Probabilistic Seismic Design of Geotechnical Systems

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ABSTRACT
Designs of geotechnical systems are typically aimed at specific lifetime reliability index targets corresponding to serviceability and ultimate limit states. The lifetime reliability targets for extreme limit states, such as earthquakes, should be consistent with ultimate limit state reliability targets. However, current design practice of structural and geotechnical systems against extreme limit states do not necessarily result in lifetime reliability indices which agree with those aimed at for static conditions. This paper investigates some possible approaches to calibrating an agreement between static and seismic design code target reliability indices for geotechnical systems. The total probability theorem is used to estimate the reliability of a geotechnical seismic design according to the performance-based design specifications of the Canadian Highway Bridge Design Code and to determine the required target reliabilities associated with each return period design check. The results are preliminary but suggest that seismic design may have a higher failure probability than current static designs.

RÉSUMÉ
Les conceptions des systèmes géotechniques visent généralement des cibles spécifiques d'indice de fiabilité à vie correspondant à la facilité d'utilisation et aux états limite ultimes. Les cibles de fiabilité à vie pour les états limites extrêmes, comme les tremblements de terre, devraient être compatibles avec les cibles de fiabilité de l'état limite. Cependant, la pratique de conception actuelle des systèmes structurels et géotechniques contre les états limites extrêmes n'entraîne pas nécessairement des indices de fiabilité à vie qui concordent avec ceux destinés à des conditions statiques. Cet article étudie certaines approches possibles pour étalonner un accord entre les codes de code statique et sismique cible des indices de fiabilité pour les systèmes géotechniques. Le théorème de probabilité totale est utilisé pour estimer la fiabilité d'une conception sismique géotechnique selon les spécifications de conception basées sur la performance du Code de conception du pont routier canadien et pour déterminer les fiabilités cibles requises associées à chaque vérification de la conception de la période de retour. Les résultats sont préliminaires mais suggèrent que la conception sismique peut avoir une probabilité d'échec plus élevée que les conceptions statiques actuelles.

1 INTRODUCTION

The performance-based seismic design provisions of the 2014 edition of the Canadian Highway Bridge Design Code (CHBDC, CSA 2015) now specifies required performance levels for each of three levels of seismic ground motion. Table 1 is an extract from the CHBDC for the middle importance level of bridges referred to as Major-route bridges. The table states that if the bridge is subjected to seismic ground motion corresponding to earthquakes having at least a 475 year return period, the bridge "must" remain in service and damage "must" be minimal. If the bridge is subjected to an earthquake having return period in excess of 2475 years, we accept that service may be disrupted and that damage may be extensive. We put the word "must" in quotes above, since of course there is always a non-zero probability that even if only the 475 year return period earthquake takes place that service is nevertheless delayed.

Modern design codes are concerned with producing engineered systems having societally acceptable failure probabilities. The challenge, of course, is to determine the failure probabilities of our designed systems and to adjust our design requirements accordingly.

At the moment, the Canadian design codes of practice do not specify target failure probabilities associated with each of the required performance levels.

Table 1: Bridge performance levels required by the 2014 CHBDC.

Seismic ground motion probability of exceedance in 50 years (return period)	**Major-route bridges**	
	Service	**Damage**
10% (475 years)	Immediate	Minimal
5% (975 years)	Service limited*	Repairable*
2% (2475 years)	Service disruption	Extensive

It is thus difficult to know whether current seismic design practice is safer or less safe than current static design practice (the latter of which is typically associated with target failure probabilities).

This paper takes a preliminary, theory-based, look at how the three performance level specification approach currently adopted by the CHBDC for seismic design compares, in terms of overall failure probability, to that predicted by the total probability theorem (see, e.g., Cornell 1968). The goal of the paper is to arrive at reasonable target failure probabilities to assign to the words “Immediate”, “Service limited”, etc., in the performance requirements of Table 1. These probabilities should lead to a total lifetime failure probability which is approximately equal to the maximum acceptable failure probabilities aimed at in static design. It is assumed, for example, that the “Immediate” service requirement actually corresponds to a low probability of failure (where failure is defined as loss of service), perhaps something like 0.0001 in the event that a 475 year return period earthquake is the largest earthquake experienced by the geotechnical system over its design lifetime. Similarly, “Service disruption” under a 2475 year return period might correspond to a probability of failure which is well above 50%.

Once reasonable target failure probabilities are assigned to each of the three return-period performance targets, the geotechnical resistance factors required to achieve all of the desired performance requirements can be estimated.

