# Probabilistic stability analysis of a shallow passive trapdoor

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**Abstract**

The stability of a trapdoor has long been an important benchmark solution in theoretical soil mechanics, and it is also of considerable practical interest in many geotechnical applications. It is well known that natural soils always exhibit spatial variability with the properties varying from point to point. This paper uses the Random Finite Element Method (RFEM) to investigate the influence of spatial variability on the limit loads for a shallow passive trapdoor embedded in a purely cohesive soil. RFEM is an advanced numerical tool for probabilistic geotechnical analysis which merges finite-element methodologies with random field theory in a Monto-Carlo framework. In the current parametric study, the mean undrained shear strength has been held constant while the coefficient of variation and spatial correlation length have been varied systematically. As might be expected, for trapdoors with low values of the coefficient of variation, the mean limit loads agree well with the results from deterministic analysis. For higher values of the coefficient of variation, the mean limit loads fall quite steeply. Failure is defined as occuring when the computed limit load is less than the deterministic solution based on the mean strength, reduced by an appropriate factor of safety. By interpreting the Monte-Carlo simulations in a probabilistic context, the probability of failure is assessed as a function of the factor of safety and the spatial variability of the soil. It is found, for example, that a factor of safety of 2.5 is required to avoid the probability of failure exceeding 5% for soils with strength variability within typical ranges.

**Keywords:** Trapdoor Stability, Statistical Analysis, RFEM, Spatial Variability, Limit State Design

## 1. Introduction

The trapdoor problem, originally studied by Terzaghi [1], has become an important benchmark solution in theoretical soil mechanics. This problem has two kinds of displacement pattern, depending on whether the movement of the trapdoor is upward (passive) or downward (active). The passive mode corresponds to the uplift problem of anchors (e.g. Merifield et al. [2]), while the active mode can be applied to analyze the gravitational flow of granular material between vertical walls, which has been used to design tunnels (e.g. Koutsabeloulis and Griffiths [3]). In the past, the trapdoor problem has received considerable attention deterministically [3-8]. This paper will investigate the influence of soil strength variability on the limit load for a rough rigid strip trapdoor embedded in an undrained clay by the Random Finite Element Method (RFEM). The program merges finite-element methodologies [9] with random field theory [10] within a Monte-Carlo framework. In this study, the undrained shear strength $c_u$ is supposed to be characterized by a lognormal distribution with four parameters given in Table 1.

With the purpose of nondimensionalizing the input parameters, the variability of the undrained shear strength can be expressed by the coefficient of variation $v_{c_u} = \sigma_{c_u} / \mu_{c_u}$. For the random field modeling, a dimensionless and isotropic (i.e. $\theta_\mathrm{h}$ is assumed to be $\theta_\mathrm{v}$) spatial correlation length $\Theta = \theta / B$ is used, where $B$ is the width of the strip trapdoor.

In the present study, the mean strength $\mu_{c_u}$ is fixed at a value of 100 kN/m2, and the other two input parameters the coefficient of variation $v_{c_u}$ and the spatial correlation length $\Theta$ vary systematically.

Table 1 Undrained shear strength properties

| Statistical property | Symbol | Units |
|---|---|---|
| Mean | $\mu_{c_u}$ | Stress |
| Standard deviation | $\sigma_{c_u}$ | Stress |
| Horizontal and vertical spatial correlation length | $\theta_{\mathrm{h}}$ and $\theta_{\mathrm{v}}$ | Length |

## 2. Results

2.1 Finite element method

The stability analysis of a shallow passive trapdoor is performed by the elastic-perfectly plastic finite element method (Smith and Griffiths [9]). With the assumption that the soil is weightless and no surcharge pressure acts on the surface, the behavior of a trapdoor is affected by undrained shear strength ($c_u$), Young's modulus ($E$) and Poisson's ratio ($\nu$). The RFEM can model random distributions of these three parameters however, for simplifying the analyses, only the undrained shear strength is considered to be randomised in the present study.

