Effect of spatial variability on failure mechanism location in random undrained slopes

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ABSTRACT: Since the charts of Taylor, it has been well known that the location of the critical failure mechanism in a homogeneous undrained clay slope goes either deep (tangent to a firm base) or shallow (through the toe) depending on whether the slope angle is, respectively, less than or greater than about 53^{o}. When slopes are made up of variable soils however, these expectations no longer hold true for all cases. In this paper, the influence of random soil strength and slope angle on the location of the critical failure mechanism and probability of failure is examined using the random finite element method (RFEM). It is found following Monte-Carlo simulation, that there exists a critical value of slope angle above which it would be unconservative to assume high spatial correlation length and below which it would be conservative to assume high spatial correlation length. For $\beta > 48^{o}$, both correlation length and slope angle have no influence on the proportion of toe failures. For slope angle lying between 20^{o} and 40^{o}, slopes that have higher correlation length give lower proportion of toe failures. Research into the critical mechanism location forms part of a broader study of slope failure risk, in which the consequences of failure are assumed to be more serious in a deep failure, because a greater volume of soil is affected.

1 INTRODUCTION

Landslide risk assessment involves the probability of failure and the consequences of failure (e.g. Christian 2004; Ang & Tang 2007; Fenton & Griffiths 2008). The probability of failure (p_f) can be estimated from engineering analysis but the consequences of failure are site-specific (e.g. loss of life, economics etc.). As shown by the two slopes in Figure 1, both cases have failed, but the consequences are clearly different. The deeper failure mechanism can be assumed to be more serious, because a greater volume of soil is involved.

Soil shear strength properties can vary significantly from point to point (e.g. Lacasse et al. 2013) and the Random Finite Element Method (RFEM) offers a systematic way of accounting for this kind of spatial variability. The RFEM, which combines finite elements with random fields generated to account for spatial variation, has been applied successfully to slope reliability analysis (Griffiths & Fenton 2000, 2004) and is now widely used by several groups. RFEM not only estimates the value of but also delivers useful visualisations of the failure mechanism and displacement vectors. A significant advantage of RFEM is that it allows failure mechanisms to develop naturally through soil masses by following the path of least resistance.

Following the work of Fenton et al. (2013) in which the focus was on the probability of failure, this paper investigated the influence of the slope angle on the location of the critical failure mechanism and further on the probability of failure of the undrained slopes at different correlation length and coefficient of variation of undrained shear strength.

The geometry and input parameters of a test slope of undrained clay are shown in Figure 2. The boundary conditions are given as rollers on the left and right vertical boundaries and full fixity at the base. The undrained shear strength, c_u, has been non-dimensionalised as $C = c_u / (\gamma_{sat} H)$ and is assumed to be a spatial random variable characterized by a lognormal distribution defined by a mean (μ_C), a standard deviation (σ_C) and a spatial correlation length $\theta_{\ln C}$.

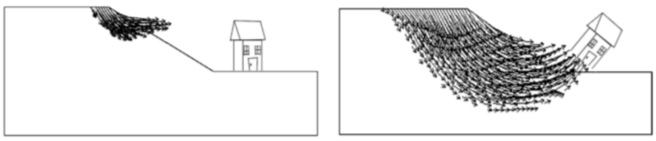

Figure 1. Shallow and deep failure mechanisms in a stratified soil. Both cases have failed, but the consequences are clearly different.

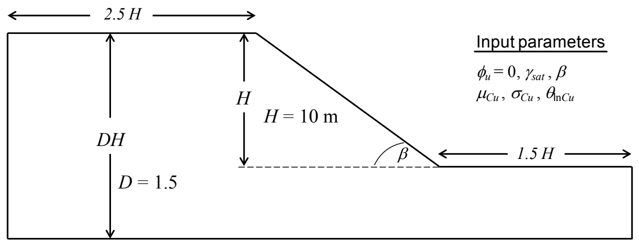

Figure 2. Geometry and input parameters for the test slope.

2 BRIEF INTRODUCTION TO THE RFEM

The lognormal distribution guarantees that all the random variables are positive and benefits from a simple transformation to the classical normal (Gaussian) distribution. It has been used and advocated by many researchers as a reasonable model for soil properties (e.g. Lacasse et al. 2013). The dimensionless coefficient of variation V_C, given by

$$V_C = \frac{\sigma_C}{\mu_C} \quad (1)$$

is a useful guide to the dispersion of the distribution about the mean.

