs as presented in Figure 3. The springs are simulating the soil response with material behavior defined by the p-y curves. Shear strength of clay, s_u, in the two layers is considered to be random and independent between the layers, but homogeneous within a single layer. Variability of shear strength is expected to significantly influence the pile-soil response due to the formulation of p-y curves for clay, where shear strength is directly related to the peak value of soil resistance (e.g., Matlock, 1970).

The statistical properties of s_u are described by a lognormal distributions with an equivalent normal mean of $\mu_{s_{u1}} = 25.0$ kPa in layer 1 and $\mu_{s_{u2}} = 50.0$ kPa in layer 2. The coefficients of variation are equal for both layers $CoV(s_{u1}) = CoV(s_{u2}) = 0.25$ (e.g., Nadim, 2007). Additional parameters of the p-y curves are equal for both layers; unit weight γ=18.0 kN/m^3, empirical model parameter J=0.25, and the strain corresponding to one half of the maximum principal stress difference y_{50}=0.005. The pile is loaded with a deterministic horizontal force of $H = 10.0$ MN and a moment of $M = 300.0$ MNm at the sea bed level.

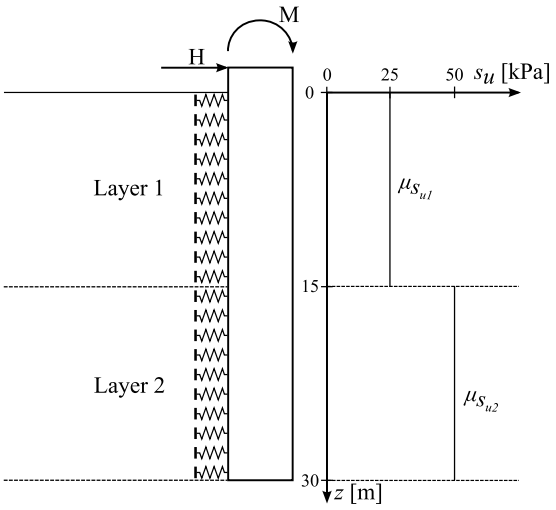


Figure 3: Laterally loaded pile with two soil layers.

The performance of the pile is classified as failure or safe by assessing plastification limit of the pile, defined by the moment $M_{lim} = f_y \cdot I/(D/2) = 397.176$ MNm, where I is a bending moment of a hollow circular cross section. The performance function has the following form:

$$g(\mathbf{s_u}, M_{lim}) = g(s_{u1}, s_{u2}, M_{lim}) = M_{lim} - M_{max} \tag{4}$$

where $\mathbf{s_u} = [s_{u1}, s_{u2}]^T$ is a vector defining shear strengths in layer 1 and 2 respectively, and M_{max} is a maximum bending moment along the pile for a given realization of the shear strength profile. Following the LSS procedure, the *pseudo pdf* has the following formulation:

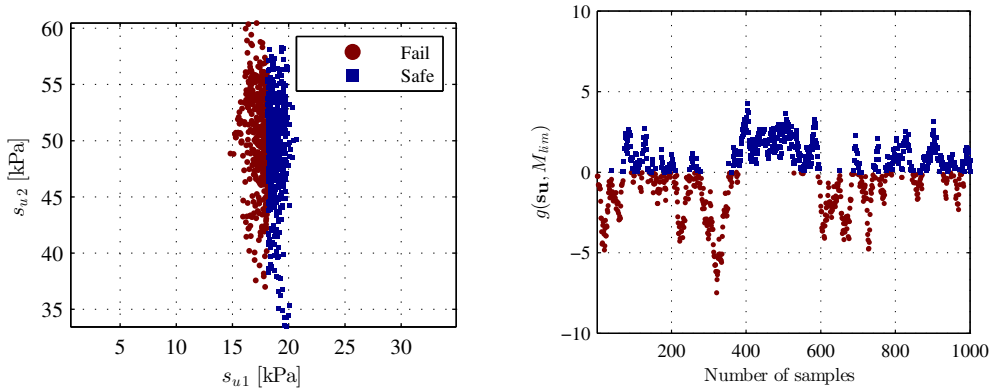
$$\varphi(g(\mathbf{s_u}, M_{lim})) = \exp(-a \cdot |g(\mathbf{s_u}, M_{lim})|) = \exp(-a \cdot |M_{lim} - M_{max}|) \tag{5}$$

where a is a shape parameter controlling the "width" of the *pseudo pdf* around the failure limit state. The *pseudo pdf*, defined in Equation 5, is sampled $g^{(k)} \sim \varphi(g(\mathbf{s_u}, M_{lim})); k = 1, ..., S$ by the MH algorithm with a uniform proposal distribution centered at the current state of the Markov Chain in the variable space, $\mathbf{s_u}^{(k-1)}$, with the

"width" defined by the parameter $\mathbf{h} = [h_1, h_2[^T$:

$$q(\cdot|\mathbf{s_u}^{(k-1)}) = U(\mathbf{s_u}^{(k-1)} - \mathbf{h}, \mathbf{s_u}^{(k-1)} + \mathbf{h}) \tag{6}$$