As shown in Fig. 1, a typical mesh used in FE analysis includes 1200 eight-noded square element,

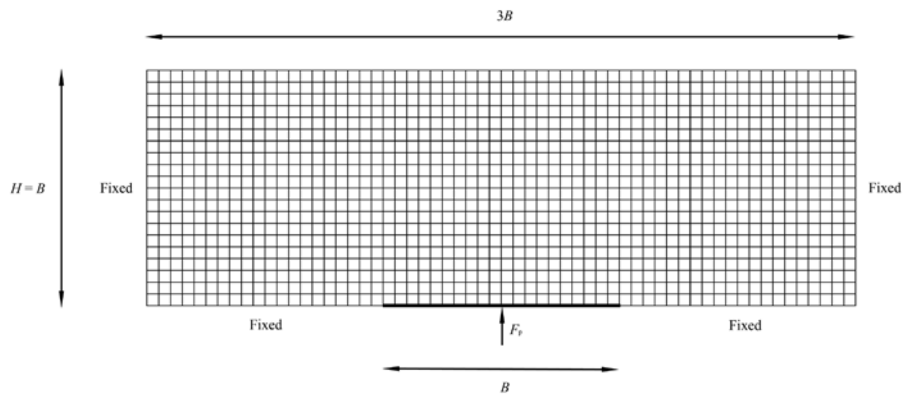



Fig. 1. Mesh used in probabilistic stability analysis of a shallow passive trapdoor

with 60 columns and 20 rows. The side length of each element is 0.05 m and the width of the strip trapdoor $B$ is equal to 1 m. The height $H$ is assumed to equal $B$. Since the trapdoor is rigid and rough, displacement control is employed. The trapdoor is incrementally displaced vertically into the soil, while the horizontal movement of the nodes which represent the trapdoor is fixed at zero. The average vertical stress $\sigma_y$ in the row of Gauss points just above the displaced nodes can be calculated after each increment. Failure of the trapdoor is said to have occurred if the calculated $\sigma_y$ levels out within a strict tolerance. Finally, the limit load is given by

$$F_{\mathrm{p}} = \sigma_y B \tag{1}$$

2.2 Monte-Carlo simulations

In the context of a Monte-Carlo analysis, each realization of an RFEM analysis of the problem shown in Fig. 1 involves generation of the $c_u$ random field and the succeeding FE analysis of the trapdoor stability. In each realization, the underlying statistical properties $v_{c_u}$ and $\Theta$ are the same however, the spatial pattern of undrained shear strength values over the region of the FE mesh is quite different, leading to a different value of limit load for the trapdoor. Following each suite of 1000 simulations, the limit loads are subjected to statistical analysis.

2.3 Deterministic analysis

Recent solutions of the deterministic shallow trapdoor stability problem shown in Fig. 1 (Martin [6]) represent the most accurate solutions. When the soil is weightless and there is no surcharge pressure, the limit load $F_{\mathrm{p}}$ for passive failure is given by

$$F_{\mathrm{p}} / B = N_c c_u \tag{2}$$

where $N_c$ is a dimensionless stability factor which is proportional to the cover ratio $H / B$ given by

$$N_c = 1.956 H / B \tag{3}$$

For cover ratio $H / B = 1$ and undrained strength $c_u = 100$ kN/m$^2$, Martin's solution gives the limit load $F_{\mathrm{p}} = N_c c_u B = (1.956)(1)(100)(1) = 195.6$ kN/m.

The results of deterministic analysis by finite element method are shown in Fig. 2. The estimated

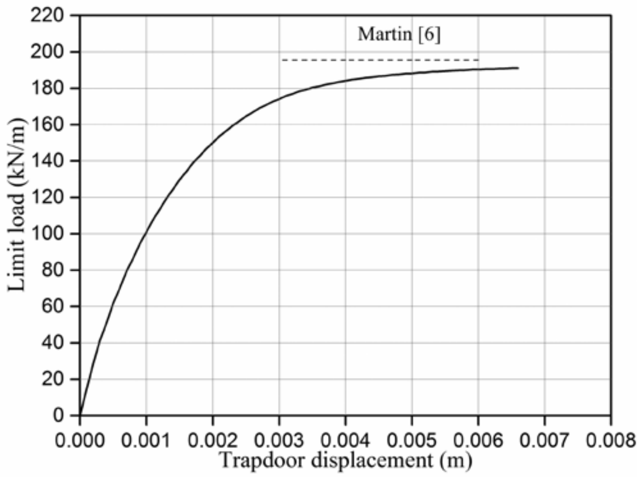



Fig. 2. Deterministic stability analysis of the trapdoor

limit load was 191.1 kN/m, which is 2.3% lower than the Martin's analytical solution of $F_{\mathrm{p}} = 195.6$ kN/m. The reason is attributed to the uniform and coarser mesh for simplifying the random field generation. In the discussion that follows, and for error consistency, the mean limit load from statistical analysis will be standardized by the deterministic value from FE analysis. The influence of this slightly lower prediction is relatively inessential. In this study, the deterministic limit load will be referred to as $F_{p_d}$, i.e., $F_{p_d} = 191.1$ kN/m.

