The Random Finite Element Method (RFEM) in Probabilistic Slope Stability Analysis with Consideration of Spatial Variability of Soil Properties

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ABSTRACT: This paper presents the results of probabilistic analyses in slope stability problems using the Random Finite Element Method (RFEM). The influence of spatially variable soil properties on design outcomes relating to slope stability analysis has been assessed through parametric studies, with focus on the "worst case" (critical) spatial correlation length that leads to a minimum reliability of the soil mass. This critical value is of particular interest, because it could be used for design in the absence of good site specific data.

INTRODUCTION

Probabilistic methods have been used in slope stability analysis since 1970s, and have received considerable attention in the literature. Starting in the early 90's, a new method called the Random Finite Element Method (RFEM), which combines random field theory and the finite element method, was developed for use in probabilistic geotechnical engineering (e.g. Griffiths and Fenton 1993). The method was subsequently applied to several areas of geotechnical engineering including probabilistic slope stability analysis by Griffiths and Fenton 2000, 2004. The Local Average Subdivision method (LAS) proposed by Fenton and Vanmarcke (1990) was used for generating the random fields. It was shown that traditional probabilistic analyses, in which spatial variability is ignored by implicitly assuming perfect correlation, does not necessarily result in a conservative estimates of the probability of failure. Later on, Griffiths et al. (2009) studied the influence of spatial variability of soils more precisely.

One of the advantages of using elastic-plastic finite elements for stability analysis is that the failure mechanism is allowed to find the weakest path through the soil. The ability of the FE approach to model the shape and location of the failure mechanism offers many benefits over traditional methods in which the shape of the failure mechanism is fixed a priori. Slope stability analysis is a good example this, in which commonly used methods such as Bishop's method, require the failure mechanism to be circular.

This paper investigates the influence of soil spatial correlation length on probabilistic slope stability analysis. Numerical results show that for a given value of the coefficient of variation of soil strength parameters, there is a critical value of the spatial correlation length which leads to a minimum reliability of the soil mass. In other words, if spatial variation is ignored or implicitly assumed to be infinite, the probability of failure can be underestimated resulting in an unconservative design.

The slope studied in this paper is shown in Figure 1 with consideration of both undrained $\phi_u = 0$, c_u and drained c', $\tan\phi'$ slopes. The slope inclination, height, and foundation ratio is given by β, H and D respectively. The saturated unit weight of the soil, γ are held constant, while the shear strength c_u of the undrained slope and c' and $\tan\phi'$ of the drained slope are assumed to be random variables. For the both drained and undrained slopes, c_u and c' were expressed in dimensionless forms as C_u and C' respectively where $C_u = c_u/(\gamma_{sat} H)$ and $C' = c'/(\gamma H)$.

