Worst-Case Spatial Correlation Length in Reliability Analysis of Excavated Clay Slopes

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Abstract: This paper investigates the reliability of slopes in undrained soils by the random finite-element method (RFEM). The RFEM is an advanced numerical tool for geotechnical reliability analysis, which merges finite-element modelling with random field theory in a Monte-Carlo framework. The emphasis of this paper is on the "worst-case" spatial correlation length, at which the probability of slope failure reaches a maximum. The RFEM outcomes indicate that slopes in undrained soils with a relatively low mean factor of safety or a relatively high coefficient of variation of undrained strength, are most likely to display the "worst-case" phenomenon. Slopes with both isotropic and anisotropic spatial correlation structures are considered. In the absence of substantial soil field data, knowledge of the "worst-case" spatial correlation length is useful, because it can be adopted for conservative reliability-based design.

Keywords: Worst case; spatial correlation length; random finite-element method; undrained shear strength.

1 Introduction

It has long been recognized that natural soil exhibits spatial variability, but it was not until the early 1990s that a computational tool called the random finite element method (RFEM) was developed to explicitly account for this variability (Griffiths and Fenton 1993; Fenton and Griffiths 1993). Since then, the method continues to be used by research groups worldwide, and applied to a wide range of geotechnical problems. The development of the method and many examples of its use in geotechnical engineering can be found in Fenton and Griffiths (2008). In some of the applications to reliability-based design, a maximum of "worst case" probability of failure has been observed at intermediate spatial correlation lengths. Baecher and Ingra (1981) noted the phenomenon in a footing settlement problem, and it has been further demonstrated using RFEM in several applications, including seepage, bearing capacity, earth pressures and settlement problems (Fenton and Griffiths 2008). Further work by Ching et al. (2017) has also shown the effect. In reliability-based design, the worst-case spatial correlation length is important (Fenton and Griffiths 2003), because in the absence of high quality and plentiful field data, it can be used in preliminary studies to ensure a conservative design.

Paice and Griffiths (1997) first applied RFEM to slope stability analysis, while Griffiths et al. (2007) first reported the worst-case phenomenon in it. This paper will extend the work of Griffiths et al. (2007), to achieve a more systematic understanding of the conditions under which a worst-case spatial correlation length occurs in probabilistic stability analysis of clay slopes. In this paper, RFEM will be applied to the stability analysis of 2D undrained slopes with both isotropic and anisotropic spatial correlation lengths. In the latter case, slopes will be assumed to have been excavated in layered soil where the horizontal spatial correlation length is significantly higher than that in the vertical direction typical of stratified deposits (Phoon and Kulhawy 1999). For the anisotropic case, only spatial variability in the vertical direction is considered, with the horizontal spatial correlation length assumed infinite (Griffiths et al. 2009; Allahverdizadeh et al. 2015).

2 RFEM Model

Figure 1 shows a typical FE mesh employed in stability analysis of a clay slope by RFEM. The RFEM merges elastic-plastic finite-element analysis with random field theory (Vanmarcke 1984; Fenton and Vanmarcke 1990). This methodology performs Monte-Carlo analysis, where each simulation involves generation of a random field of undrained strength over the mesh, followed by the application of gravity loading. If the algorithm is not able to converge within 500 iterations, the slope is deemed to have failed. Non-convergence indicates no stress redistribution can be found which simultaneously satisfies the Tresca failure criterion and global equilibrium. Preliminary studies have indicated that 500 iterations are enough to indicate failure, and 2000 Monte-Carlo

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simulations are enough to give statistically reproducible results. The probability of slope failure p_f is simply the proportion of those 2000 RFEM analyses which failed.

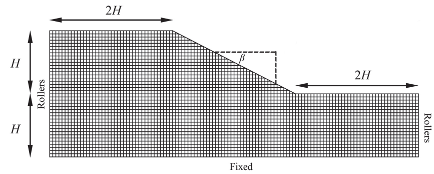

Figure 1. Typical mesh employed in stability analysis of a clay slope by RFEM.

The spatial correlation length θ is a measure of the distance over which properties are essentially similar, i.e. small correlation lengths result in rapid spatial variability, while large correlation lengths result in slow spatial variability. In this study, for an isotropic random field, an exponentially decaying correlation function is assumed as follows

$$\rho = \exp\left(\frac{-2\tau}{\theta}\right) \tag{1}$$

where ρ is the correlation coefficient and τ is the absolute distance between two points in the random field.

