Influence of different input distributions on probabilistic outcomes in geotechnical stability analysis

Influence des différentes distributions d'entrée sur les résultats statistiques des analyses de stabilité géotechnique

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ABSTRACT Reliability analysis has gained considerable popularity in practice and academe as a way of quantifying and managing geotechnical risk in the face of uncertain input parameters. The user is often required to choose a probability density function for characterizing random input without a great deal of data to support the decision. Normal and log-normal distributions are both common choices that have been applied to geotechnical problems. This paper uses the random finite element method (RFEM) to study the influence of two different input random variable distribution functions on the probability of failure of stability problems involving slopes and block compression problems. The paper presents the results of parametric studies using these distributions and makes some conclusions on their advantages and disadvantages.

RÉSUMÉ L'analyse de fiabilité a acquis une popularité considérable, sur le terrain comme dans le monde universitaire, en tant que moyen de quantifier et gérer les risques géotechniques en cas de paramètres d'entrée incertains. Il est souvent demandé à l'utilisateur de choisir une fonction de densité de probabilité pour caractériser une entrée aléatoire en l'absence d'un grand nombre de données pour appuyer la décision. Les distributions normales et log-normale sont deux choix communs qui ont été appliquées à des problèmes géotechniques. Ce document utilise la méthode des éléments finis aléatoire (RFEM) pour étudier l'influence de deux fonctions différentes de distribution de variables aléatoires sur la probabilité de défaillance des problèmes de stabilité impliquant des problèmes de pentes et de compression de blocs. Le document présente les résultats d'études paramétriques utilisant ces distributions et rend des conclusions sur leurs avantages et leurs inconvénients.

1 INTRODUCTION

In geotechnical engineering there are very few quantities than can be precisely estimated. Hence, probabilistic analysis has been used by engineering and researchers for analyzing different geotechnical problems. Starting in the early 90's, a new method called the Random Finite Element Method (RFEM), which combines random field theory and the finite element method, was developed for use in probabilistic geotechnical engineering (e.g. Griffiths and Fenton 1993). The method was subsequently applied to several areas of geotechnical engineering including probabilistic slope stability analysis by Griffiths and Fenton 2000, 2004. The Local Average Subdivision method (LAS) proposed by Fenton and Vanmarcke (1990) was used for generating the random fields. It was shown that traditional probabilistic analyses, in which spatial variability is ignored by implicitly assuming perfect correlation, does not necessarily result in a conservative estimates of the probability of failure (e.g. Griffiths et al. 2002).

In probabilistic analysis, user is often required to choose a proper distribution function for input variables. Normal and lognormal distributions are two of most popular choices in geotechnical engineering field for shear strength properties of soil. This paper investigates the influence of normal and lognormal input random variable distribution functions on probability of failure of slope stabilities and a 2D soil block compression problem.

The shear strength parameters of the soil c' and $\tan\phi'$ are treated as random variables for both problems and characterized statistically by both normal and lognormal distributions separately. c' is expressed in dimensionless form as C' where $C' = c'/(\gamma H)$ for the slope problem where H is the height of the slope and γ represent the unit weight of the soil.

In the lognormal distribution, the logarithms of the properties are normally distributed (e.g. Fenton and Griffiths, 2008). Lognormal distributions guarantee that the random variable will never have negative values.

Both distributions are defined by a mean μ and a standard deviation σ. The probability density function of C' is given by Equation 1 and 2 for normal and lognormal distribution respectively. The same equations were used for $\tan\phi'$.

$$f(C') = \frac{1}{\sigma_{C'}\sqrt{2\pi}} exp\left\{-\frac{1}{2}\left(\frac{C'-\mu_{C'}}{\sigma_{C'}}\right)^2\right\} \quad (1)$$

$$f(C') = \frac{1}{C'\,\sigma_{\ln C'}\sqrt{2\pi}} exp\left\{-\frac{1}{2}\left(\frac{\ln C'-\mu_{\ln C'}}{\sigma_{\ln C'}}\right)^2\right\} \quad (2)$$

The mean and standard deviation can conveniently be combined in terms of the dimensionless coefficient of variation V defined as:

$$V = \frac{\sigma_{C'}}{\mu_{C'}} = \frac{\sigma_{\tan\phi'}}{\mu_{\tan\phi'}} \quad (3)$$

2 RANDOM FINITE ELEMENT METHOD (RFEM)

The RFEM implementation used in this study combines elastic-plastic finite element analysis with random field theory in the block compression and slope stability analysis. The methodology has been described in details elsewhere (Fenton and Griffiths, 2008).