Probabilistic seismic design is dependent on many issues. The occurrences of earthquakes are generally assumed to follow a Poisson model, so in any design lifetime, l, there could be any number of earthquakes, especially if one starts at earthquakes of magnitude 0. We will assume in this paper that there is some minimum earthquake magnitude, m_0, below which the probability of geotechnical failure is negligible. A Poisson model can then be used to characterize the number of earthquakes that may occur at a site having magnitude in excess of m_0 over the lifetime of the designed system. Of course, each of these earthquakes can have random magnitudes, so the question becomes one of how system failure is defined. Each earthquake which may occur during the design lifetime has some potential to cause damage. Presumably, if damage is incurred, it will be repaired and how that affects the probability of damage during the next earthquake is difficult to estimate. We will assume in this paper that the design should be aimed at achieving a target maximum acceptable probability against failure due to the largest earthquake experienced by the geotechnical system during its lifetime. In other words, we will only consider the maximum earthquake occurring during the system lifetime and assume that failure may only occur during that maximum earthquake ground motion – all other earthquakes experienced by the system are assumed to not cause failure.

We start by looking at the probability of failure due to the occurrence of a single future earthquake. Since the magnitude of that earthquake is random, the failure probability can be expressed as a function of all possible return periods using the total probability theorem as follows:

$$\begin{aligned} p_m \le p_f &= \mathrm{P}[F] = \mathrm{P}[F \mid R = r_1]\mathrm{P}[R = r_1] \\ &+ \mathrm{P}[F \mid R = r_2]\mathrm{P}[R = r_2] + \ldots \\ &= \sum_{i=1}^{\infty} \mathrm{P}[F \mid R = r_i]\mathrm{P}[R = r_i] \end{aligned} \qquad [1]$$

where p_m is the maximum acceptable failure, R is the random return period of the earthquake, and r_i is a specific realization of R. Larger values of r_i imply stronger earthquakes. The above probability could be written directly in terms of earthquake magnitude, M, of the earthquake having return period R, as follows:

$$\begin{aligned} p_m \le p_f &= \mathrm{P}[F] = \mathrm{P}[F \mid M = m_1]\mathrm{P}[M = m_1] \\ &+ \mathrm{P}[F \mid M = m_2]\mathrm{P}[M = m_2] + \ldots \\ &= \sum_{i=1}^{\infty} \mathrm{P}[F \mid M = m_i]\mathrm{P}[M = m_i] \end{aligned} \qquad [2]$$

The relationship between earthquake magnitude and mean recurrence rate has been well established by the Gutenberg-Richter recurrence law (Gutenberg and Richter, 1944) to be,

$$\lambda_m = 10^{a-bm} = e^{\alpha-\beta m} \qquad [3]$$

where m is the earthquake magnitude, and $\lambda_m = 1/r_i$ is the mean annual rate of exceeding m. The parameters a and b are site specific values obtained by regression of the seismicity data from the region of interest (see, e.g., Kaila and Narain, 1971). To convert to natural logarithms at the right hand side of Eq. [3],

$$\begin{aligned} \alpha &= 2.303a \\ \beta &= 2.303b \end{aligned} \qquad [4]$$

Solving Eq. [3] for m allows directly calculating earthquake magnitude, m, for a given mean annual rate of exceedance, λ_m, as follows:

$$m = \frac{\alpha - \ln(\lambda_m)}{\beta} \qquad [5]$$

In general, in the seismic design process, we are interested in determining the probability of system failure over some lifetime, l. During that lifetime, there may be more than a single earthquake and the magnitudes of those earthquakes will be random. We are thus interested in the distribution of the magnitude of the maximum earthquake occurring during lifetime, l. Let N_l be the number of earthquakes occurring over lifetime l having magnitude $M \ge m_0$, where m_0 is a lower threshold magnitude below which damage will be insignificant. m_0 is normally selected to have a value between 4.0 and 5.0.