## 2.4 Parametric study

Parametric analyses were carried out with the following input parameters:

$$\Theta = 0.01,\ 0.1,\ 0.2,\ 0.5,\ 1.0,\ 2.0,\ 5.0,\ 10.0$$
$$v_{c_u} = 0.1,\ 0.3,\ 0.5,\ 0.75,\ 1.0,\ 1.5$$

On completion of 1000 Monte-Carlo simulations, the mean and standard deviation of the limit load was obtained.

Figure 3 shows how the mean limit load, standardized by the deterministic value from FE analysis,

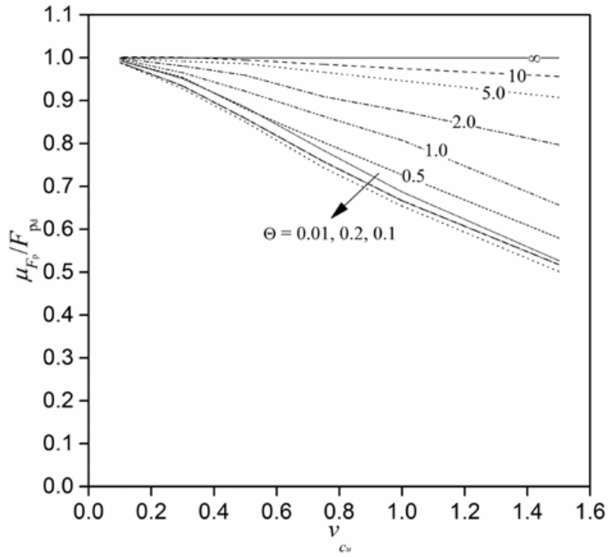



Fig. 3. Estimated mean limit load as a function of undrained shear strength statistic $\Theta$

changes with $\Theta$ and $v_{c_u}$. For low values of $v_{c_u}$, the mean limit load $\mu_{F_p}$ is apt to be the deterministic value. As the value of $v_{c_u}$ increases, the mean limit load falls quite sharply, particularly for spatial correlation length $\Theta \approx 0.1$. For example, for a highly variable soil with $\Theta = 0.1$ and $v_{c_u} = 1.5$, the mean limit load is just approximate 50% of the deterministic value. For the recommended upper limit of $v_{c_u} = 0.5$ (e.g. Lee et al. [11]) with $\Theta = 1.0$, the $\mu_{F_p}$ is about 90% of the deterministic value. The horizontal line included in Fig. 3 represents the analytical solution for the limiting case of $\Theta = \infty$. This case implies that each Monte-Carlo realization generates an essentially homogeneous soil, although the properties are different from one realization to the next. For this limiting case, the distribution of the limit load $F_p$ will be statistically analogous to the lognormal distribution of $c_u$ but magnified by $F_{pd}$, i.e., $\mu_{F_p} = F_{p_d}$ for all values of $v_{c_u}$.

As shown in Fig. 4, a minimum mean limit load is reached when $\Theta \approx 0.1$. At the next lower value of $\Theta$, $\mu_{F_p}$ starts to increase in all cases. It is inferred that for the limiting case of $\Theta \to 0$, there are no "preferential" low strengths to attract the failure mechanism. As a result, the mean limit load will tend to the deterministic value based on the median of the undrained shear strength. Hence, as $\Theta \to 0$ $\mu_{F_p} / F_{pd} \to \left(1+v_{c_u}^2\right)^{-1/2}$ (Griffiths et al. [12]). The interested reader can refer to this paper for further discussion on this limiting case.

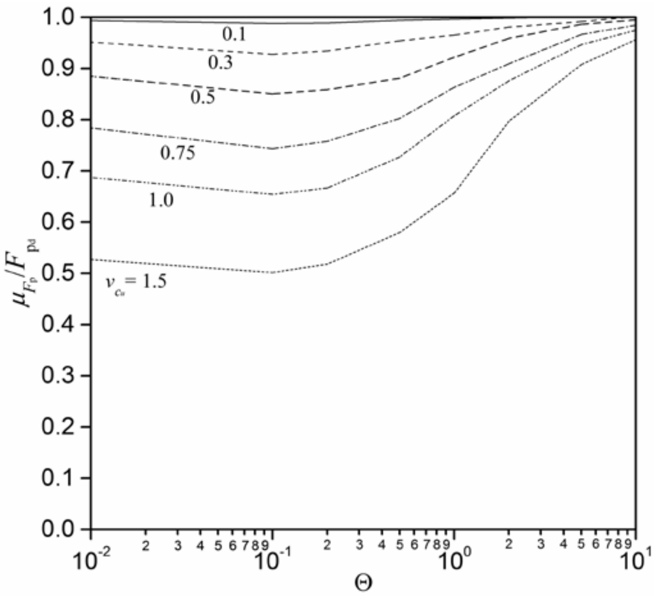



Fig. 4. Estimated mean limit load as a function of undrained shear strength statistic $v_{c_u}$