The RFEM is used in conjunction with Monte Carlo simulations in which the stability analysis is repeated until the probabilities relating to output quantities of interest become statistically reproducible. In the case of slope stability analysis, the probability of failure is defined by dividing the number of realizations in which the slope failed by the total number of realizations. For the block problem, which is a 2D model of a square soil mass under pressure on the top, the average compressive strength for all Monte-Carlo realizations is calculated.

2.1 Spatial correlation

Generally, the mean and standard deviation of a variable are well understood by engineers. However, the spatial correlation length θ of a random property is less well understood. This property, called the “scale of fluctuation” or “spatial correlation length”, has units of length, and represents the distance over which the soil or rock property in question is reasonably well-correlated to its neighbors. In this study, a “Markovian” correlation function is used where the spatial correlation is assumed to decay exponentially with distance (Vanmarcke 1984).

$$\rho = e^{-2|\tau|/\theta} \quad (4)$$

In Eq (4) which is for an isotropic material, τ is the absolute distance between any two points in the random field, and ρ is the correlation coefficient between properties assigned to two points in the random field separated by τ. In this study, the spatial correlation length has been non-dimensionalized by dividing it by the height of the slope H and block B for the slope and block problems respectively. The non-dimensionalized spatial correlation length is presented as Θ in this paper.

3 RFEM RESULTS

The results of the RFEM analysis for the block compression problem and drained slope are presented in this section.

3.1 Block compression

As a simple geotechnical probabilistic problem, a block compression model has been analyzed using the RFEM. The block has an equal width and height represented by B and equal to 1. The generation and mapping of random field variables including c' and $\tan\phi'$ properties onto a finite element mesh were performed. Both parameters have the same Θ. RFEM takes full account of local averaging and variance reduction over each element, and the exponentially decaying spatial correlation function was incorporated. An elastic-plastic finite element analysis using the Mohr-Coulomb failure criterion was then performed. Axial loading was then applied to the mesh until a maximum failure stress, q_u was reached. Using Monte-Carlo simulations this procedure was repeated 2000 times. Although each simulation of the Monte-Carlo process involves the same mean, standard deviation and spatial correlation length of soil properties, the spatial distribution of properties varies from one realization to the next which led to a different value for q_u for each simulation. Following a sufficient number of realizations, the statistics including mean and standard deviation of the output quantity q_u were computed. Figure 1 shows a typical failure mechanism with the soil's cohesion distribution in the form of a grey scale in which weaker regions are lighter, and stronger regions are darker. The soils properties c' and $\tan\phi'$ were assumed to be uncorrelated to each other.

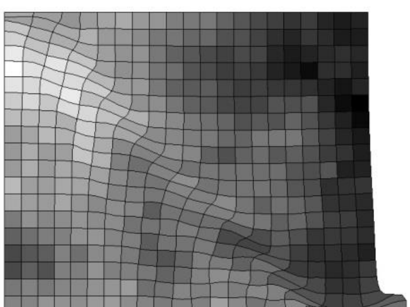

Figure 1. Typical random field realization and failure mechanism for the block compression problem with Θ=5.

A series of analyses were performed in which the mean of c' and $\tan \phi'$ were kept constant and equal to $\mu_{c'}$ = 100kPa and $\mu_{\tan \phi'}$ = tan 30° = 0.577. The coefficient of variation, V was 0.3 and 0.5 for both random variables while the spatial correlation length Θ was varied. Both normal and lognormal distributions were applied on input random variables distributions. The mean compressive strength μ_{qu}, was calculated by averaging the compressive strength values for all 2000 Monte-Carlo simulations and normalized with respect to the deterministic value q_u ($\mu_{c'}$, $\mu_{\tan \phi'}$) = 346.5 based on the mean values of input parameters using Eq 4 (Griffiths et al. 2002).

$$q_u = 2c' \tan\left(45 + \frac{\phi'}{2}\right) = 2c'[\tan \phi' + (1 + \tan^2 \phi')^{0.5}] \qquad (4)$$

The result of these analyses has been plotted in Figure 2 for 4 different analyses. As it can be seen in this picture by increasing V, regardless of the spatial correlation length, μ_{qu} decreases. There is also a minimum μ_{qu} value when Θ = 0.2 for the both normal and lognormal cases. This is similar to what Griffiths et al. (2002) found for a pillar stability problem. Another observation from this graph is that when V = 0.3, normal and lognormal distribution results in the same values for μ_{qu}. However, by increasing the V to 0.5 there is a difference in μ_{qu} values for these two cases. This difference reaches to the maximum value when Θ has small or intermediate values; however, by increasing the Θ the μ_{qu} converge to similar values.

