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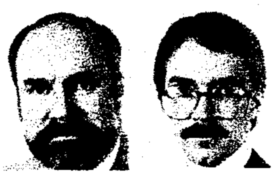
Flexural Capacity of Reinforced Masonry Members



by G. T. Suter and G. A. Fenton

An analysis of a reinforced masonry flexural testing program carried out at Carleton University is presented with data obtained from other published sources. Practical design relationships are proposed in keeping with the current limit state design philosophy to deal with flexural capacity and minimum and maximum reinforcement ratios.

Keywords: compressive strength; ductility; **flexural strength**; flexural tests; **reinforced masonry**; reinforcing steels; **strains**; **stress block**; structural design.

Experimental work has been carried out over the past seven years at Carleton University to study the flexural behavior of reinforced masonry (RM) beams. A total of 97 beams were tested, of which 78 were reinforced concrete masonry (RCM), 5 were grouted reinforced brick masonry (GRBM), and 14 were reinforced brick masonry (RBM). The primary goals of the program were to determine modes of failure, ultimate flexural capacities, and ultimate masonry compressive strains.

This paper first presents details of the Carleton University results and discusses test observations in light of the failure modes and ultimate compressive strains. In addition to the Carleton University data, a further 140 ultimate flexural capacity test results were obtained from several other published sources.[1-6] Finally, all the test results are analyzed and useful design relationships are developed according to the current limit state design philosophy. These relationships deal with ultimate flexural capacity and ductility considerations, i.e., maximum and minimum reinforcement ratios.

About three-quarters of the tested sections were constructed from concrete masonry; thus, most of the following discussion is concerned with RCM members. However, the behavior of the brick masonry was found to be very similar to that of the concrete masonry and so a unified design approach is proposed.

RESEARCH SIGNIFICANCE

The paper deals with experimental and analytical work on reinforced flexural members at the ultimate strength level; it thus contributes to establishing limit state design criteria for reinforced masonry.

TEST PROGRAM

Within the test program at Carleton University, such parameters as effective depth, reinforcement ratio, masonry unit type and strength, and infill type and strength were varied to evaluate their effect on the flexural behavior of RM beams. Most of the beams were continuously grouted and under-reinforced with reinforcement ratios ranging from 0.4 to 2.5 percent. The appendix shows the beam cross sections along with geometries and material strengths as well as the ultimate moment capacities and, in many cases, the ultimate compressive strains.

The beams were constructed at different times during the period from 1977 to 1983 by experienced masons; their quality of workmanship was judged to be average. To prevent premature anchorage failures, bearing plates were welded to the ends of the reinforcing bars prior to placement. The following construction procedure was used:

1. For the RCM and RBM beams, the first, and in many cases only, course of masonry units was laid. For GRBM beams, the two brick wythes were constructed.
2. The reinforcement was placed on suitable chairs in the grout space of RCM and GRBM beams with the end plates mortared tight against the masonry. The RBM reinforcement was placed directly on the mortar bed and covered with a further layer of mortar.
3. The upper courses of masonry were laid for the two-course RCM and multicourse RBM beams.
4. After allowing a period of two days for the mortar joints to harden, plywood forms were secured to the RCM and GRBM beam ends and the fill was poured and vibrated in 3 stages.
5. The beams were then covered with polyethylene for a period of 7 days, at which point the plastic was removed. Curing continued for 21 to 25 days under

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normal laboratory conditions of about 21 C and 50 percent relative humidity.

To obtain a measure of the inherent compressive and flexural strength of the masonry beams, a set of secondary specimens accompanied each beam test series. In general the secondary specimens consisted of:

Block units — At least 5 units were selected at random from the lot provided for each beam series.

Mortar cubes — At least six 50 mm mortar cubes were taken at random during the construction of each beam series and cured in the same manner as the beams themselves.

Absorbent prisms — At least five 100 x 100 x 200 mm absorbent prisms were taken at random from each different fill material used, generally from within the compression region of the beam. These specimens were cured in the same manner as the beams.

Grouted prisms — At least five 2-stack grouted concrete block prisms (h/t = 2.0) were constructed from each type of block and fill used. The prisms were cured in the same manner as the beams and tested in the direction in which beam compressive stresses would occur.

Brick prisms — At least 3 brick prisms were constructed from each type of brick and mortar used. Two types of prisms were employed: (a) a double brick prism constructed of four bricks laid 2 bricks high in a stack bond pattern and loaded parallel to the bed joint (Beams CU79-CU83), and (b) a five-stack brick prism loaded perpendicular to the bed joints (Beams CU84-CU97). These specimens were cured in the same manner as the beams.

Reinforcement — At least three coupons were selected at random from each bar diameter and lot.

The beams were tested between 28 and 32 days after construction using a two-point loading system. The a/d ratio ranged from 1 to 6, since many of the beam series were also designed to investigate shear capacity. This paper presents only those beams which were carried on to a flexural failure.

TEST RESULTS

The Appendix lists beam details, ultimate moment capacities, and the average strains observed on the top surface just prior to failure. The last column indicates the actual mode of failure of the beams. In the cases where a shear failure occurred, the beam was bandaged with threaded external stirrups outside the maximum moment region and loading was continued until a flexural failure was achieved.

Prior to testing, the beams were inspected and one surface was whitewashed to highlight the crack progression during loading. In general, for the grouted beams shrinkage cracks were observed in the upper surface of the fill both in the transverse direction and along the joint between the fill and shell. The latter indicated that some debonding occurred between the fill and the shell although it was not known how deep these cracks penetrated. If the debonding does not extend below the beam's neutral axis position, which is not unlikely since most of the shrinkage occurs near the drying surface, it may in fact be beneficial. This potentially beneficial effect occurs because as stresses approach ultimate, the Poisson ratio of mortar and grout is often quite a bit higher than that of the enclosing shell. Thus, within the compression zone, the mortar or grout fill will tend to push outward on the shell and failure will likely occur through spalling of the shell. However, if a small crack is present between the fill and shell, it must be closed before the shell feels this outward force, effectively reducing its magnitude.

On the other hand, the cracking observed in the transverse direction is probably detrimental to the beam's flexural capacity. Since the compressive forces occurring above the neutral axis of the beam cannot be transmitted through a crack until it has closed, they must be supported by the shell. This may result in much higher stresses arising in the shell than in the fill and failure again will likely be due to spalling or crushing of the shell.