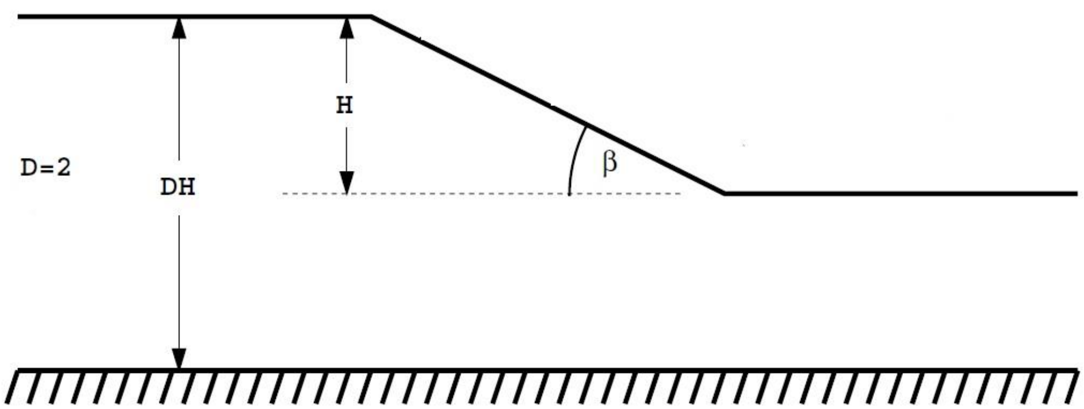

FIG. 1. Slope profile

The shear strength parameters of the soil C_u and C' and $\tan\phi'$ are treated as random variables, characterized statistically by lognormal distributions; in other words, the logarithms of the properties are normally distributed. The lognormal distribution is one of many possible choices (e.g. Fenton and Griffiths, 2008) that has been advocated and used by several other investigators as a reasonable model for variable soil properties (e.g. Massih et al., 2008). Lognormal distributions guarantee that the random variable will never have negative values.

The lognormal distribution is defined by a mean μ and a standard deviation σ. The probability density function of C_u is given by Equation 1 and an equivalent equation is applied to C' and $\tan\phi'$.

$$f(C_u) = \frac{1}{C_u\,\sigma_{\ln C_u}\sqrt{2\pi}} exp\left\{-\frac{1}{2}\left(\frac{\ln C_u - \mu_{\ln C_u}}{\sigma_{\ln C_u}}\right)^2\right\} \tag{1}$$

The mean and standard deviation can conveniently be combined in terms of the dimensionless coefficient of variation defined as:

$$V_{C_u} = \frac{\sigma_{C_u}}{\mu_{C_u}} \tag{2}$$

RANDOM FINITE ELEMENT METHOD

The RFEM implementation used in this study combines elastic-plastic finite element analysis with random field theory in slope stability analysis. The methodology has been described in details elsewhere (e.g. Fenton and Griffiths, 2008).

The RFEM is used in conjunction with Monte Carlo simulations in which the stability analysis is repeated until the probabilities relating to output quantities of interest become statistically reproducible. In the case of slope stability analysis, the probability of failure is defined by dividing the number of realizations in which the slope failed by the total number of realizations.

Spatial Correlation

Generally, the mean and standard deviation of a variable are well understood by engineers. However, the spatial correlation length θ of a random property is less well understood. This property, called the "scale of fluctuation" or "spatial correlation length", has units of length, and represents the distance over which the soil or rock property in question is reasonably well-correlated to its neighbors. In this research, a "Markovian" correlation function is used where the spatial correlation is assumed to decay exponentially with distance (Vanmarcke 1984).

$$\rho = e^{-2|\tau|/\theta} \tag{3}$$

In Eq (3) which is for an isotropic material, τ is the absolute distance between any two points in the random field, and ρ is the correlation coefficient between properties assigned to two points in the random field separated by τ.

Since the actual undrained shear strength field is lognormally distributed, its logarithm yields an underlying normal distributed (or Gaussian) field. The spatial correlation length is measured with respect to this underlying field, that is, with respect to $\ln C_u$. In particular, the spatial correlation length ($\theta_{\ln Cu}$) describes the distance over which the spatially random values tend to be significantly correlated in the underlying Gaussian field. Thus, a large value of $\theta_{\ln Cu}$ implies a smoothly varying field, while a small value will imply a ragged field. In this study, the spatial correlation length has been non-dimensionalized by dividing it by the height of the slope H and will be expressed in the form:

$$\Theta_{C_u} = \frac{\theta_{\ln C_u}}{H} \tag{4}$$

The influence of θ on a wide range of geotechnical systems has been assessed through parametric studies (e.g. Griffiths and Fenton 2004, Griffiths et al. 2009, Huang et al. 2010, Kasama and Whittle 2011, Al-Bitar and Soubra 2013) and has been shown to have a significant influence on probabilistic quantities under considerations.