The effect of $\Theta$ and $v_{c_u}$ on the sample coefficient of variation of the calculated limit load, $v_{F_p} = \sigma_{F_p} / \mu_{F_p}$ is shown in Fig. 5. This figure indicates that $v_{F_p}$ is positively correlated with both $v_{c_u}$ and $\Theta$. For the limiting case of $\Theta = \infty$, $v_{F_p}$ would be equal to $v_{c_u}$.

Figure 6 shows two representative deformed meshes at failure above the trapdoor with parametric combinations indicated in the figure caption. Dark and light regions indicate higher and lower soil strengths, respectively. Due to the spatial variability of the soil, the failure mechanism is no longer symmetrical.

## 2.5 Probabilistic interpretation

Figure 7 can be used to choose a required factor of safety to satisfy the desired probability of failure. Failure is defined here as occurring if the computed limit load is smaller than the deterministic solution calculated using the mean undrained shear strength, reduced by an appropriate factor of safety. For example, if an objective probability of failure of 5% is desired for an undrained clay with

$v_{c_u} = 0.1$ (a recommended lower range of Lee et al [11]), a factor of safety of at least $FS = 1.2$ would be required (Fig. 7(a)). For an intermediate value of $v_{c_u} = 0.3$, the required factor of safety of at least $FS = 1.7$ is needed (Fig. 7(b)), and for an undrained clay with $v_{c_u} = 0.5$, the required factor of safety would be at least 2.5, as shown in Fig. 7(c).

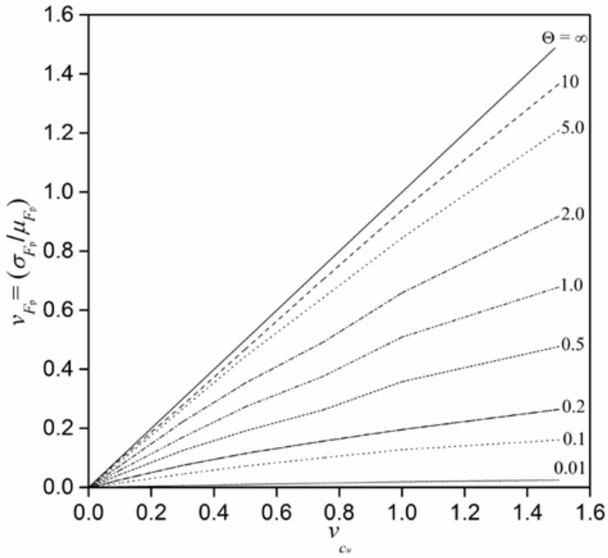
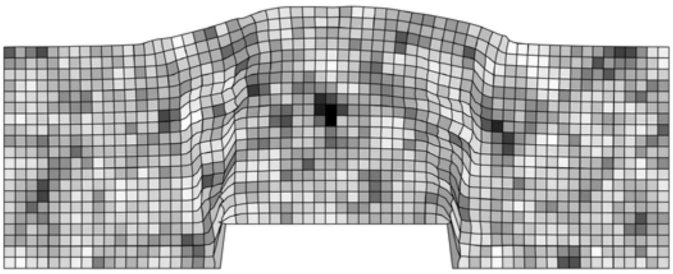
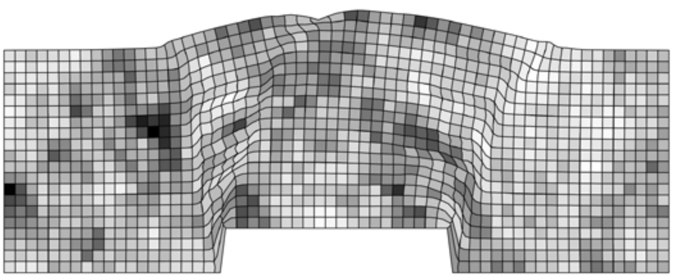