Failure of the beams was generally observed to be initiated by crushing or spalling of the shell. Nevertheless, the flexural behavior of the beams was very similar to that of reinforced concrete (RC), displaying both an over-reinforced mode of failure typified by crushing of the masonry prior to yielding of the steel and an under-reinforced mode of failure where initial tensile yielding of the reinforcement is followed by crushing of the masonry. In one of the under-reinforced cases (Beam CU57), the beam deflection was so large that loading was discontinued before crushing was achieved in the masonry, but for all practical purposes the beam had achieved its ultimate flexural capacity. The mortar for the RBM Beams CU93 and CU94 was an excessively weak mix of 1 part masonry cement, ½ part lime, and 4½ parts sand by volume. It was believed that the ultimate strain measurement taken on Beam CU93 was unrepresentative; therefore, the value was not considered in later analyses. The central reinforcing bar of Beam CU96 was left free to slide though the end plates and bond failure resulted prior to yielding of the bar. Failure of Beam CU97 was by rupture of the truss tie reinforcement rather than by crushing of the masonry. The results of the latter two beams were ignored in this analysis.

To determine the ultimate compressive strain at the upper surface of the beams, 59 of the tested beams were instrumented for Demec extensometer measurements. The number of measurement positions on the upper surface varied from two to six, half on the shell and half on the fill where present. Strain measurements were taken at each load increment until failure was imminent.

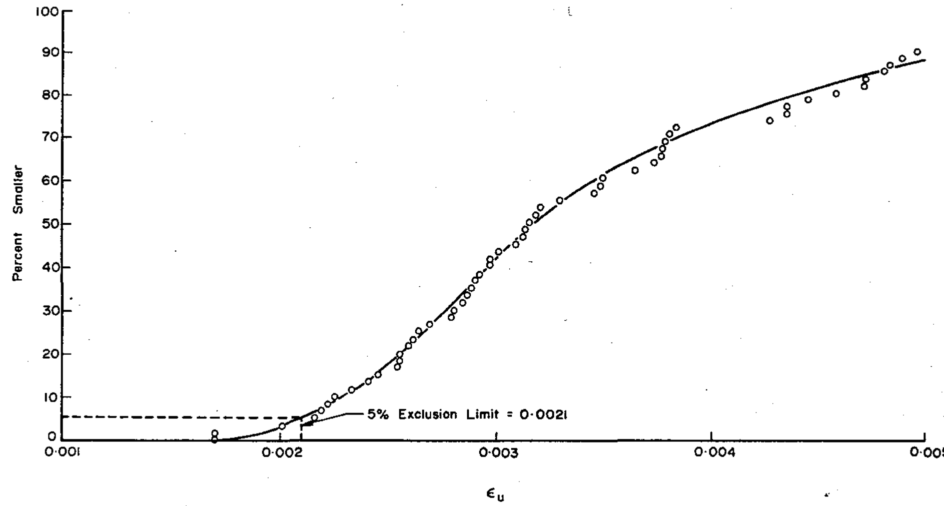

Fig. 1 — Ultimate masonry strain distribution

Two comments can be made about these "ultimate" strain measurements: first, since the final measurement was taken prior to the actual crushing failure, these values are not true measures of the ultimate compressive strains, but rather could be considered to be conservative estimates; second, if a shrinkage crack crosses between the Demec points, the measurements will not accurately reflect the stress-strain relationship of the masonry, i.e., the strains will be larger than they should be for a given stress state.

In general, the strain measurements were taken over what were deemed to be crack-free areas.

In a composite material such as masonry, there is some question as to which strain value measured on a given beam should be used to represent the ultimate compressive strain if, as was the case, the strains were measured at more than one location. One line of thought favors selecting the largest fill and shell values and using both in the ultimate strain distribution. This method is based on the premise that masonry crushing is most likely to occur where the strains are the highest; unfortunately, this could not be verified during the testing. Using this method, the lower 5 percent exclusion limit on the ultimate strain of RCM members was determined to be about 0.003.[7]

Another approach is to average all the surface strain values to arrive at a single value for each beam. The advantage to this method is that it minimizes possible errors due to the presence of shrinkage cracks. For RCM members, this approach yields a 5 percent exclusion limit of about 0.0024. The combined ultimate strain distribution for both concrete block and clay brick beams is shown in Fig. 1. The 5 percent exclusion limit can be seen from this figure to have a slightly reduced value of 0.0021. It appears that a unified approach that uses an ultimate masonry strain value of 0.002 may be best. Note that the average ultimate strain in this case is about 0.0034.

TEST RESULTS FROM OTHER SOURCES

The beam tests carried out at Carleton University represented a rather narrow range of the possible RM flexural members. Therefore, before general design criteria can be proposed it is first necessary to study masonry beams with a wider range of effective depths, then to determine the effect of the number of courses on flexural capacity, and finally to test reinforced masonry wall sections.

A further 140 flexural test results were obtained from a variety of sources.[1-6] These include 55 RCM beams ranging from 1 to 5 courses in depth, 38 RCM walls, and 47 GRBM beams. In total, 237 RM flexural test results are presented in this paper.

Before a direct comparison of the flexural data can be made, the compressive strength of the masonry assemblage f'_m in each test must be brought to a common denominator, as if determined from a common test procedure. In the flexural tests performed by the authors, Khalaf,[3] and Sinha,[6] the value of f'_m was based on masonry prism tests. For those experimental programs that did not carry out this particular prism test,[1,2,4,5] the equivalent concrete masonry prism strength was derived from the work of Eskenazi[8] and Drysdale[9] to be

$$f'_{mg} = \xi\,(0.644\,f'_{bl} + 1.184\,\sqrt{f'_{mort}} - 3.405) + (1\text{-}\xi)\,(1\text{-}1.15\xi)\,\sigma_{cg} \qquad (1)$$

The equivalent brick masonry prism strength was estimated from Grimm* to be

$$f'_m = 9.105\,(3.37 + ln\,(c/s))\,(ln(f'_{br}) - 2.253)\ \text{MPa} \qquad (2)$$

LIMIT STATE DESIGN ANALYSIS

Since, as noted previously, the flexural behavior of RM members is very similar to that of RC beams, the design of reinforced masonry should follow as closely as possible the current practice used for the design of reinforced concrete.[10] The major justification for this approach lies in its simplicity for the practicing engi-

*Grimm, C. T., "Dimensional Stability of Clay Masonry," unpublished internal report, Austin, Nov. 1981.