According to Cornell (1968), N_l follows a Poisson distribution with arrival rate $\lambda = e^{\alpha-\beta m_0}$. Of these earthquakes, some fraction, r_m, have magnitude $M > m \ge m_0$. If N_{lm} denotes the number of earthquakes over time l having magnitude $M > m \ge m_0$ then N_{lm} has mean rate $\lambda_m = r_m \lambda$ and distribution

$$\mathrm{P}[N_{lm} = k] = \frac{(\lambda_m l)^k}{k!} e^{-\lambda_m l} \qquad [6]$$

Note that the fraction r_m is the ratio of mean rates,

$$r_m = \frac{\lambda_m}{\lambda} = \frac{e^{\alpha-\beta m}}{e^{\alpha-\beta m_0}} = e^{-\beta(m-m_0)} \quad [7]$$

Now let $M_{\max}(l)$ be the maximum earthquake magnitude experienced over lifetime l. The distribution of $M_{\max}(l)$ can be determined as follows. If $M_{\max}(l) \le m$ then this must mean that $N_{lm} = 0$ so that

$$F_{M_{\max}}(m) = \mathrm{P}\left[M_{\max}(l) \le m\right] = \mathrm{P}\left[N_{lm} = 0\right] = \exp\left\{-\lambda_m l\right\} \quad [8]$$

Substituting Eq.[3] in Eq. [7] gives

$$F_{M_{\max}}(m) = \exp\left\{-le^{\alpha-\beta m}\right\} \quad [9]$$

which is a Type 1 extreme value distribution, also referred to as a Gumbel distribution.

Differentiating the CDF in Eq. [9] with respect to earthquake magnitude, m, leads to the following probability density function (PDF):

$$f_{M_{\max}}(m) = \frac{\partial F_{M_{\max}}(m)}{\partial m} = \beta l \exp\left\{\alpha - \beta m\right\} \exp\left\{-le^{\alpha-\beta m}\right\} \quad [10]$$

Note that the extreme value distribution of maximum earthquake magnitude is purely a function of lifetime period, l, and site specific constants α and β obtained by regression in the Gutenberg-Richter Law.

2 SEISMIC FAILURE PROBABILITY

In this section, a methodology is developed to assess the failure probability of a system due to earthquakes occurring having magnitudes exceeding m_0. Using the Total Probability Theorem, the failure probability of such a system is defined as

$$\mathrm{P}[F] = \int_{m_0}^{\infty} \mathrm{P}\left[F \mid M_{\max}(l) = m\right] f_{M_{\max}}(m)\, dm \quad [11]$$

where $\mathrm{P}\left[F \mid M_{\max}(l) = m\right]$ is the conditional probability of system failure due to the largest earthquake, having magnitude m, to occur over lifetime l and $f_{M_{\max}}(m)dm$ is the probability of that earthquake event.

The main problem at this point is determining the conditional probability of failure in Eq. [11] for each given earthquake magnitude. Clearly, the system failure probability increases from zero, for earthquake magnitudes below m_0, to one as the earthquake magnitude increases. A possible approximation to this increasing conditional failure probability is to simply use a cumulative distribution function (CDF) which also increases from zero to one. In this paper a lognormal distribution is selected as a preliminary choice for the conditional failure probability. The lognormal distribution has two parameters μ and σ which can be obtained by fitting the lognormal CDF to the failure probabilities corresponding to the lowest and highest of the three typical return periods considered in design codes, that is, to the 475 and 2475 return period earthquakes. Using the target failure probabilities mentioned above, this corresponds to matching the CDF to

$$\begin{aligned} p_{f_{475}} &= \mathrm{P}\left[F \mid M_{\max}(l) = m_{475}\right] \\ p_{f_{2475}} &= \mathrm{P}\left[F \mid M_{\max}(l) = m_{2475}\right] \end{aligned} \quad [12]$$

where $p_{f_{475}}$ and $p_{f_{2475}}$ are the target probabilities that we will select for these two return period performance specifications and where the conditional probability is expressed in terms of the lognormal CDF as:

$$\mathrm{P}\left[F \mid M_{\max}(l) = m\right] = \phi\left(\frac{\ln(m-m_0)-\mu}{\sigma}\right) = \phi\left(\frac{\ln\Delta_m - \mu}{\sigma}\right) \quad [13]$$

where $\Delta_m = m - m_0$. The terms m_{475} and m_{2475} in Eq. [12] are the earthquake magnitudes obtained via Eq. [3] corresponding to return periods 475 and 2475:

$$\begin{aligned} m_{475} &= \frac{\alpha - \ln(\lambda_{475})}{\beta} = \frac{\alpha - \ln(1/475)}{\beta} \\ m_{2475} &= \frac{\alpha - \ln(\lambda_{2475})}{\beta} = \frac{\alpha - \ln(1/2475)}{\beta} \end{aligned} \quad [14]$$

Solving for the mean and standard deviation of the lognormal distribution gives us

$$\begin{aligned} \mu &= \frac{\ln\Delta_{m_{2475}} - c\ln\Delta_{m_{475}}}{1-c} \\ \sigma &= \frac{\ln\Delta_{m_{475}} - \mu}{\phi^{-1}\left(p_{f_{475}}\right)} \end{aligned} \quad [15]$$

where

$$c = \frac{\phi^{-1}\left(p_{f_{2475}}\right)}{\phi^{-1}\left(p_{f_{475}}\right)} \quad [16]$$

and $\Delta_{m_{475}} = m_{475} - m_0$, $\Delta_{m_{2475}} = m_{2475} - m_0$.