Figure 2 (a and b) indicate two random field realizations and the associated failure mechanisms. Figure 2(a) shows a relatively low spatial correlation length of $\Theta_{C'}$=0.5 and Figure 2(b) shows a high spatial correlation length of $\Theta_{C'}$=100. The figures depict the variation in lnC' and have been scaled in such a way that the dark and light regions depict "strong" and "weak" soils, respectively. Black represents the strongest element and white is the weakest in a particular realization. Although both cases shown in Figure 2 had the same mean and variance, the different spatial correlation lengths have led to quite different failure characteristics. In the case with a high spatial correlation length, a much smoother mechanism was observed, more like the type of mechanism that would be observed in a homogeneous soil.

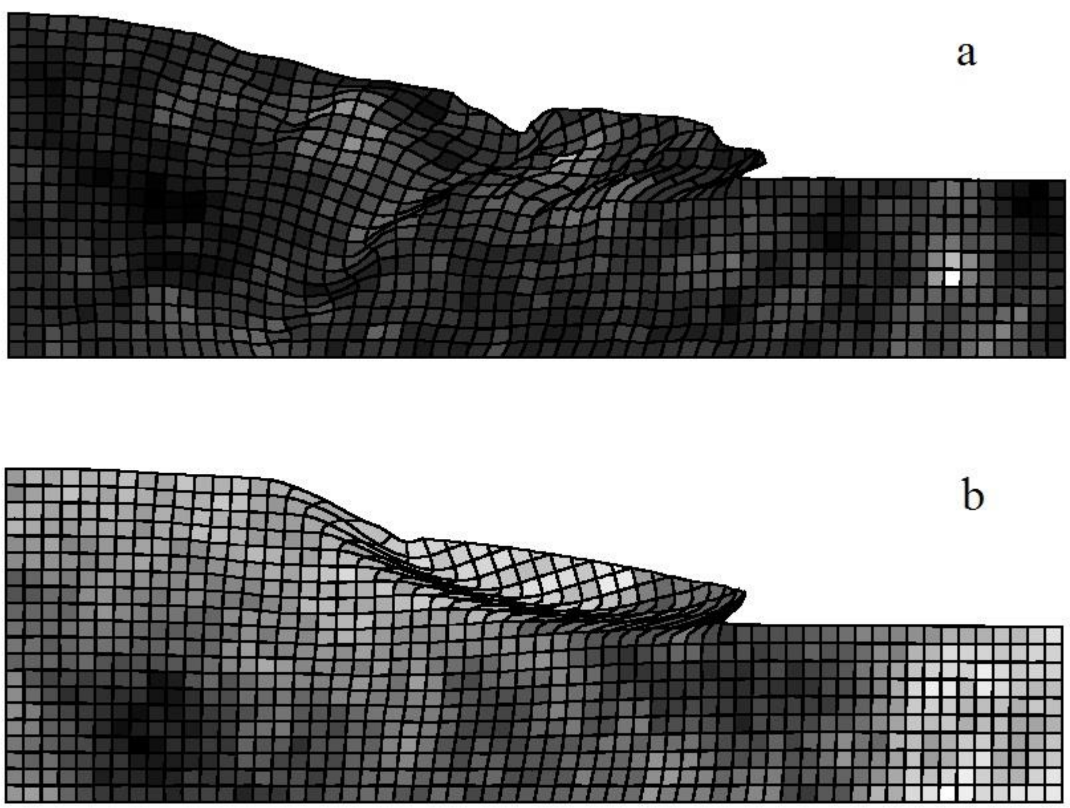

FIG. 2. Typical random field realizations and deformed mesh at slope failure for two different spatial correlation length, a) $\Theta_{C'}$ =0.5, b) $\Theta_C{}'$ =100

RFEM RESULTS

The results of the RFEM analysis for the undrained and drained slopes are presented in this section. Using this method, random fields are generated and assigned to each element. Gravity loads are then applied, and if the algorithm could not converge within a specific number of iterations, failure is said to have occurred. Lack of convergence means that no stress redistribution could be found that is simultaneously able to satisfy both the Mohr Coulomb failure criterion and global equilibrium. The analysis is repeated numerous times, using Monte Carlo simulations, using the same mean, standard deviation, and spatial correlation length of soil properties. The spatial distribution of properties, however, varies from one realization to the next. Following a "sufficient" number of realizations, the probability of failure p_f is estimated by dividing the number of failures by the total number of simulations.