The undrained shear strength c_u is modelled as a random variable characterized by a lognormal distribution. The variability of c_u can be expressed in the form of coefficient of variation v_{c_u}, given by

$$v_{c_u} = \frac{\sigma_{c_u}}{\mu_{c_u}} \tag{2}$$

where σ_{c_u} and μ_{c_u} are the standard deviation and mean of c_u respectively. It has been suggested that v_{c_u} typically lies in the range 0.1 to 0.5 (e.g., Lee et al. 1983). In this study, the deterministic parameters include the saturated unit weight $\gamma = 20$ kN/m^3, the undrained friction angle $\phi_u = 0$, the slope height $H = 10$ m and the slope angle β, which is varied in the parametric studies.

3 Worst-Case Spatial Correlation Length

In simple slope reliability analyses, the "single random variable" (SRV) approach, where spatial variability is ignored, has been widely used. The SRV approach assumes infinite horizontal and vertical spatial correlation lengths ($\theta_x = \theta_y \to \infty$). For undrained slopes in such cases, the probability of failure can be derived analytically, as shown in Fig. 4 in Griffiths and Fenton (2004) based on the formula

$$p_f = \Phi\left[\frac{\ln\left(1+v_{c_u}^2\right) - 2\ln\left(\overline{\mathrm{FS}}\right)}{2\sqrt{\ln\left(1+v_{c_u}^2\right)}}\right] \tag{3}$$

where $\Phi[.]$ is the standard normal cumulative distribution function and $\overline{\mathrm{FS}}$ is the deterministic mean factor of safety of the slope assuming a uniform soil with its strength set equal to μ_{c_u}.

Griffiths and Fenton (2004) warned about the dangers of using the SRV approach, because it can lead to an underestimation of p_f (i.e. unconservative) when the mean factor of safety is relatively low or the coefficient of variation is relatively high, i.e. the worst-case phenomenon may be observed under these conditions. To further investigate, a test slope with $\beta = 26.6°$ shown in Fig. 1 is first considered. For this test slope, the factor of safety with uniform properties, can be computed by traditional deterministic approaches (e.g. stability chart of Taylor 1937), to give a factor of safety of $\mathrm{FS} = 1$ when $c_u/(\gamma H) = 0.17$. Since the factor of safety is proportional to the undrained strength in a uniform slope, the mean factor of safety $\overline{\mathrm{FS}}$ is given by

$$\overline{\mathrm{FS}} = \frac{\mu_{c_u}/(\gamma H)}{0.17} \tag{4}$$

3.1 Isotropic case

In this subsection, a dimensionless spatial correlation length $\Theta = \theta / H$ is used for parametric analyses, with the following selected values: $\Theta = 0.01,\ 0.1,\ 0.2,\ 0.5,\ 1.0,\ 2.0,\ 5.0,\ 10.0,\ 100.0$.

3.1.1 Influence of $\overline{\mathrm{FS}}$ *and* v_{c_u}

Figure 2 shows the p_f versus Θ for the $\beta = 26.6°$ slope with different mean factors of safety when $v_{c_u} = 0.5$. When the mean factor of safety is $\overline{\mathrm{FS}} = 1.2$, a pronounced worst case occurs at a spatial correlation length of about $\Theta = 0.2$. As $\overline{\mathrm{FS}}$ is increased, the worst-case spatial correlation length also increases, but the worst-case effect becomes less pronounced, and is barely noticeable for $\overline{\mathrm{FS}} = 1.4$. When $\Theta \to 0$, because the c_u is lognormal, the slope becomes essentially "deterministic" due to local averaging, with a uniform strength fixed at the median of c_u given by $\mu_{c_u} \Big/ \left(1+v_{c_u}^2\right)^{1/2}$. For the undrained slopes shown in Fig. 2, the median corresponds to $\mathrm{FS} > 1$, so $p_f \to 0$. When $\Theta \to \infty$, as expected the RFEM solutions converge on the analytical solutions from Eq. (3) given as horizontal dotted lines. The reason for the worst-case phenomenon is that at extreme values of Θ the results of p_f are fixed as explained above, however intermediate spatial correlation lengths can facilitate the formation of additional failure mechanisms, leading to more failure simulations in the Monte-Carlo process, and a higher p_f. Figure 2 also demonstrates that the SRV approach may be unconservative when $\overline{\mathrm{FS}}$ is relatively low ($\overline{\mathrm{FS}} < 1.4$) for undrained slopes with $v_{c_u} = 0.5$.