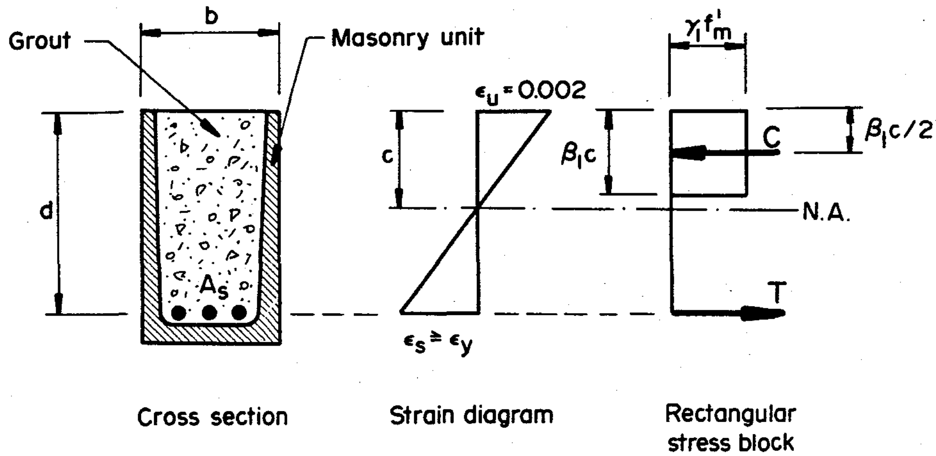

Fig. 2 — Typical under-reinforced ultimate strength conditions for RM beam

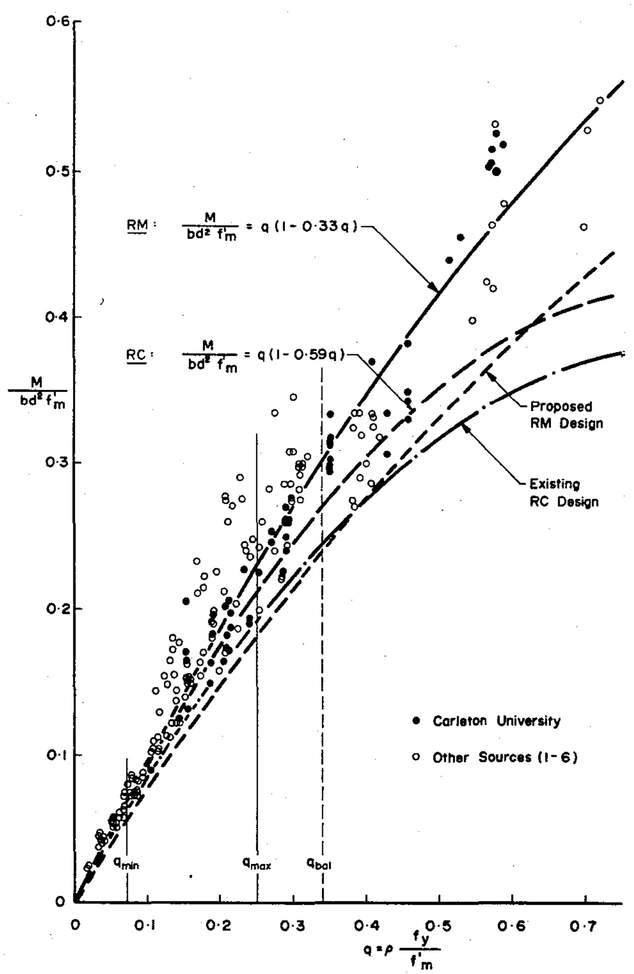

Fig. 3 — Ultimate flexural capacity of RM test sections

neer. In this section the parameters of the equivalent rectangular stress block, shown for an RCM beam in Fig. 2, will be determined in a statistical fashion from the test results given in Appendix B and References 1 through 6. Design relations for singly reinforced rectangular masonry beams will be proposed from this data and comparisons will be made with traditional RC requirements. For under-reinforced design no particular advantage is gained through the use of a nonrectangular stress block.

Failure moment capacities M_u of all under-reinforced test results have been plotted in Fig. 3 in the convenient nondimensional form used for reinforced concrete test results by Mattock et al.[11] in 1961. Although there are only about one-half as many RM as RC results, the scatter and trend of the data is quite similar. In both cases the line of best fit was found to be a parabola of the form

$$\frac{M_u}{bd^2 f'_m} = q\,(1 - \alpha q) \qquad (3)$$

where $q = \rho f_y / f'_m$. Mattock found the line of the best fit to give $\alpha = 0.59$ for reinforced concrete. Using a least square parabolic fit method, α was found to have a value of 0.33 for reinforced masonry and 0.32 for RCM results alone.

Considering Fig. 2 and applying the design theory derived for reinforced concrete, it can be determined that

$$\frac{M_u}{bd^2 f'_m} = q(1 - \frac{l}{2\gamma_1} q) \qquad (4)$$

Note that only γ_1 can be determined from the under-reinforced failure data. β_1 drops out of the relationships and so as long as $\beta_1 \neq 0$, its actual magnitude has no effect on the predicted ultimate moment capacity of an under-reinforced, singly reinforced, rectangular section.

Comparing Eq. (3) and (4), it can be seen that $\gamma_1 = \frac{1}{2}\alpha$, which for reinforced concrete gives γ_1 equal to 0.85 as expected. For reinforced masonry, however, γ_1 is found to be equal to 1.50. The fact that this value is so much higher than that used in reinforced concrete design has some interesting implications. If the true stress distribution is assumed to be parabolic, the maximum masonry stress reaches 1.70 f'_m as compared to 0.96 f'_c in reinforced concrete. The value of f'_m, as a measure of flexural strength, may be underestimated by as much as 75 percent when compared to the corresponding measure f'_c.

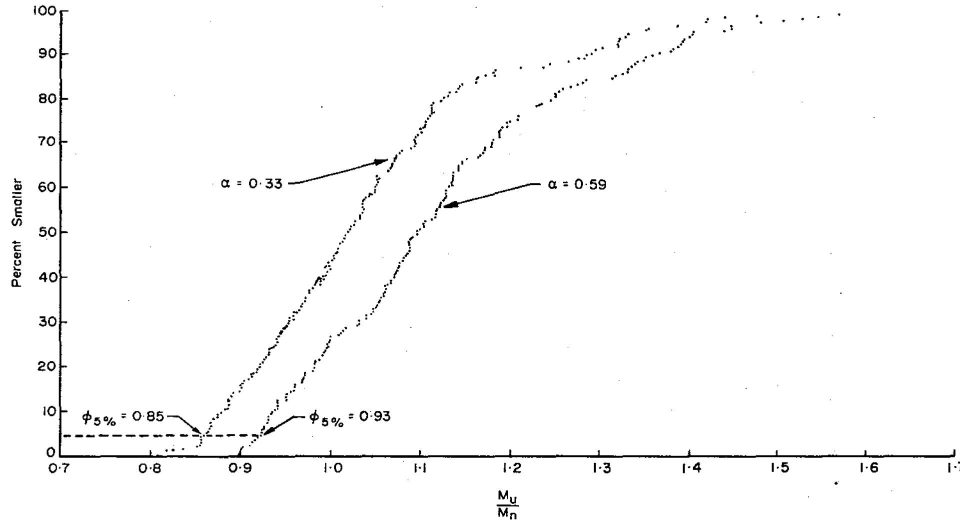

Fig. 4 — Observed over predicted failure moment distribution for under-reinforced RM sections

In any case, there appears to be a considerable discrepancy between the mechanics of pure axial compression and pure bending in masonry which is not present in the more homogeneous reinforced concrete.