A typical finite element mesh used for this problem is shown in Figure 3. The majority of the elements are 8-node square except the elements adjacent to the slope which are degenerated into triangles. The slope model has 910 total elements which results in 910 random variables for the undrained slope with C_u and 1820 random variables for the drained slope with C' and $\tan\phi'$ in each simulation. 4000 simulations were used to ensure the reproducibility of the model.

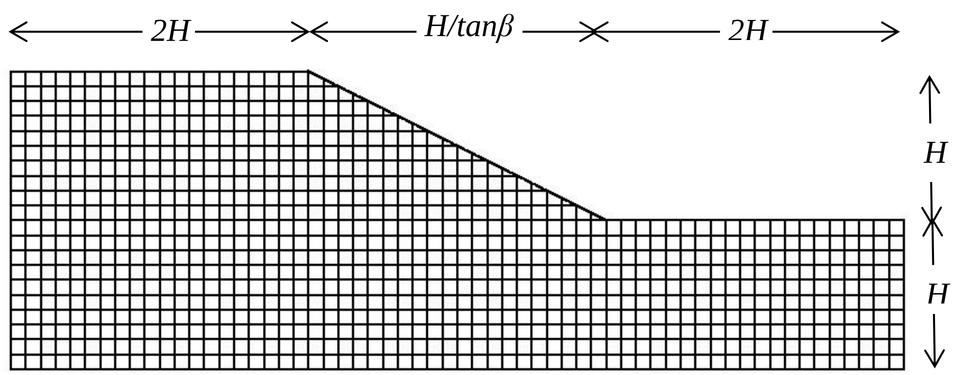

FIG. 3. Typical mesh used for the RFEM slope stability analysis

Table 1 and 2 show the strength parameters and dimensions of the undrained and drained slopes respectively. Young's modulus (E) and Poisson ratio (υ) are set to 10^5 (kPa) and 0.3 respectively for all analyses. The unit weight of the soil is also considered as a deterministic parameter equal to 20 (kN/m^3).

The value of μ_{Cu} and V was fixed at 5 (kPa) and 0.3 respectively for the undrained slope. The 0.3 value for the coefficient of variation was selected, as the typical V values for undrained shear strength lie in the range 0.13-0.5 (e.g. Duncan 2000). Four slopes with different slope angles were modeled with different spatial correlation lengths to study the effect of soil spatial variability and slope angle on the probability of failure of slopes. The height of the slope, H is equal to 1. Figure 4 illustrates the variation of the probability of failure with non-dimensionalized spatial correlation length and slope angle for the undrained slope.

Table 1. Parameter used in this study for the undrained slope. The coefficient of variation was kept constant, $V = \sigma_{Cu}/\mu_{Cu} = 0.3$.

μ_{Cu} (kPa)	β (slope angle)	$\Theta_{Cu} = \theta/H$
5	26.6	0.1, 0.25, 0.5, 1, 2, 4, 8
5	45	0.1, 0.25, 0.5, 1, 2, 4, 8
5	55	0.1, 0.25, 0.5, 1, 2, 4, 8
5	63.5	0.1, 0.25, 0.5, 1, 2, 4, 8

Table 2. Parameter used in this study for the drained slope. Slope angle is kept constant to, β = 26.6° (2:1 slope).