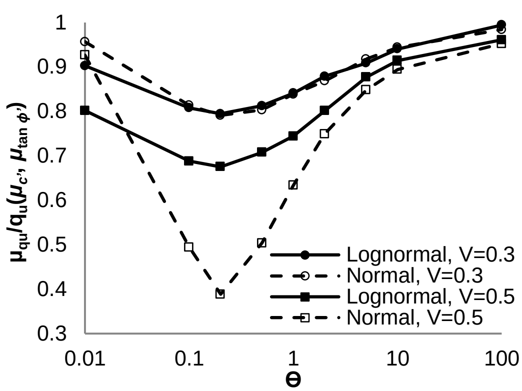

Figure 2. Variation of normalized μ_{qu} with Θ and V with normal and lognormal distributions for input variables.

A reason for this difference could be that by increasing coefficient of variation, the standard deviation increases, therefore, some values for the input variables with normal distribution became negative. For example, for V = 0.3, 0.04% of each input values are negative, however when V changes to 0.5, this number increases to 2.27%. However, these values are small and cannot be the main reason for different outputs of normal and lognormal distributions. Figure 3 indicates the distribution of the c' values when V is equal to 0.1, 0.3 and 0.5. As it has been shown it this figure, by increasing the coefficient of variation, the normal and lognormal input functions result in more different distributions. This explains the different values of μ_{qu} for the normal and lognormal cases, especially when V=0.5

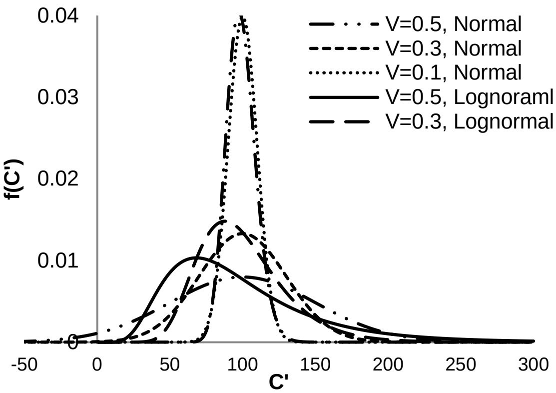

Figure 3. Normal and lognormal distribution of *c'*.

3.2 Slope stability analysis

The slope studied in this paper is shown in Figure 4 with consideration of a drained slope. The slope inclination, height, and foundation ratio is given by β, H and D respectively.

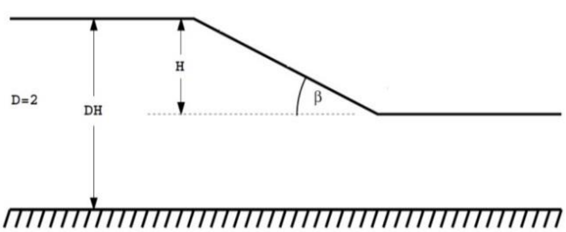

Figure 4. Slope profile.

In this study, H and β are equal to 10 and 26.6° respectively and the $\mu_{C'}$ and $\mu_{\tan\phi'}$ of the soil has been chosen to be 0.025 and tan 20° = 0.364. C' and tan ϕ' have considered non-correlated to each other. The slope has been modeled with 4 different coefficients of variation equal to 0.3, 0.4 and 0.5. Θ has varied from 0.01 to 100 for each slope. Both normal and lognormal distributions were taken into account for these analyses. Figure 5 illustrates the meshing and a typical failure of the drained slope.

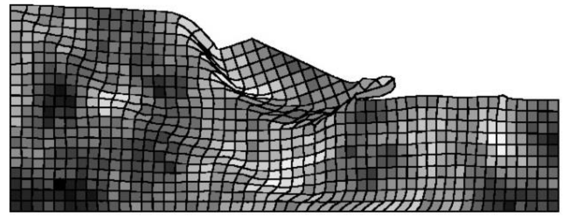
Figure 5. Typical random field realization and failure mechanism for the slope stability analysis with Θ=1.

Figures 6 to 8 show the probability of failure p_f versus non-dimensionless spatial correlation length Θ for the slope with different coefficient of variation for each figure. The spatial correlation lengths in the both X and Y directions are equal. Each figure has two graphs one corresponding to a slope stability analysis when the input random variables C' and tan ϕ' have normal distribution and another with lognormal distribution.