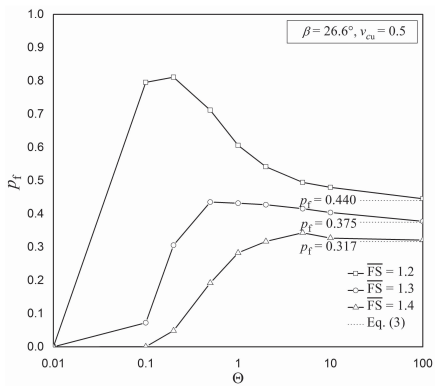

Figure 2. p_f versus Θ with different mean factors of safety for isotropic case.

Figure 3 shows the p_f versus Θ for the $\beta = 26.6°$ slope with different coefficients of variation when

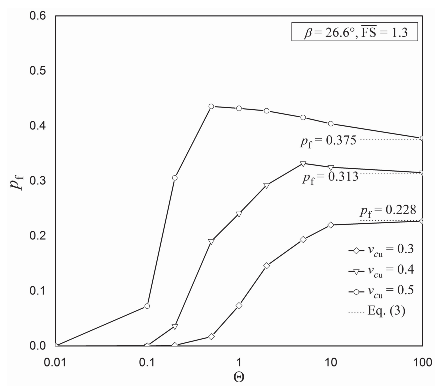

Figure 3. p_f versus Θ with different coefficients of variation for isotropic case.

$\overline{\mathrm{FS}} = 1.3$. The $v_{c_u} = 0.5$ result is the same as the middle plot in Fig. 2. As v_{c_u} is decreased, the worst-case spatial correlation length increases, but the worst-case effect becomes less pronounced, and is not evident for $v_{c_u} = 0.3$. Figure 3 demonstrates that the SRV approach may be unconservative when v_{c_u} is relatively high ($v_{c_u} > 0.3$) for undrained slopes with $\overline{\mathrm{FS}} = 1.3$.

3.1.2 Influence of slope angle

Figure 4 shows the p_f versus Θ for different slope angles with $\overline{\mathrm{FS}} = 1.3$ and $v_{c_u} = 0.5$. It can be seen from Fig. 4 that a worst-case spatial correlation length was observed for all cases, however, the $\beta = 60°$ result gives the most pronounced worst case. Taylor (1948) indicated that 53° slopes represented the transition between deep and shallow critical failure mechanism for uniform undrained slopes. Apparently, for the undrained slopes under consideration, the 60° slope allows more paths than other slope angle cases. For large spatial correlation lengths ($\Theta \to \infty$), probabilities of failure in all cases approach the value given by Eq. (3) of $p_f = 0.375$.

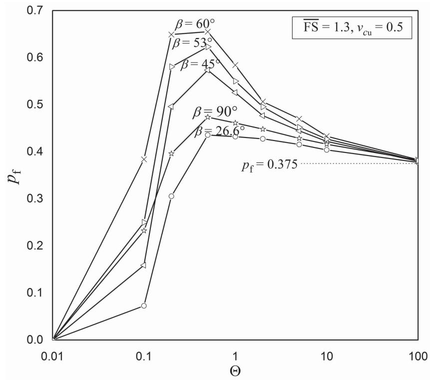

Figure 4. p_f versus Θ with different slope angles for isotropic case.

3.2 Anisotropic case

In this subsection, a dimensionless vertical spatial correlation length $\Theta_y = \theta_y / H$ ($\Theta_x = \infty$) is adopted, with the following selected values: $\Theta_y = 0.01,\ 0.1,\ 0.2,\ 0.5,\ 1.0,\ 2.0,\ 5.0,\ 10.0,\ 100.0$. The horizontal correlation length is set to infinity.