$\mu_{C'}$ (kPa)	$\tan\phi'$	$V = \sigma_{C'}/\mu_{C'} = \sigma_{\tan\phi'}/\mu_{\tan\phi'}$	$\Theta = \theta/H$
5	0.364	0.25	0.01, 0.05, 0.1, 0.2, 0.5, 1, 2, 10, 100
5	0.364	0.3	0.01, 0.05, 0.1, 0.2, 0.5, 1, 2, 10, 100
5	0.364	0.35	0.01, 0.05, 0.1, 0.2, 0.5, 1, 2, 10, 100
5	0.364	0.45	0.01, 0.05, 0.1, 0.2, 0.5, 1, 2, 10, 100

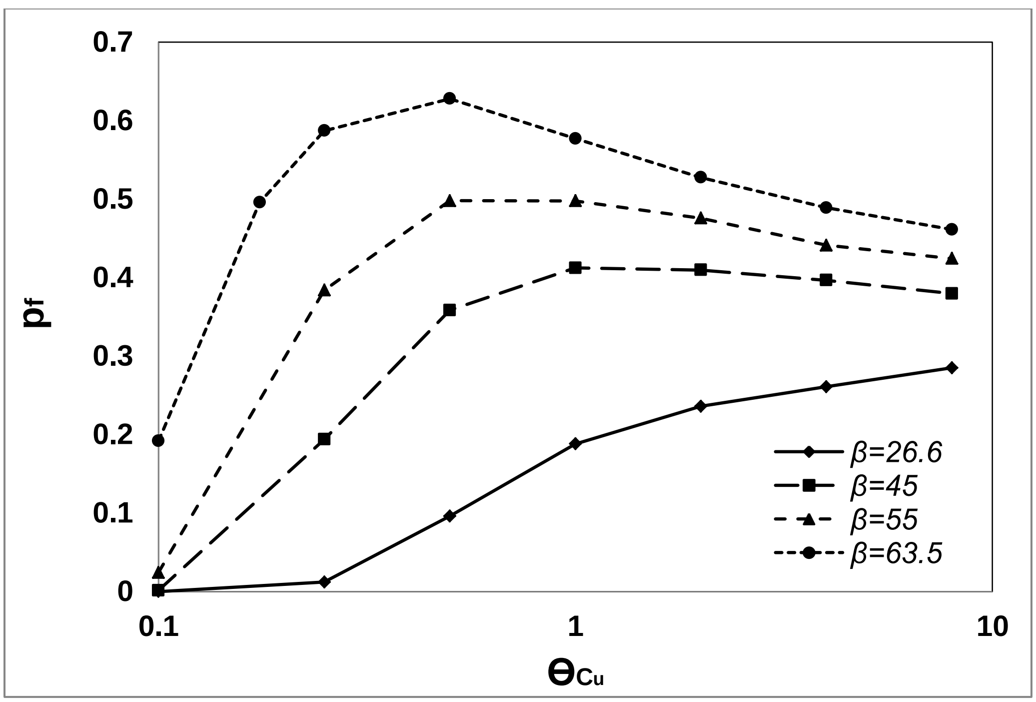

FIG. 4. Probability of failure vs. spatial correlation length for different slope angles for the undrained slope, $\phi_u = 0$.

As it can be seen in Figure 4, by increasing the slope inclination, the probability of failure for the slope increases. This fact was expected based on traditional theories.

An important observation from Figure 4 is the influence of spatial correlation length on probability of failure of the slopes. By increasing the spatial correlation length for the 2:1 (β = 26.6°) slope, the p_f increases. This observation was addressed in previous studies on slopes (e.g. Griffiths and Fenton 2004). For the steeper slopes, however, a maximum p_f happens when the Θ_{Cu} is between 1 and 0.5. This spatial correlation length called the "worst case" correlation length has been reported by other researchers for bearing capacity and retaining wall problems (e.g. Griffiths and Fenton 2001, Fenton and Griffiths 2003, Fenton et al. 2005, Massih et al. 2008). However, for slopes with small coefficient of variation ($V \leq 0.5$) the maximum probability of failure was observed to happen when the slope has an infinite spatial correlation length.

Another 2:1 slope was modeled with drained soil properties. The spatial correlation and coefficient of variation of C' and $\tan\phi'$ were assumed to be the same:

$$\Theta = \Theta_{C'} = \Theta_{\tan\phi'} \tag{5}$$

$$V = \sigma_{C'}/\mu_{C'} = \sigma_{\tan\phi'}/\mu_{\tan\phi'} \tag{6}$$

The height of the slope H, is equal to 10m. Three correlations ρ, between C' and $\tan\phi'$ were considered to evaluate the effect of ρ on p_f for the drained slope. Figure 5 shows the variation of the p_f with spatial correlation length and ρ.