Using the predicted ultimate moment capacity for masonry flexural members given by

$$M_n = \rho f_y b d^2 (1 - 0.33 \rho f_y/f'_m) \quad (5)$$

the distribution of the observed over the predicted failure moment ratio has been plotted in Fig. 4. At the lower 5 percentile, the capacity reduction factor ϕ can be estimated directly from the test results as 0.85. However, the variability of workmanship obtained under laboratory conditions is probably lower than that obtained in the field. Grimm[12] estimates that a coefficient of variation of 10 percent or less represents excellent quality control, 10 to 15 percent is good control, and 20 percent is the upper limit for fair control. Mirza and MacGregor[13] found that safety index values between 3.0 and 3.5 are representative of current RC design in Canada. If the safety index for RM design is conservatively selected at 3.5, the capacity reduction factor has been determined on a statistical basis by Lim[7] to be about 0.82 for a workmanship variability of 10 percent and 0.60 for a workmanship variability of 20 percent. Such typical North American masonry codes as the Canadian masonry code[14] require a minimum level of supervision insuring reasonably good control and so a capacity reduction factor of 0.80 would be recommended. Fig. 4 shows that all the test results lie above the observed to predicted moment capacity ratio of 0.80.

Fig. 4 also shows the distribution of the observed to predicted moment capacity ratio using the RC design relationship with $\alpha = 0.59$. The capacity reduction factor corresponding to the lower 5 percentile is in this case about 0.93 and all points lie above the ϕ factor of 0.90 for reinforced concrete.

In summary, the ultimate flexural capacity of a rectangular under-reinforced masonry section is best predicted by Eq. (5). However, the design capacity can be predicted by either

$$M_d = \phi \rho f_y b d^2 (1 - 0.33 \rho f_y/f'_m) \quad (6)$$
$$\phi = 0.8$$

or the RC relationship

$$M_d = \phi \rho f_y b d^2 (1 - 0.59 \rho f_y/f'_m)$$
$$\phi = 0.9$$

The nominal and design curves for both methods are shown in Fig. 3, where it can be seen that for the lower values of q the two design methods are almost identical despite the large difference in γ_1.

DEPTH OF EQUIVALENT RECTANGULAR STRESS BLOCK

As pointed out earlier, the parameter governing the depth of the equivalent rectangular stress block β_1 cannot be determined directly from the under-reinforced failure data. Although the design of under-reinforced rectangular sections is independent of β_1, β_1 becomes important in the design of doubly reinforced, nonrectangular, or over-reinforced sections and is particularly important in determining balanced conditions (simultaneous yielding of the steel and crushing of the masonry).

During the 1950's the parameters of the equivalent rectangular stress block used in RC design were determined experimentally using special U-shaped, eccentrically loaded prismatic concrete specimens, which allowed the determination of both γ_1 and β_1 directly. To the authors' knowledge, this type of test has not yet been performed using masonry specimens and so β_1 must be determined by other means.

Since the stress in the reinforcement at the time of failure of an over-reinforced section is dependent on the position of the neutral axis, it is possible to determine β_1 explicitly from the over-reinforced failure data. This premise rests on two assumptions: first, that the

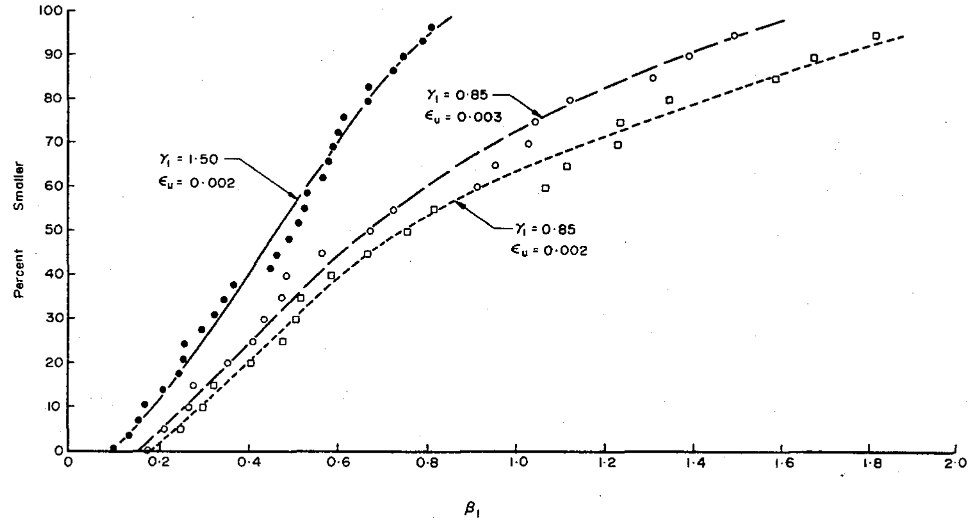

Fig. 5 — Distribution of β_1 values derived from over-reinforced test data using Eq. (9)

strain in the masonry and steel is directly proportional to the distance from the neutral axis, and second, that the parameters ϵ_u and γ_1 are deterministic, that is, not random variables themselves.

The first assumption has been found largely true for reinforced masonry, even after cracking, from several tests carried out at Carleton University. The second assumption is obviously in error since considerable scatter is present in both the values of ϵ_u and γ_1. This is particularly true of γ_1, whose value has been determined using under-reinforced test results. It is likely that the value of γ_1 will change under the lesser strain gradients associated with over-reinforced failures. Note that for pure axial compression, as in prism tests, $\gamma_1 = 1.0$. In design, however, these parameters are assumed to be constant or deterministic and independent of the actual mode of failure, and scatter being taken into account by the capacity reduction factor. Thus, it is possible to proceed using assumed deterministic values for ϵ_u and γ_1, bearing in mind that the resultant values of β_1 will satisfy the empirical design relations, but will only partially reflect the true neutral axis position. When calculating balanced conditions, the satisfaction of the design relations is the more important criterion.

By considering the strain gradient shown in Fig. 2 and applying static equilibrium equations for over-reinforced sections, it can be shown that the depth to the neutral axis c is given by

$$c = \frac{\sqrt{R^2 + 4\beta_1 RSd} - R}{2\beta_1 S} \tag{7}$$

where $R = A_s E_s \epsilon_u$ and $S = \gamma_1 \, bf'_m$. Note that only the positive root has been considered. Also from statics

Table 1 — Average β_1 values derived from over-reinforced RM test data

ϵ_u \ γ_1	1.50	0.85
0.002	0.45	0.84
0.003	0.38	0.71

$$M_u = R\left(\frac{d}{c} - 1\right)(d - \tfrac{1}{2}\beta_1 c) \tag{8}$$

which after substituting for c using Eq. (7) and simplifying gives

$$\beta_1^2(Rd)^2 + \beta_1(4RdM_u - 2R^2d^2 - 4RSd^3) + \left(4M_u^2 + 4RdM_u + \frac{2M_uR^2}{S}\right) = 0 \tag{9}$$

In general, the solution of Eq. (9) leads to two positive β_1 values for each over-reinforced test result, one corresponding to $f_s < f_y$ and the other to $f_s > f_y$. Since the beam failures were known to be over-reinforced, only the former value was considered. In some cases both roots were found to correspond to steel stresses greater than yield, indicating that the beam would be predicted to be under-reinforced using the assumed values of γ_1 and ϵ_u. In a few other cases no solution was possible, which indicated that the average masonry stress was in fact greater than that predicted by the assumed value of γ_1.