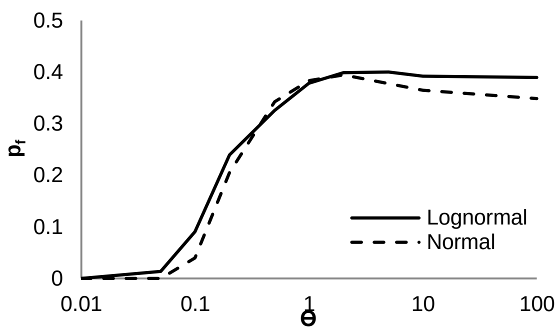

Figure 6. Variation of p_f with Θ for V = 0.3 with normal and lognormal distributions for input variables.

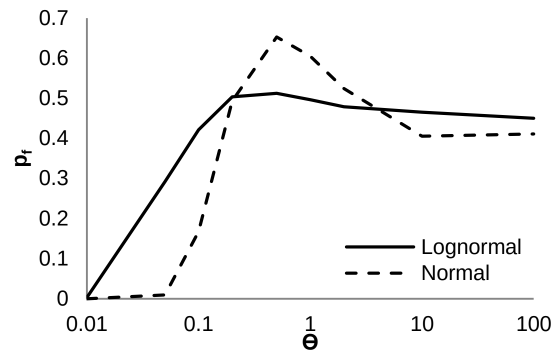

Figure 7. Variation of p_f with Θ for V = 0.4 with normal and lognormal distributions for input variables.

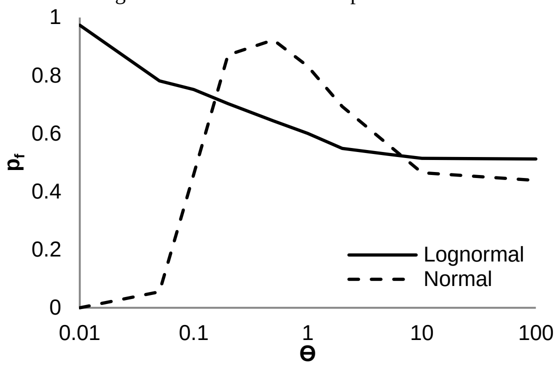

Figure 8. Variation of p_f with Θ for V = 0.5 with normal and lognormal distributions for input variables.

Figure 6 shows that when the coefficient of variation is equal to 0.3 input variables with normal and lognormal distributions result in almost a same probability of failure for the slope. By increasing the V to 0.4 there is a difference in the results of two analyses with different input distribution density functions. This difference decreases by increasing the spatial correlation length. As it discussed for the block problem, different distribution functions of C' and tan ϕ' in normal and lognormal distribution cases (Figure 3) can result in different results in finite element and consequently different probability of failure for the slope stability analysis.

For the slope with V = 0.5 for C' and tan ϕ', on the other hand, this difference is more considerable. For the small values of Θ this difference is the maximum. This incompatibility is more visible when the Θ is 0.01 in which the normal distribution results in $p_f = 0$ while with the lognormal distribution the probability of failure is equal to 1. When Θ is small, the slope becomes increasingly uniform with essentially constant strength at each simulation. For the slope with the normal distribution function these constant values tend to the mean of C' and tan ϕ', however, in the lognormal distribution the values tend to the median of input random variables (Griffiths and Fenton 2004). The means of C' and tan ϕ' result in a Factor of Safety (FS) of 1.1 for the slope, therefore the probability of failure of the slope is equal to 0 for the normal distribution case. On the other hand, FS is calculated to be essentially 1.0 with the median values of the soil strength parameters, hence $p_f = 1$.

4 CONCLUSIONS

This paper has studied the probability of failure for a block compression problem and drained slope stabilty using the RFEM. The influence of the coefficient of variation V, spatial correlation length Θ, and the choice of input random variable distribution functions (normal or lognormal) on the probability of failure p_f of the block compression and slope stability problem was studied. For both the block and slope problems results from both distributions were similar when the coefficient of variation was small band increasingly differed as the coefficient of variation increases. This is to be expected from the nature of the normal and lognormal distributions. Another reason which could have a small impact on the different behavior with normal and lognormal distribution functions, is that with a higher coefficient of variations, a small percentage of the input variables in the normal case will have negative values. The biggest differences between the results of normal and lognormal distributions occurred at low spatial correlation lengths, when local averaging resulted in properties being “safe” with the normal distribution and “unsafe” with the lognormal. This paper concludes that users should give careful consideration to the choice of input probability density functions for geotechnical analysis. Distributions should be based on field data if available, but if that is not available, a conservative approach should always be followed.

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