According to Figure 5, a slope with a positive correlation between C' and $\tan\phi'$ leads to the highest probabilities of failure. It has been suggested by some investigators (e.g. Cherubini 2000) that C' and $\tan\phi'$ may have a negative correlation which results in lower values for the p_f. Thus, modeling the slope with no correlation between the C' and $\tan\phi'$ would be on the conservative side.

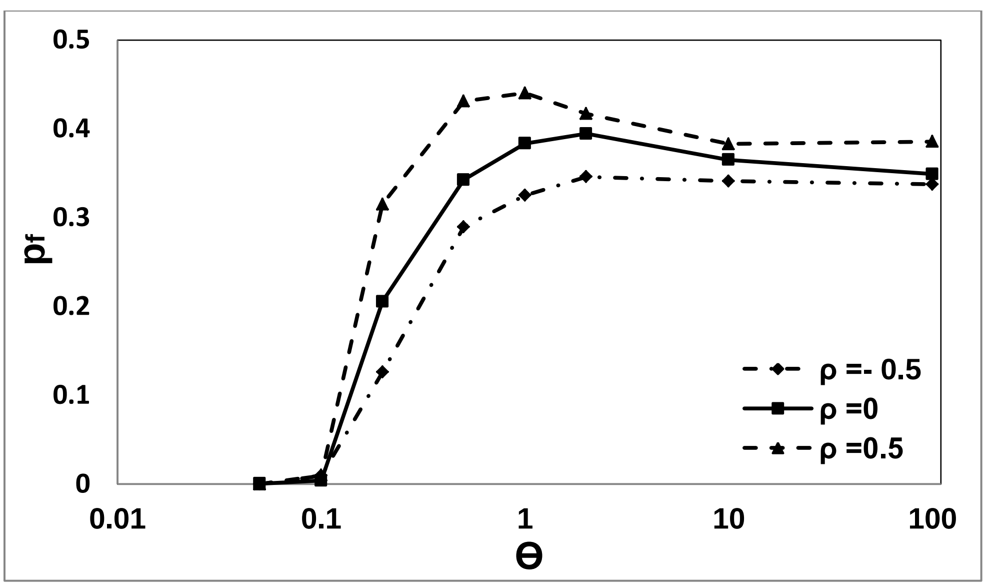

FIG. 5. Probability of failure vs. spatial correlation length for different cross-correlations for the drained slope.

For the drained slope, the slope angle β, was kept constant while the coefficient of variation V, and spatial correlation length varied to investigate the worst case Θ based on different *V values*. Figure 6 illustrates the variation of the p_f with Θ and V. The mean of C' and $\tan\phi'$ is kept constant while the standard deviation of these variables changed with coefficient of variation.