The distribution of the β_1 values derived from the over-reinforced test data are plotted in Fig. 5 for various assumed values of ϵ_u and γ_1. Table 1 lists the average β_1 value obtained under each set of assumptions. A review of Table 1 indicates that for the proposed RM parameters γ_1 and ϵ_u of 1.50 and 0.002, respectively, the optimum value for β_1 is 0.45. This appears to be too small until one remembers that γ_1 appears to be too high, leading again to the conclusion that perhaps f'_m underestimates the true flexural compressive strength. Interestingly, when an ultimate masonry strain of 0.002 is assumed along with the RC value for γ_1 of 0.85 the average value of β_1 is very nearly 0.85.

The suitability of the various sets of parameter values can be partially ascertained by considering the reliability of the predicted over-reinforced failure moments. Using Eq. (7) and (8), the ratios of the observed

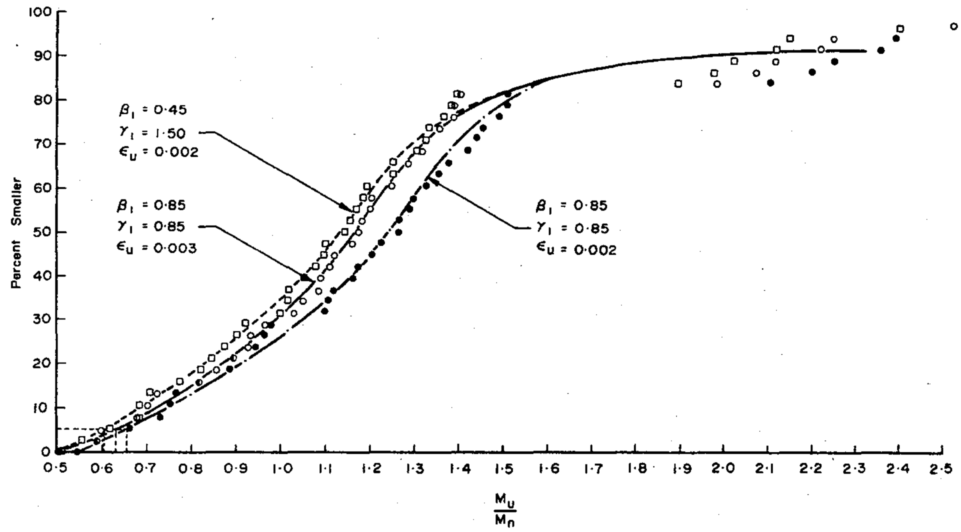

Fig. 6 — Distribution of observed over predicted over-reinforced failure moment ratios for three different sets of design parameters

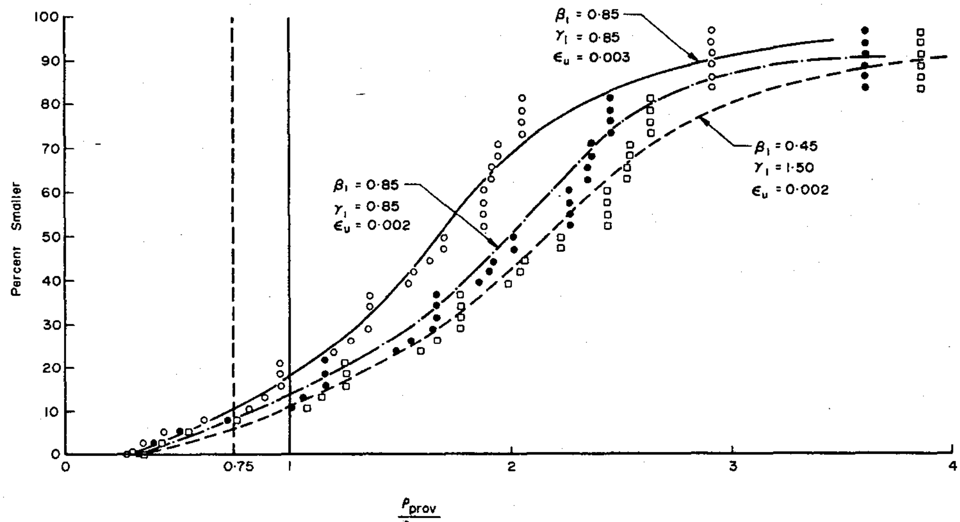

Fig. 7 — Distribution of provided over predicted balanced reinforcement ratios for over-reinforced test results

to predicted failure moments are plotted in Fig. 6 for three sets of parameter values: (1) those determined experimentally from RM results: $\gamma_1 = 1.50$, $\beta_1 = 0.45$, and $\epsilon_u = 0.002$, (2) those in use for RC design: $\gamma_1 = 0.85$, $\beta_1 = 0.85$, and $\epsilon_u = 0.003$, and (3) a combination of the above: $\gamma_1 = 0.85$, $\beta_1 = 0.85$, and $\epsilon_u = 0.002$. The third curve [Item (3)] has the highest 5 percentile exclusion limit of about 0.65; however, the parameter set giving the best agreement between predicted and observed over-reinforced failure capacity is that of Item (1). This is primarily because the parameters $\gamma_1 = 1.50$ and $\epsilon_u = 0.002$ resulted in the greatest number of usable solutions to Eq. (9). Under any assumption, the over-reinforced flexural capacity is overestimated in about 30 percent of the cases and a capacity reduction factor of about 0.6 would be recommended for over-reinforced RM design.