Fig. 5. Estimated coefficient of variation of limit load ($v_{F_p}$) as function of undrained shear strength statistics $\Theta$ and $v_{c_u}$

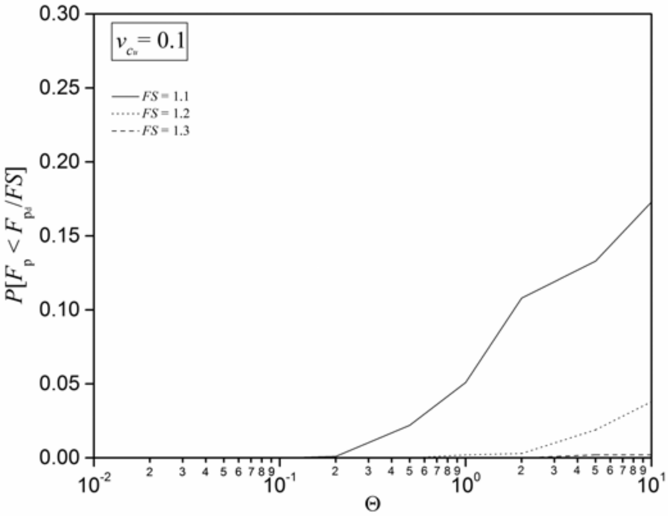


(a)

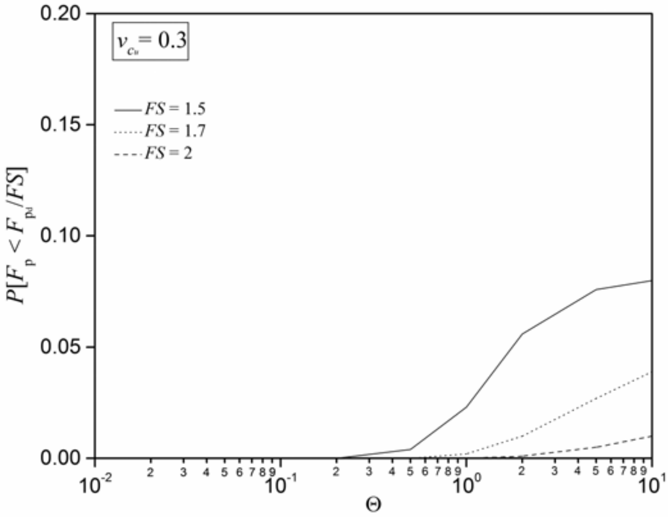


(b)

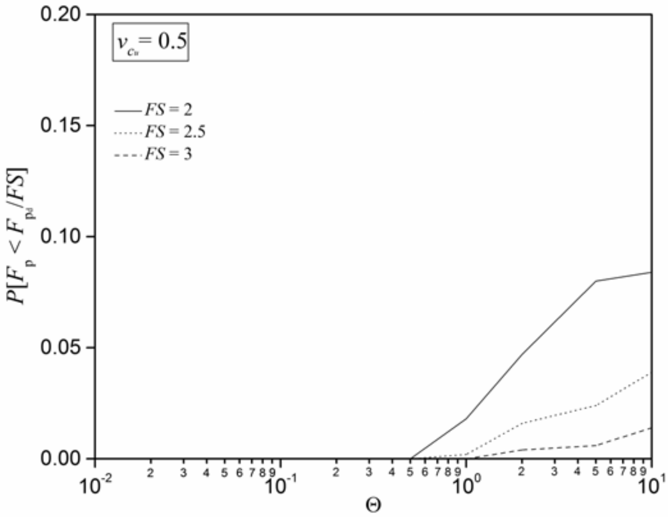
Fig. 6. Two typical deformed meshes at failure with: (a) $v_{c_u} = 0.5$ and $\Theta = 0.1$, (b) $v_{c_u} = 0.5$ and $\Theta = 0.2$




(a)




(b)




(c)

Fig. 7. Probability of design failure as a function of $\Theta$ for different factor of safety:
(a) $v_{c_u} = 0.1$, (b) $v_{c_u} = 0.3$, (c) $v_{c_u} = 0.5$

## 3. Conclusions

A probabilistic analysis of the limit load for a shallow passive trapdoor embedded in a spatially varying undrained clay has been performed. The minimum mean limit load is reached for higher values of $v_{c_u}$ and the spatial correlation length $\Theta \approx 0.1$. The results show that a factor of safety of $FS = 2.5$ would be required to avoid the probability of design failure exceeding 5% for soils with $v_{c_u} \leq 0.5$.

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