Further substituting Eq. [13] into Eq. [11] leads to the following expression for the failure probability of a system due to seismic loading:

$$\mathrm{P}[F] = \int_{m_0}^{\infty} \phi\left(\frac{\ln\Delta_m - \mu}{\sigma}\right) f_{M_{\max}}(m)\, dx \quad [17]$$

If the conditional failure probabilities given by Eq. [13] are correct, then Eq. [17] gives an accurate estimate of the overall lifetime failure probability. The numerical evaluation of Eq. [17] is performed using Gaussian quadrature with 16 Gauss points from $m_0 = 4$ to $m = 10$.

3 CURRENT DESIGN PRACTICE

In CHBDC's current seismic design practice, only three return periods 475-, 975-, and 2475-years are considered.

If failure probabilities are assigned to each return period scenario then it becomes possible to estimate the failure probability of a system using the total probability theoem:

$$\mathrm{P}[F] = \mathrm{P}[F \mid E_{475}]\mathrm{P}[E_{475}] + \mathrm{P}[F \mid E_{975}]\mathrm{P}[E_{975}] + \mathrm{P}[F \mid E_{2475}]\mathrm{P}[E_{2475}] \quad [18]$$

where the conditional probabilities $\mathrm{P}[F \mid E_{475}]$, etc., appearing on the right-hand side are the target design probabilities, $p_{f_{475}}$, $p_{f_{975}}$, and $p_{f_{2475}}$, which are yet to be determined. In Eq. [18], E_r are the events that the maximum earthquake over lifetime l has return period r. In fact, the probability that the maximum earthquake has return period exactly equal to r is zero. In other words, in order to use Eq. [18] according to the total probability theorem, the events E_r must be defined over ranges in return period (or equivalently in ranges over magnitude). We have arbitrarily selected ranges that are approximately centered on CHBDC's selected return periods of 475, 975, and 2475 years. The resulting probabilities appearing in Eq. [18] are as follows:

$$\begin{aligned} \mathrm{P}[E_{475}] &= \mathrm{P}[m_{300} \le M_{\max}(l) < m_{650}] \\ \mathrm{P}[E_{975}] &= \mathrm{P}[m_{650} \le M_{\max}(l) < m_{1300}] \\ \mathrm{P}[E_{2475}] &= \mathrm{P}[M_{\max}(l) > m_{3000}] \end{aligned} \quad [19]$$

4 COMPARISON OF CONTINUOUS AND DISCRETE FAILURE PROBABILITIES

It is of interest to investigate how well the three return period discrete approximation to the failure given by Eq. [18] compares to the continuous failure probability approximation given by Eq. [17]. To do so, we need to decide on values for the target design probabilities, $p_{f_{475}}$, $p_{f_{975}}$, and $p_{f_{2475}}$. Table 2 provides the parameters selected for this comparison.

Table 2. Values Considered in this paper

Parameter	Value
a^1	5.05
b^1	1.09
m_0	4

[1]Kaila and Narain (1971) for Western Canada obtained by regression for shallow earthquakes during a time interval of 14 years

The values of a and b provided in Table 2 are derived from earthquake records over a period of 14 years. The constants α and β in Eq. [4], which are used in the calculation of the mean annual rate λ_m in Eq. [3], need to be adjusted to shift to an annual basis as follows:

$$\begin{aligned} \alpha &= 2.303a - \ln(14) \\ \beta &= 2.303b \end{aligned} \quad [20]$$

Table 3 presents the comparison between the more exact continuous approximation to the failure probability, Eq. [17], with the discrete approximation in Eq. [18], which is developed based on current CHBDC practice.