DUCTILITY CONSIDERATIONS

To insure against brittle failure and to provide some ductility in a given system, North American reinforced concrete codes[10,15] limit the amount of reinforcement provided in a member to 75 percent the amount required to achieve balanced conditions (simultaneous yielding of the steel and crushing of the concrete). Applying the same concept to reinforced masonry, the balanced reinforcement ratio for a singly reinforced rectangular beam can be determined from

$$\rho_{bal} = \beta_1 \gamma_1 (f'_m/f_y) \left(\frac{\epsilon_u}{\epsilon_u + \epsilon_y}\right) \tag{10}$$

The distribution of the ratio ρ_{prov}/ρ_{bal} is plotted in Fig. 7 for all the over-reinforced failure test data. Special note should be made of the over-reinforced failures that occur at a provided steel ratio less than that predicted by ρ_{bal} ($\rho_{prov}/\rho_{bal} < 1.0$). According to Fig. 7, some 6 to 11 percent of the masonry beams designed and built using a maximum steel ratio of $\rho_{max} = 0.75\ \rho_{bal}$ will still experience over-reinforced failures. Allen[16] found that reinforced concrete beams designed using $\rho_{max} = 0.75\ \rho_{bal}$ will still be over-reinforced 8 percent of the time for good workmanship and 18 percent of the time for poor workmanship, based on a 4 hr load duration. Thus

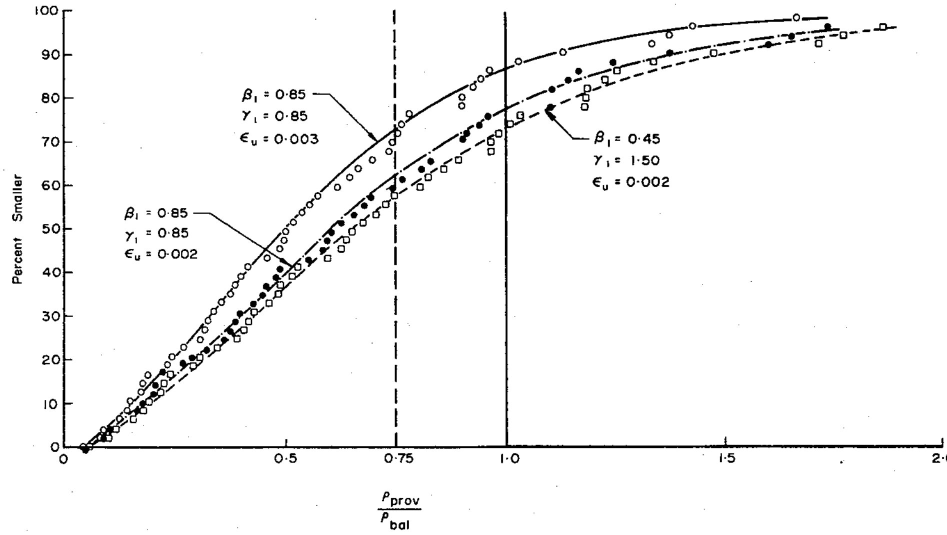

Fig. 8 — Distribution of provided over predicted balanced reinforcement ratios for under-reinforced test results (every fourth point plotted)

Table 2 — Percentages of under-reinforced failures with $\rho > 0.75\ \rho_{bal}$ (P_1) and over-reinforced failures with $\rho < 0.75\ \rho_{bal}$ (P_2)

	$\beta_1 = 0.85$, $\gamma_1 = 0.85$, $\epsilon_u = 0.003$	$\beta_1 = 0.85$, $\gamma_1 = 0.85$, $\epsilon_u = 0.002$	$\beta_1 = 0.45$, $\gamma_1 = 1.50$, $\epsilon_u = 0.002$
P_1, percent	27	37	42
P_2, percent	11	8	6

Table 3 — Relative merits of three design parameter sets

Criteria	$\beta_1 = 0.45$, $\gamma_1 = 1.50$, $\epsilon_u = 0.002$, $\phi = 0.80$	$\beta_1 = 0.85$, $\gamma_1 = 0.85$, $\epsilon_u = 0.002$, $\phi = 0.90$	$\beta_1 = 0.85$, $\gamma_1 = 0.85$, $\epsilon_u = 0.003$, $\phi = 0.90$
Prediction of under-reinforced ultimate flexural capacity	○	◑	◑
Prediction of under-reinforced design flexural capacity	○	○	○
Agreement between β_1 and over-reinforced failure data	○	◑	●
Prediction of over-reinforced flexural capacity	○	●	◑
Fewest unexpected over-reinforced failures ($\rho < \rho_{bal}$)	○	◑	●
Fewest unexpected under-reinforced failures ($\rho > \rho_{bal}$)	●	◑	○

Key: ○ ◑ ● (Best → Worst)

it appears that RM flexural members have about the same probability of being ductile as RC members when ρ_{bal} is calculated according to Eq. (10) and $\rho_{max} \leqslant 0.75\ \rho_{bal}$. To insure ductility for earthquake design in RC, Allen recommends that ρ_{max} be reduced to at most 0.50 ρ_{bal}. This recommendation would also be appropriate for RM members. Note that the parameter set $\beta_1 = 0.45$, $\gamma_1 = 1.50$, and $\epsilon_u = 0.002$ gives the lowest probability, about 6 percent, of a member failing in an over-reinforced mode when designed with $\rho_{max} = 0.75\ \rho_{bal}$. However, all three parameter sets are very close.

The distribution of the ratio ρ_{prov}/ρ_{bal} derived from the under-reinforced test data is shown in Fig. 8 for the three different parameter sets. Note that 15 to 25 percent of the beams exhibiting under-reinforced modes of failure have reinforcement ratios exceeding ρ_{bal}. The optimum design appears to involve some trade-off between the percentage of under-reinforced failures with $\rho > 0.75\rho_{bal}$ (P_1) and the percentage of over-reinforced failures with $\rho < 0.75\ \rho_{bal}$ (P_2). Since over-reinforced failures are less desirable from a safety standpoint, it is better to minimize P_2 than P_1. Table 2 lists the values of P_1 and P_2 for the three parameter sets.

Although the authors know of no definitive study on acceptable values of P_1 and P_2, the parameter set $\beta_1 = 0.85$, $\gamma_1 = 0.85$, and $\epsilon_u = 0.002$ gives a P_2 value of 8 percent which corresponds to that for reinforced concrete built with good workmanship.

The relative merits of the three parameter sets are summarized in Table 3. The best-worst ratings only strictly apply in the horizontal direction. In the vertical direction it must be left to a code committee to decide on the relative importance of the various criteria. Although the parameter set $\beta_1 = 0.45$, $\gamma_1 = 1.50$, and $\epsilon_u = 0.002$ appears to be the best, it gives a fairly low ρ_{max} value of

$$\rho_{max} = (0.75)\,(0.45)\,(1.50)\left(\frac{0.002}{0.002 + 0.002}\right)(f'_m/f_y)$$
$$= 0.25\, f'_m/f_y$$

which is shown in Fig. 3 and is fairly restrictive to the designer. Alternatively, the parameters used in RC design, $\beta_1 = 0.85$, $\gamma_1 = 0.85$, and $\epsilon_u = 0.003$ result in a 30 percent increase in ρ_{max} with only a 5 percent increase in the number of unexpected over-reinforced failures.

MINIMUM REINFORCEMENT

To insure a ductile failure in a beam after initial cracking, the area of tension reinforcement provided must be at least sufficient to develop the same tensile capacity as that of the surrounding masonry. Due to the presence of weaker joints, the flexural tensile strength of masonry will generally be less than that found in concrete. Tests performed by Drysdale*[18] indicate flexural tensile strengths for tension parallel to the bed joints of about 1.7 MPa (cov = 11 percent) for grouted concrete masonry and 2.35 MPa (cov = 20 percent) for brick masonry. Again choosing a unified design approach, the upper 5 percentile limit of the brick masonry tensile strength would be about 2.8 MPa, which can be assumed to be the maximum tensile strength of masonry.