In addition to μ_C, σ_C (and V_C), the spatial correlation length can be included to describe the correlation between random variables at two spatial locations. The correlation length denotes the distance over which random values tend to be correlated. In the interests of generality, the correlation length has been non-dimensionlised by dividing it by the slope height (see Figure 1) as follows:

$$\Theta = \theta_{\ln C} / H \quad (2)$$

It has been suggested that the typical range of V_C for undrained shear strength lies between 0.05 and 0.5 (e.g. Lacasse et al. 2013). The spatial correlation length is assumed isotropic throughout this paper and soil anisotropy is not considered (e.g. Zhu and Zhang, 2013). An exponential decaying autocorrelation function of the following form is used:

$$\rho = \exp(-2\tau / \theta_{\ln C}) \quad (3)$$

where ρ is correlation coefficient; τ is absolute distance between two points in a random field. The correlation function is merely a way of representing field observations where soil samples taken close together are more likely to have similar properties than if they are far apart.

Parameters such as the mean, variance and coefficient of variation at the point level are hard if not impossible to measure in practice. They represent the inherent variability of soil properties which can be corrected through local averaging to take account of sample size. In the context of RFEM, each finite element within each simulation of the Monte-Carlo process is assigned a constant soil property. The sample is represented by the size of finite element. If the distribution is normal, local averaging reduces the variance but the mean is unaffected. If a lognormal distribution is assumed, both the mean and the variance are reduced by local averaging. Adjustment to the statistics by local averaging should be implemented before each finite element analysis takes place.

The slope stability analyses use an elastic-perfectly plastic stress-strain law with a Mohr-Coulomb failure criterion. It involves applying gravity loading and monitoring the stresses at all the Gauss points. If the Mohr-Coulomb criterion at a Gauss point is violated, the algorithm will try to redistribute the stress to neighboring elements that still have reserves of shear strength. This is an iterative process that continues until both the Mohr-Coulomb criterion and the global equilibrium are satisfied at all Gauss points. In this study, failure is said to have occurred if the algorithm is unable to satisfy these criteria within an iteration ceiling (typically set to 500). Following a sufficient number of Monte-Carlo simulations, the p_f is obtained as the proportion of the total number of simulations that required 500 iterations or more.

In this paper, results of RFEM analyses are presented, demonstrating the influence of slope angle on the p_f and failure mechanisms of slopes in random undrained clay. A range of non-dimensionlised spatial correlation lengths and different V_C values are also considered. 2000 simulations are determined to be sufficient to give reliable and reproducible estimates of probability of failure in each analysis case.

3 RESULTS AND DISCUSSIONS

3.1 *Probabilistic stability analysis of the test slope (D = 1.5)*

Figure 3 demonstrates that a $\beta = 30^o$ slope (based on the mean) can result in a p_f as high as 0.38. It should be emphasized that a factor of safety based on the mean strength of a variable soil will generally lead to optimistic estimates of the factor of safety. A lower value of the characteristic strength would typically be used in practice as discussed by Griffiths and Fenton (2004). Figure 3 also demonstrates that the slope will be essentially safe when FS>1.38 and FS>1.57 for $V_C = 0.2$ and 0.4, respectively.

Figure 4 shows the influence of slope angle on the p_f of a slope with $D = 1.5$, $\mu_C = 0.2$, $V_C = 0.2$. The influence of the correlation length on the p_f versus β relationship is also demonstrated. There clearly exists a cross-over point, previously noted by Griffiths and Fenton (2004), corresponding to approximately $\beta = 48^o$ at which p_f starts to decrease with increasing correlation length. It is interesting to note the step function that corresponds to $\Theta = 0$ where p_f at $\beta = 63^o$ suddenly changes from zero to one. When Θ equals zero, all the variances are removed and the problem becomes deterministic with a uniform strength given by the Median. It is obtained analytically that slope of $\beta < 63^o$ will have a FS greater than one ($p_f = 0$). It seems that the value of the slope angle corresponding to 63^o lies to the right of the crossover point. Results show that when $\beta < 48^o$, perfect correlation tends to overestimate p_f which is conservative for engineering design, while the opposite effect is observed for $\beta > 48^o$ which is unconservative.