Table 3. Comparison of failure probability estimates from Eq.'s [17] and [18]

$p_{f_{475}}$	$p_{f_{975}}$	$p_{f_{2475}}$	p_f (Eq. [17])	p_f (Eq. [18])
0.001	0.01	0.1	0.012	0.0062
0.001	0.05	0.1	0.012	0.0084
0.001	0.1	0.1	0.012	0.011
0.0001	0.001	0.01	0.0025	0.00062
0.0001	0.005	0.01	0.0025	0.00084
0.0001	0.01	0.01	0.0025	0.0011
0.00001	0.0001	0.001	0.00044	0.000062
0.00001	0.0005	0.001	0.00044	0.000084
0.00001	0.001	0.001	0.00044	0.00011

The results presented in Table 3 suggest that the discrete approximation of Eq. [18] consistently underestimates the failure probability predicted by the more accurate Eq. [17]. However, the two probabilities are well within the same order of magnitude so that the discrete approximation appears to be reasonable.

5 DETERMINATION OF TARGET PROBABILITIES

At this point, target probabilities can be assigned to the three design return periods used in the CHBDC by computing the total failure probability of seismic design and comparing that to the target failure probability under static design. The CHBDC specifies an annual target reliability index of 3.75 for ultimate limit states (ULS). Fenton et al. (2016) use a 75-year lifetime reliability index of 3.5 which corresponds to an annual reliability index of 4.5 if years are assumed independent. Thus the lifetime reliability index of 3.5 is considered to be conservative and will be adopted here as the target system reliability. This means that both seismic and static designs should target a failure probability of $\phi^{-1}(-3.5) = 0.00023$. To achieve this target failure probability using the discrete three return period approximation employed by the CHBDC, we must investigate trial values of $p_{f_{475}}$, $p_{f_{975}}$, and $p_{f_{2475}}$. Possible values which lead to failure probabilities of approximately 0.00023 are shown in Table 4.

One immediate observation from Table 4 is that all three probabilities must be small in order to achieve a reliability index of 3.5. In practice, this implies that even though a performance of "Service disruption" is specified for the 2475 year return period earthquake, its probability of occurrence must be small in order to achieve overall small target probability of failure. This is true even if the values of $p_{f_{475}}$ and $p_{f_{975}}$ are set to zero. In other words, the product $\mathrm{P}[F \mid E_{2475}]\mathrm{P}[E_{2475}]$ governs the failure probability at these low probability levels. The problem may be that we are assuming that the occurrence of a "2475 year return period earthquake" actually corresponds to the occurrence of a maximum earthquake having return period in excess of 1300 years. We note that choosing ranges which correspond to exceedance probabilities of 10%, 5%, and 2% may make more sense but the calibration of the lognormal CDF must be done for a specific return period, not for a range. As a result, we attempted to match the discrete and continuous probability estimates using return period ranges centered on the return periods specified in the CHBDC. These ranges result in required failure probabilities that appear to be lower than suggested by current practice. In other words, if the current probabilistic analysis is correct, then current practice is unsafe. However, the probabilistic model suggested here is still very preliminary and needs further work.

Table 4. Target failure probability estimates to achieve lifetime failure probability of $\phi^{-1}(-3.5) = 0.00023$

$p_{f_{475}}$	$p_{f_{975}}$	$p_{f_{2475}}$	p_f (Eq. [17])	p_f (Eq. [18])
0.001	0.01	0.1	0.012	0.0062
0.0001	0.001	0.01	0.0025	0.00062
0.00001	0.0001	0.001	0.00044	0.000062

6 CONCLUSION AND FUTURE WORK

A methodology is developed in this work to probabilistically assess the reliability of a geotechnical system due to the earthquake excitations over a 75-year lifetime by the means of the total probability theorem in two ways; one by looking at all possible return periods over the lifetime, and two by looking at just three representative return periods as is done by the CHBDC. The latter "three-return-period" approximation assumes that the maximum earthquake to occur over the design lifetime has return periods somewhere in ranges centered on 475, 975, and 2475 years. This assumption leads to required target design probabilities which are much smaller than expected for performance specifications such as "Service disruption". The implication of these results is that the performance-based design criteria, if set to probabilities which are reasonable with respect to the definitions of the performance critera, will lead to lifetime failure probabilities which are greater than those assumed for static loading design. In other words, these results suggest that seismic design is less safe than static design. This conclusion needs significantly more study, especially since seismic design appears to be quite safe in practice.

The results of this paper depend on the assumptions made regarding the ranges in return periods for the three-return-period approximation. In particular, it probably makes more sense to use earthquake return period ranges with lower bounds as given in the CHBDC (475, 975, and 2475 years). Unfortunately, this leads to problems with the calibration of the so called improved accuracy model of Eq. [17] and needs further study.

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