Equating the flexural capacities of the reinforced and unreinforced sections gives

$$\phi A_s f_y (d - \tfrac{1}{2} \beta_1 c) = \frac{f_r I_g}{y} \qquad (11)$$

Assuming $h = 1.1d$, $\frac{1}{2}\beta_1 c = 0.03d$, and $y = \frac{1}{2}h$

$$\rho_{min} = \frac{0.208 f_r}{\phi f_y}$$

which for $f_r = 2.8$ MPa, and choosing ϕ conservatively as 0.80 gives

$$\rho_{min} = \frac{0.73}{f_y}$$

Comparing this value with the ρ_{min} of $1.4/f_y$ used in RC design indicates that the lower tensile strength of masonry obviously requires a lower ρ_{min} value. The corresponding q_{min} value of 0.073, assuming a typical f'_m of 10 MPa, has been shown in Fig. 3.

CONCLUSIONS

1. Reinforced masonry flexural members behave similarly to reinforced concrete members and do not appear to be significantly affected by normal shrinkage cracking of the infill.

2. Based on a 5 percent exclusion limit, the combined ultimate masonry compressive strain was found to be 0.002. This value increased to 0.0024 when concrete masonry alone was considered. The average combined ultimate masonry compressive strain was found to be 0.0034.

3. Where the determination of f'_m is based on the results of prism tests, the use of an equivalent rectangular stress block with a constant stress of 1.50 f'_m gives good agreement between the predicted and observed ultimate moment capacities of the under-reinforced masonry sections.

4. The design moment capacity of an under-reinforced masonry section is equally well predicted using either $\gamma_1 = 1.50$ with a capacity reduction factor $\phi = 0.80$ or by using the RC relationship with $\gamma_1 = 0.85$ and $\phi = 0.90$, within the usable range of reinforcement ratios. All of the observed under-reinforced moment capacities were in excess of those predicted using either method.

5. The ratio of depth of the equivalent stress block to depth of the neutral axis β_1 was found to have an average value of 0.45 for $\gamma_1 = 1.50$ and $\epsilon_u = 0.002$. An average value of 0.85 was determined for $\gamma_1 = 0.85$ and $\epsilon_u = 0.002$.

6. The maximum reinforcement ratio, $\rho_{max} = 0.75\ \rho_{bal}$ was found to range from $0.25\ f'_m/f_y$ to $0.33\ f'_m/f_y$, depending on the choice of design parameters. The corresponding numbers of unexpected over-reinforced failures varied from 6 to 11 percent, respectively.

7. Based on the more conservative capacity reduction factor of 0.80, the minimum reinforcement ratio for masonry ρ_{min} is proposed as $0.73/f_y$.

NOTATION

A_s	=	area of tension reinforcement, mm²
a	=	shear span from load point to adjacent support, mm
b	=	width of compression face of member, mm
c	=	distance from extreme compression fiber to neutral axis, mm
c/s	=	volumetric ratio of cement to sand in mortar, dimensionless
d	=	distance from extreme compression fiber to centroid of tension reinforcement, mm
E_s	=	modulus of elasticity of steel, MPa
$f'_{b,t}$	=	compressive strength of concrete block unit based on net area, MPa
$f'_{b,r}$	=	compressive strength of clay brick unit, MPa
f'_c	=	specified compressive strength of concrete, MPa
f'_m	=	mean compressive strength of masonry assemblage, MPa
f'_{mg}	=	compressive strength of grouted concrete block masonry, MPa
f'_{mort}	=	compressive strength of mortar, MPa
f_r	=	modulus of rupture of masonry, MPa
f_s	=	stress acting in tension reinforcement, MPa
f_y	=	yield stress of tension reinforcement, MPa
h	=	overall depth of section, mm
I_g	=	moment of inertia of gross masonry section about the centroidal axis, mm⁴
M_d	=	design moment capacity of RM members, kN·m
M_n	=	predicted or nominal ultimate moment capacity of RM members, kN·m
M_u	=	ultimate test moment capacity of RM members, kN·m
P_1	=	percentage of masonry members failing in a under-reinforced mode with $\rho > 0.75\ \rho_{bal}$
P_2	=	percentage of masonry members failing in an over-reinforced mode with $\rho < 0.75\ \rho_{bal}$
q	=	reinforcement index, dimensionless
q_{bal}	=	reinforcement index at balanced conditions
$q_{max,min}$	=	maximum and minimum reinforcement indexes
y	=	distance from the centroidal axis of gross section, neglecting reinforcement, to the extreme fiber in tension, mm
α	=	ultimate moment capacity coefficient
β	=	safety index
β_1	=	ratio of the depth of the equivalent rectangular stress block to the total depth of the compressive stress distribution
γ_1	=	ratio of the effective compressive stress acting in the equivalent stress block to f'_m
ϵ_s	=	strain in the tension reinforcement
ϵ_u	=	ultimate masonry compressive strain
ϵ_y	=	yield strain of the tension reinforcement
ξ	=	ratio of the net to gross block unit area

*Drysdale, R. G., and Hamid, A. A., "Effect of Grouting on the Flexural Tensile Strength of Concrete Block Masonry," submitted to *Masonry Society Journal,* 1984.

ρ = ratio of tension reinforcement area to masonry area, *bd*
ρ_{bal} = ratio of tension reinforcement area to masonry area *bd* under balanced conditions
$\rho_{max,min}$ = maximum and minimum ratio of tension reinforcement area to masonry area, *bd*
ρ_{prov} = provided ratio of tension reinforcement area to masonry area *bd*
σ_{cg} = compressive strength of grout as determined from absorptive prism tests, MPa
ϕ = capacity reduction factor
B = bond failure between reinforcement and masonry
ED = testing discontinued due to excessive deflection
FO = over-reinforced flexural failure
FU = under-reinforced flexural failure

SI CONVERSIONS

1 ksi = 6.895 MPa
1 in. = 25.4 mm
1 ft-kip = 1.356 kN·m

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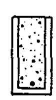
APPENDIX — CARLETON UNIVERSITY FLEXURAL TEST RESULTS