After the corresponding proposal state in the variable space is drawn $\hat{\mathbf{s}}_\mathbf{u} \sim q(\cdot|\mathbf{s_u}^{(k-1)})$, the proposal state $\hat{g}$ is generated by evaluating the performance function $\hat{g} = g(\hat{\mathbf{s}}_\mathbf{u}, M_{lim})$. The value of the shape parameter and the selection of the proposal distribution can be optimized so that the MH algorithm has good mixing properties as discussed in previous sections. Optimal selection of the shape parameter a in the *pseudo pdf* and the parameter h in the proposal *pdf* can significantly reduce the number of samples S necessary to adequately cover the failure limit state in the variable space. In Figure 4, $S = 10^3$ samples $g^{(k)} \sim \varphi(g(\mathbf{s_u}, M_{lim})); k = 1, ..., S$ with the corresponding $\mathbf{s_u}^{(k)}$ are presented. In Figure 4 (a), a scatter plot of simulation samples is presented where red color is associated with failed samples while blue color is associated with samples in the safe domain. It can be observed that the samples are distributed around the failure limit, following the formulation of the *pseudo pdf*. In Figure 4 (b), the value of the performance function with the number of samples is presented. It can be observed that the MH sampling procedure provides samples in a narrow band around the failure limit $g(\mathbf{s_u}, M_{lim}) = 0$ as expected from the formulation of the *pseudo pdf*.



(a) Scatter plot of s_{u1} and s_{u2} values.

(b) $g(\mathbf{s_u}, M_{lim})$ with the number of samples.

Figure 4: Simulation results for $a = 0.4$, $h_1 = 0.5$ and $h_2 = 2$.

After locating the failure limit state by a cloud of samples as presented in Figure 4 (a), a surrogate model of the performance function can be built. In this example the performance function is approximated by employing a k-Nearest Neighbor Classification (kNN) method from the group of statistical learning methods (Hastie et al., 2001). In general, the task of approximating the performance can be considered as a classification problem, if the samples in the failure domain are considered as one group or class, and the samples in the safe domain as the second group or class. In the problem of evaluating the P_F, the conversion to a classification problem is natural, as we are

only interested in the ratio of the samples in the failure domain over the total number of samples. The classification model (e.g., kNN) is predicting the class of a sample $\mathbf{s_u}^{(j)} \sim f(\mathbf{s_u}); j = 1, ..., K$ by utilizing the information obtained by sampling the *pseudo pdf*. The kNN method predicts the class of the sample of interest, $\mathbf{s_u}^{(j)}$, by locating and performing statistical analysis on k-nearest neighbors around the sample of interest. In this example, a formulation of the kNN algorithm where a class is predicted by a majority vote among 4 nearest neighbors with equal weights on all neighbors, is selected. After drawing $K = 10^6$ samples from $f(\mathbf{s_u})$ and performing classification by the kNN, the failure probability is estimated to be $\hat{P}_{F_{LSS}} = 8.29 \cdot 10^{-4}$.

The LSS approach is validated by performing a crude MC sampling on $N = 5 \cdot 10^5$ samples, $\mathbf{s_u}^{(i)} \sim f(\mathbf{s_u}); i = 1, ..., N$, to obtain a reasonably low coefficient of variation of $\hat{P}_F$. Failure probability is estimated by the MC method to be $\hat{P}_{F_{MC}} = 7.47 \cdot 10^{-4}$ and it agrees well with the estimated of P_F by the LSS method.

If it is noticed that the computational efforts necessary to perform the kNN classification on $K = 10^6$ samples are usually on a magnitude of one simulation of a structural model, the reduction in computational time of the LSS method when compared to the crude MC is approximately $N/S = 500$ times. Since the formulation of the *pseudo pdf* is independent of the formulation of the joint *pdf*, the reductions in computational efforts are expected to be higher in problems with lower $\hat{P}_F$ (e.g., 10^{-7},10^{-8}).

CONCLUSION

Estimation of the failure probability of structures by sampling methods can be a computationally challenging task if the failure probability is low and/or if the structural model is computationally complex. The Limit State Sampling approach has shown a potential of reducing the computational efforts in these situations by introducing a concept of the pseudo probability density function. Samples from the pseudo probability density function can be used to build an approximate model of the performance function around the failure limit, referred to as surrogate model. The surrogate model can be implemented by performing interpolation, regression or classification methods on samples from the pseudo probability density function to predict the structural performance at the failure limit. The surrogate model is selected in a such way that it requires significantly lower computational expense when compared to the original structural model.

Special attention has to be given to the formulation and sampling efficiency of the pseudo probability density function, since the quality and the accuracy of the surrogate models depends on the samples from the pseudo probability density function. The Limit State Sampling approach has shown significant reduction in computational efforts when applied in low-dimensional problems. Future research will be conducted on the applicability of the approach in higher dimensional problems, formulation of the pseudo probability density function, and selection of the methods for building the surrogate model.

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