3.2.1 Influence of $\overline{\mathrm{FS}}$ *and* v_{c_u}

Figure 5 shows the p_f versus Θ_y with different mean factors of safety when $\beta = 26.6°$ and $v_{c_u} = 0.5$. It can be

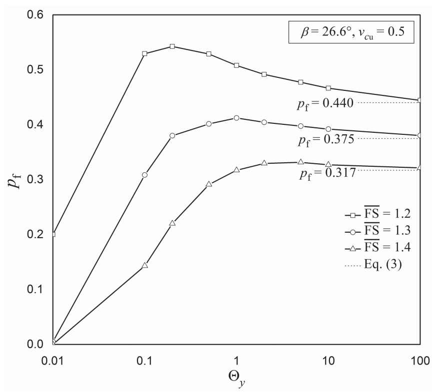

Figure 5. p_f versus Θ_y with different mean factors of safety for anisotropic case.

observed from Fig. 5 that there exists a pronounced worst case occurring at about $\Theta_y = 0.2$ for $\overline{\mathrm{FS}} = 1.2$. For higher values of $\overline{\mathrm{FS}}$, the value of Θ_y corresponding to the maximum p_f increases, but the maximum also becomes less pronounced. For the case of $\overline{\mathrm{FS}} = 1.4$, the maximum is barely noticeable. Figure 6 shows the effect of Θ_y on p_f for different coefficients of variation with $\beta = 26.6°$ and $\overline{\mathrm{FS}} = 1.3$. The result for the case of $v_{c_\mathrm{u}} = 0.5$ corresponds to the middle plot in Fig. 5. It can be observed that, with a decrease of v_{c_u}, the worst-case phenomenon becomes less noticeable. For the case of $v_{c_\mathrm{u}} = 0.3$, the analytical solution is greater than all RFEM results and may be considered conservative. In summary the results shown in Figs. 5 and 6 indicate that: for layered excavated slopes with $v_{c_\mathrm{u}} = 0.5$, the SRV approach gives unconservative solutions if the mean factor of safety is relatively low (i.e. $\overline{\mathrm{FS}} < 1.4$); for layered excavated slopes with $\overline{\mathrm{FS}} = 1.3$, the SRV approach may give unconservative solutions, but only for relatively high coefficient of variation ($v_{c_\mathrm{u}} > 0.3$).

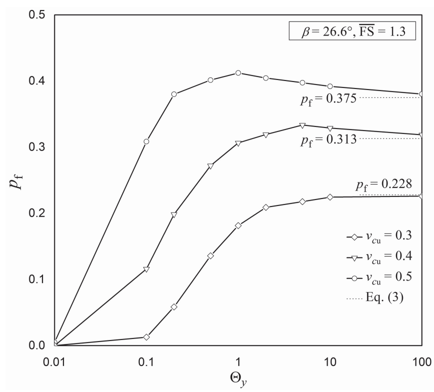

Figure 6. p_f versus Θ_y with different coefficients of variation for anisotropic case.

3.2.2 *Influence of slope angle*

Figure 7 shows the effect of Θ_y on p_f for different slope angles with $\overline{\mathrm{FS}} = 1.3$ and $v_{c_\mathrm{u}} = 0.5$. Similarly, the result for the case of $\beta = 26.6°$ is the same as those with circle symbols in Figs. 5 and 6. It can be observed that the critical Θ_y occurs at the same position for all cases, however, the $\beta = 60°$ result gives the most obvious maxima in p_f.

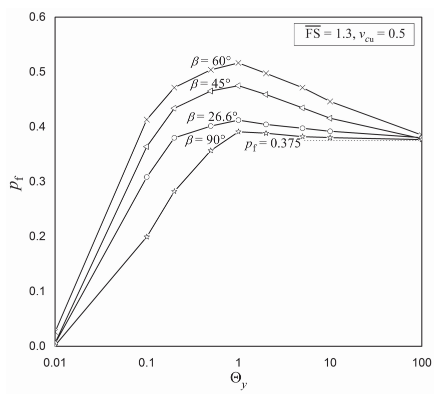

Figure 7. p_f versus Θ_y with different coefficients of variation for anisotropic case.

4 Concluding Remarks

The paper has investigated the worst-case spatial correlation length for 2D excavated clay slopes by RFEM. The worst-case effect has been studied for both isotropic and anisotropic cases. Apart from the location of the maximum p_{f} for $\overline{\mathrm{FS}} = 1.3$ in these two cases, other conclusions are similar. It was shown that the worst-case phenomenon is most pronounced when the mean factor of safety is relatively low (e.g. $\overline{\mathrm{FS}} < 1.4$) and the coefficient of variation of the undrained strength is relatively high (e.g. $v_{c_{\mathrm{u}}} > 0.3$). It should be noted that, a worst-case slope angle close to 60° was also observed (with all other parameters held constant) in both isotropic and anisotropic cases, which has implications for reliability-based design of excavated clay slopes.

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