Beam No.	Cross-section	Reinf't.	ρ (%)	f_y (MPa)	d (mm)	b (mm)	f'_{bl} (MPa)	f'_{gr} (MPa)	f'_{mort} (MPa)	f'_m (MPa)	ϵ_u (mm/m)	M_u (kN·m)	Failure Mode
CU01		1-15M	0.802	510	129	194	26.3	17.7	20.3	11.6	3.49	11.94	FU
CU02								17.7	16.9	11.6	3.73	11.39	FU
CU03								25.8	9.4	19.0	4.45	12.12	FU
CU04								25.8	9.4	19.0	4.58	11.49	FU
CU05							21.2	17.7	20.3	11.7	2.69	11.15	FU
CU06											2.80	11.02	FU
CU07											3.77	11.19	FU
CU08											3.29	12.74	FU
CU09							26.3				3.15	11.86	FU
CU10							26.3				3.12	11.74	FU
CU11			0.793	467	130		22.8	24.6	10.1	16.0	5.28	11.96	FU
CU12		2-15M	1.762	467	117		22.8	24.6	10.1	16.0	2.55	18.71	FU
CU13		2-#5,1-#3	2.220	551	109		19.8	19.7	13.7	11.2	-	23.03	FO
CU14											-	19.39	FO
CU15											-	18.97	FO
CU16								22.6			-	20.61	FO
CU17											-	20.32	FO
CU18											-	18.16	FO
CU19				394				14.0		9.8	-	10.23	FO
CU20											-	7.56	FO
CU21											2.33	5.91	FO
CU22											3.18	7.29	FO
CU23				334				12.8			3.20	9.48	FO
CU24		1-#7	1.580	481	126			19.7		11.2	3.09	16.16	FO
CU25								22.6		11.2	4.72	16.33	FO
CU26								14.0		9.8	4.89	13.40	FO
CU27								14.0		9.8	-	12.53	FO
CU28		2-10M	0.660	538	156	194	22.0	17.7	20.3	12.2	3.48	13.86	FU
CU29								17.7	16.9	12.2	3.80	14.42	FU
CU30								25.8	9.4	18.9	3.78	14.59	FU
CU31								25.8	9.4	18.9	-	13.44	FU
CU32		1-#4	0.437	461	152		21.5	31.4	14.4	13.2	-	12.14	FU
CU33											-	10.14	FU
CU34											-	9.75	FU
CU35		2-#6	1.901	454	154			14.0	13.7	11.2	4.83	18.84	FO
CU36											-	19.52	FO
CU37											-	18.50	FO
CU38											-	21.24	FO
CU39		2-#5	1.412	471	146			19.7		12.9	-	22.87	FO
CU40		2-20M	0.896	470	345	194	15.8	22.8	10.4	9.2	-	70.28	FU
CU41									19.4		-	73.04	FU
CU42									14.1		-	74.27	FU
CU43									14.1		-	81.32	FU
CU44		2-15M	0.577	510	357		22.0	17.7	16.9	12.2	-	58.62	FU
CU45								17.7	16.9	12.2	2.97	57.58	FU
CU46								25.8	9.4	18.9	2.97	61.95	FU
CU47								25.8	9.4	18.9	3.45	71.01	FU
CU48		1-#8	0.773	459	340		21.5	31.4	14.4	13.2	-	74.86	FU
CU49		1-#8	0.773	459	340						-	72.59	FU
CU50		1-#6	0.427	441	343						-	37.73	FU
CU51		2-#6	0.854								-	67.08	FU
CU52		3-#6	1.280	441	343	194	21.5	31.4	14.4	13.2	-	100.62	FU
CU53		2-#6	0.854								-	67.92	FU
CU54		3-#6	1.280								-	92.23	FU
CU55		1-#6	0.427					37.8		9.1	-	41.92	FU
CU56		1-#6	0.427	441				16.9		8.9	-	41.92	FU
CU57		2-15M	0.601	467			20.3	19.8	10.1	11.1	2.25	57.21	ED
CU58		3-20M	1.364	430	340		20.3	19.8	10.1		2.79	113.18	FU
CU59			1.385	470	335		19.8	27.6	13.8		5.24	125.20	FU
CU60			1.353		343						4.35	130.50	FU
CU61			1.345		345						4.71	129.10	FU
CU62			1.364		340						3.83	130.90	FU
CU63			1.345		345						4.96	129.50	FU
CU64			1.364		340						5.11	124.80	FU
CU65		2-15M	0.599	538	344						-	66.60	FU
CU66			0.592		348						2.90	67.30	FU
CU67			0.612		337						-	67.70	FU
CU68			0.599		344						-	66.60	FU
CU69			0.599		344						2.88	68.90	FU
CU70			0.601		343						2.64	66.40	FU
CU71		1-#5,2-#7	1.464	479			21.5	22.6	13.7	12.9	4.27	119.77	FO
CU72								22.6		12.9	3.13	133.43	FO
CU73								12.8		11.2	-	112.52	FO
CU74		1-10M,2-15M	0.868	480	297	194	23.4	19.8	10.1	10.2	2.16	64.45	FU
CU75		3-20M	1.536	430	302		23.4	19.8	10.1	10.2	2.86	83.68	FO
CU76		1-#5,2-#7	1.439	479	349		14.5	19.7	13.7	17.9	2.54	63.65	FO
CU77											4.80	85.51	FO
CU78											-	88.40	FO

Beam No.	Cross-section	Reinf't.	ρ (%)	f_y (MPa)	d (mm)	b (mm)	f'_{br} (MPa)	f'_{gr} (MPa)	f'_{mort} (MPa)	f'_m (MPa)	ϵ_u (mm/m)	M_u (kN·m)	Failure Mode
CU79		4-25M	2.567	523	265	294	-	28.5	27.0	18.2	2.55	142.3	FO
CU80		4-25M	2.567	523			-		27.0		2.22	149.5	FO
CU81		3-15M	0.770	447			-		35.0		5.01	71.7	FU
CU82		3-15M	0.773		264		-		30.0		3.64	73.1	FU
CU83		3-15M	0.773		264		-		25.0		4.35	68.5	FU
CU84		3-15M	2.299		274	95	-	-	35.0		2.92	46.8	FO
CU85		3-15M	2.291		275		-	-	47.0		2.45	52.1	FO
CU86		3-15M	2.299		274		-	-	40.0		3.01	55.1	FU
CU87		2-10M	0.769	500	273		-	-	35.0		3.76	21.9	FU
CU88		2-10M	0.766		274		-	-	45.0		1.70	23.7	FU
CU89		2-10M	0.769		273		-	-	40.0		2.19	22.4	FU
CU90		1-10M	0.394	521	267	95	83.3	-	9.1	36.8	2.41	14.4	FU
CU91		1-10M	0.393	501	268	95		-	12.2	36.8	2.01	14.5	FU
CU92		2-10M	0.788	491	267			-	12.6	36.8	2.59	22.1	FU
CU93		2-15M	1.531	552	275			-	1.1	18.7	1.28	17.1	FO
CU94		2-15M	1.554	547	271			-	1.1	18.7	2.84	24.1	FO
CU95		2-10M	1.548	489	136			-	12.1	36.8	2.61	10.7	FU
CU96		3-10M	1.187	511	266			-	12.6	36.8	1.70	26.0	B
CU97		Truss Tie	0.110	605	80			-	14.6	36.8	0.78	0.64	FU