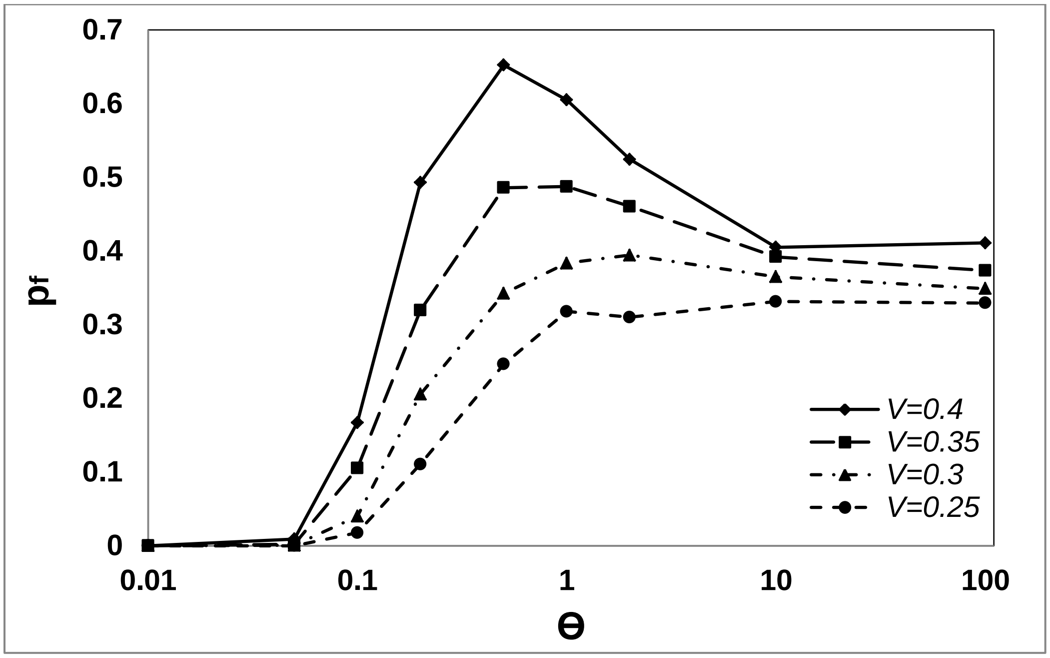

FIG. 6. Probability of failure vs. spatial correlation length for different coefficient of variations for the drained slope.

By increasing the coefficient of variation, the p_f of the slope increases. A maximum probability of failure also occurs by increasing V similar to that observed in the undrained slope. The worst case spatial correlation length also has a value between 0.5H and H. This means that when the slope has a spatial correlation length close to the height of the slope, it has the highest value for the probability of failure or lowest reliability.

When θ/H is large, the field is more strongly correlated, so that it appears smoother with less variability in each realization. The slope consequently tends to behave as a homogeneous slope more like that predicted by traditional methods. Conversely, when θ/H is small, the random field is typically rough in appearance; however, as the variability is high, the soil behaves like a uniform mass (with properties approaching the median). This results in having a slope with homogeneous soil in each realization as well. Thus, for very large and very small spatial correlation lengths, fewer failures are expected.

In finite element analysis of slope stability, as mentioned before, failure is free to seek out the weakest path through the soil. For intermediate correlation lengths within the scale of the slope height, the soil properties measured at one location may be quite different from those actually present at other locations. It gives the failure the opportunity to find the weakest path through the soil which could have a non-circular or non-linear shape. Figure 2(a) shows a failure mechanism for a drained slope with intermediate spatial correlation length, θ/H= 0.5 which is the worst case Θ for this slope. As it can be seen, failure doesn't follow a specific path. The failure is be able to find its path where the soil has the weaker parameters (lighter color). Therefore, for intermediate correlation lengths, more failures are observed. Following this reasoning, the maximum probability of failure occurs when the slope has an intermediate spatial correlation length as shown in Figures 4 and 6.

CONCLUSIONS

This paper has investigated the probability of failure for both drained and undrained slopes using the RFEM. The RFEM combines the FEM with Monte Carlo simulation in which spatial variability is properly taken into account. The RFEM enables the failure mechanism to seek out a weakest path through heterogeneous soil which can lead to higher probabilities of failure than might be predicted spatial variability is ignored. The influence of the coefficient of variation V, slope angle β, and spatial correlation length Θ, on the probability of failure p_f was studied. It was shown clearly that a worst case spatial correlation length exists for the both drained and undrained slopes. This worst case spatial correlation length, leading to a maximum probability of failure was shown to be of the order of $0.5H$ to H, where H is the slope height. The implication of this result is that the spatial correlation length need not be estimated if there is insufficient data, since the worst case Θ can be used to yield a conservative design aimed at a target reliability. This result is a practical and important finding, as the soil spatial variability is generally difficult and expensive to estimate accurately and requires a large number of samples.

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