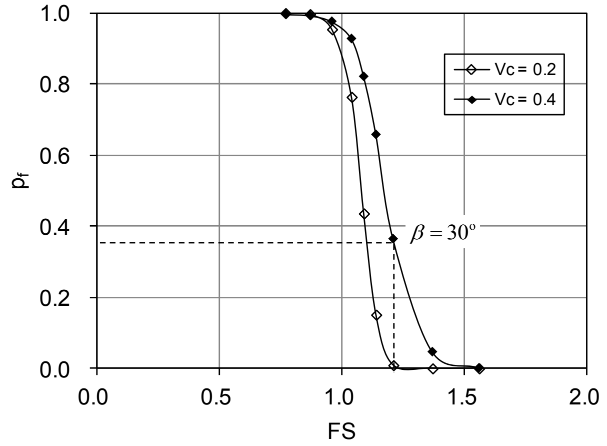

Figure 3. Probability of failure versus factor of safety (based on the mean) of undrained slopes with D = 1.5, μ_C = 0.2, Θ_{lnC} = 0.5.

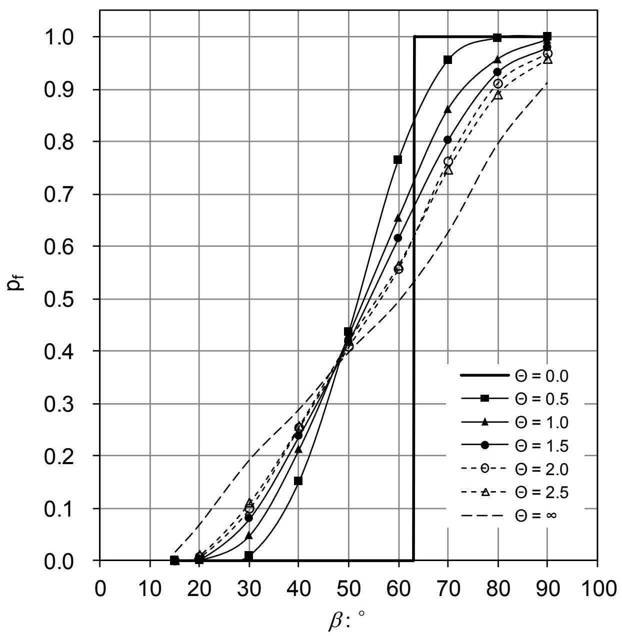

Figure 4. Influence of slope angle on the probability of failure of undrained slopes; D = 1.5, μ_C = 0.2, V_C = 0.2.

Figure 5 contains the same information as that in Figure 4, but is arranged in a different way with the correlation length along the horizontal axis. This figure shows two tails, with p_f tending to one as Θ decreases for all the slope angles greater than about 48^o and tending to zero with decrease in Θ for slope angle less than about 48°, which emphasises the importance of the slope angle in the relationship between p_f and Θ.

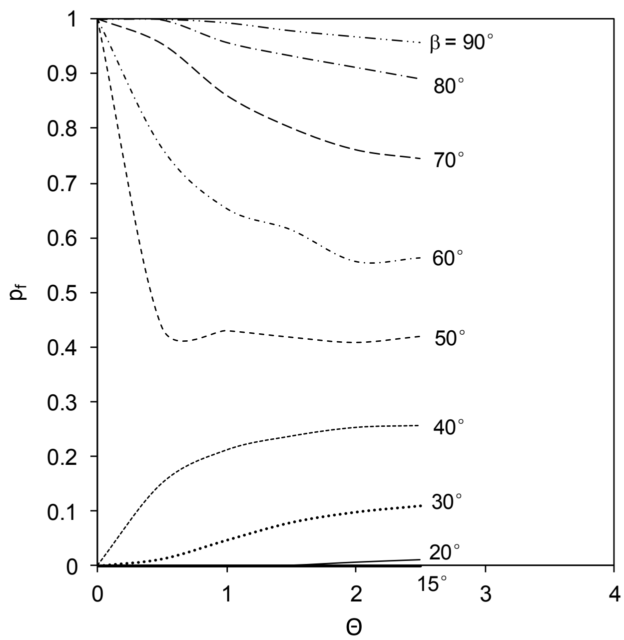

Figure 5. Influence of slope angle on the probability of failure of undrained slopes; D = 1.5, μ_C = 0.2, V_C = 0.2.

3.2 *Influence of slope angle on the failure mechanism location (D = 1.5)*

Figure 6 shows undeformed and deformed meshes at failure for several combinations of parameters. Unlike deterministic case, when the slope consists of spatially random soil, a vertical cut may display a deep mechanism (Figure 6(g)) and a flat slope may display a shallow mechanism (Figure 6(h)). Following Monte-Carlo simulation which involves 2000 realizations for each parametric combination, the proportion of toe failures is defined as the number of toe failures divided by the total number of failure simulations.

Figure 7 presents the influence of slope angle on the proportion of toe failures affected by different correlation length. For $\beta > 48^o$, both correlation length and slope angle have no influence on the proportion of toe failures. For slope angle lying between 20^o and 40^o, slopes that have higher correlation length give lower proportion of toe failures. This is due to that higher correlation length implies a more statistically homogeneous field, so that most of the pattern of mechanism is deep, closer to the deterministic case.

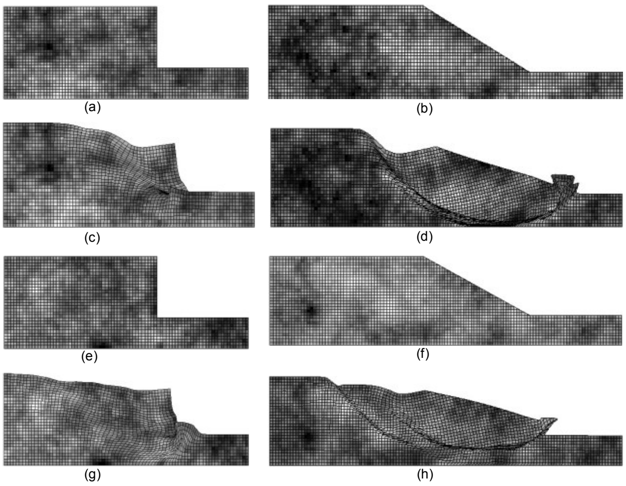

Figure 6. Failure mechanism of undrained slope with fixed C = 0.2 and Θ = 0.5; undeformed mesh: (a) $\mu_C = 0.2$, $V_C = 0.2$, $\beta = 90^o$, (b) $\mu_C = 0.2$, $V_C = 0.2$, $\beta = 30^o$; deformed mesh at failure: (c) $\mu_C = 0.2$, $V_C = 0.2$, $\beta = 90^o$, (d) $\mu_C = 0.2$, $V_C = 0.2$, $\beta = 30^o$; undeformed mesh: (e) $\mu_C = 0.2$, $V_C = 0.3$, $\beta = 90^{o\circ}$, (f) $\mu_C = 0.2$, $V_C = 0.3$, $\beta = 30^o$; deformed mesh: (g) $\mu_C = 0.2$, $V_C = 0.3$, $\beta = 90^o$, (h) $\mu_C = 0.2$, $V_C = 0.3$, $\beta = 30^o$.

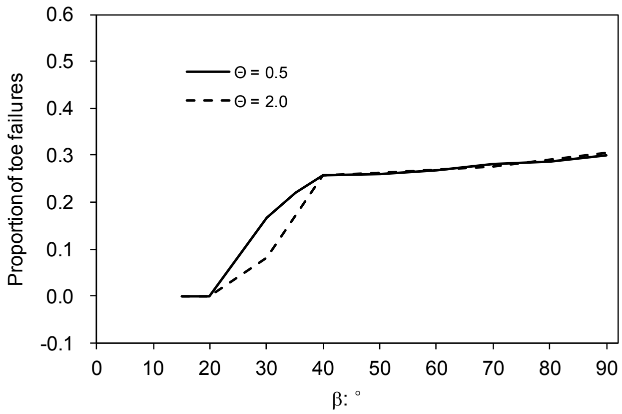

Figure 7. Influence of slope angle on the proportion of toe failures of undrained slopes with $D = 1.5$, $\mu_C = 0.2$, $V_C = 0.2$.

4 CONCLUDING REMARKS

This paper investigates the p_f and failure mechanism location of undrained slopes with spatially random soil. The following conclusions can be drawn.

The influence of slope angle on p_f indicated a critical value of slope angle above which it would be unconservative to assume high spatial correlation length and below which it would be conservative to assume high spatial correlation length.

The influence of slope angle on the proportion of toe failures is affected by different correlation length. For $\beta > 48^o$, both correlation length and slope angle have no influence on the proportion of toe failures. For slope angle lying between 20^o and 40^o, slopes that have higher correlation length give lower proportion of toe failures. This is due to that higher correlation length implies a more statistically homogeneous field, so that the pattern of mechanism is deep, closer to the deterministic case.

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