Bearing Capacity of Rough Rigid Strip Footing on Cohesive Soil: Probabilistic Study

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Abstract: A probabilistic study on the bearing capacity of a rough rigid strip footing on a weightless cohesive soil is carried out to assess the influence of randomly distributed undrained shear strength. Nonlinear finite element analysis is merged with random field theory in conjunction with a Monte Carlo method. In a parametric study, the mean shear strength is held constant while the coefficient of variation and spatial correlation length of cohesion are varied systematically. The influence of the spatial variation of cohesion on the mean bearing capacity is discussed. The results are also presented in a probabilistic context to determine the probability of failure. A comparison between rough and smooth footing conditions is also made.

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Introduction

Most geotechnical analyses in general practice are treated as deterministic. These involve analyses using representative values of design parameters, usually an average or the lowest value obtained from field and/or laboratory test results, and application of a suitable factor of safety to arrive at an allowable loading condition. However, in nature, soil parameters such as physical strength and hydraulic properties generally vary spatially in both the horizontal and vertical directions. The distribution of these soil properties at a site depends on the heterogeneity of constituent materials forming the soil matrix, the geological history of soil formation, and its continuous modification by nature. A uniform soil condition is seldom, if ever, encountered in practical problems. In most site conditions, soil properties show a significant variation over space.

Geotechnical analyses are generally carried out by treating the soil as a single homogeneous layer with uniform soil properties or as a multilayered medium with layerwise uniform properties. Numerical techniques such as finite difference or finite element methods have facilitated modeling the layerwise uniform material—variation of soil properties in the horizontal direction is generally ignored. This may be due to the fact that the variation in the horizontal direction is not so significant in many situations, and a greater number of boreholes is required to establish this horizontal variation, which is impractical due to economical considerations.

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The results of such deterministic analyses are a first order approximation to the mean response, but may easily miss the true failure mechanics, particularly where failure surfaces follow the weakest path through the soil. Thus, the results of deterministic analyses are only approximations which may vary widely from reality. Common causes of discrepancy between the estimated and actual performance of any geotechnical system may be summarized as (Cambou 1975; Lee et al. 1983; Mostyn and Li 1993; Phoon and Kulhawy 1999)

1. Variability of the soil properties at a specific site;
2. Sampling techniques;
3. Laboratory test conditions;
4. Selection of design parameters from limited field and laboratory test results;
5. Assumptions used to simplify the problem for analytical or numerical study;
6. Model error; and
7. Construction methods and materials used.

Among the above, only the randomness of the soil strength is considered in this work but such consideration may have implications for several of the other sources of error. In particular, a potentially significant source of error in the traditional model is just that spatial variability is traditionally ignored. So the consideration of spatial variability may very well reduce model error.

The deterministic approach with a suitable factor of safety has been found to be adequate to essentially eliminate the possibility of failure of geotechnical systems due to these sources of variability. However, for major projects, reporting the probability of geotechnical failure and/or risk involved in any such failure is becoming popular among engineers (Mostyn and Li 1993; Phoon et al. 2000). Such probabilistic studies may be carried out by treating some of the key soil properties as random fields. The soil parameters that do not cause any significant variation in the analyses may be treated deterministically to reduce the complexity of the problem. Based on parametric studies, engineers could further refine their design and construction requirements to minimize the project cost.

Probabilistic studies on a wide range of geotechnical applications have been reported in the literature (see, e.g., Li and Lo 1993; Lemaire et al. 1995; Shackelford et al. 1996; Pande et al.

Table 1. Undrained Shear Strength Properties

Statistical property	Symbol	Units
Mean	μ_{c_u}	Stress
Standard deviation	σ_{c_u}	Stress
Spatial correlation length	$\theta_{\ln c_u}$	Length

2000). Probabilistic studies on the bearing capacity of smooth footings have been reported previously by Fenton and Griffiths (2000, 2001) for c'-ϕ' soils and by Griffiths and Fenton (2001) for $\phi_u=0$ soils. In all these studies, a lognormal distribution was assumed for cohesion and a bounded distribution for the friction angle. In the c'-ϕ' bearing capacity analyses, the influence of cross correlation between the cohesion and friction angle was also investigated.

The smooth footing condition assumed in the previous works is an ideal case. In reality, footings are usually constructed by pouring concrete directly on a firm surface of soil or lean concrete, and the footing-soil interface is rough enough to restrain the tendency for slip. In the present study, the influence of a randomly distributed shear strength on the bearing capacity of a rough rigid strip footing at the surface of a weightless cohesive soil is assessed. The study combines a conventional nonlinear elastoplastic finite element analysis with random field theory in conjunction with a Monte Carlo method. The results from the probabilistic study for a rough footing are interpreted statistically, and then compared with similar results from a smooth footing.

Random Field Model

The behavior of a footing on a weightless cohesive soil is influenced by the following three soil parameters:

1. Young's modulus E;
2. Poisson's ratio ν; and
3. Undrained shear strength c_u.

While the parameters E and ν influence the computed settlement, the bearing capacity of a footing depends primarily on the undrained shear strength c_u. Hence in the present study, to simplify the analyses, the Young's modulus and Poisson's ratio of the soil are held constant while the undrained shear strength is modeled as a random field.

The variability of the undrained shear strength is assumed to be characterized by a lognormal distribution with the three parameters given in Table 1.

The spatial correlation length, also known as the scale of fluctuation, describes the distance over which the spatially random values will tend to be correlated in the underlying Gaussian field. Thus, a large value will imply a smoothly varying field, while a small value will imply a ragged field. For more discussion of the spatial correlation length, the reader is referred to Vanmarcke (1977). In order to nondimensionalize the input, the shear strength variability is expressed in terms of the coefficient of variation $\mathrm{COV}_{c_u}=\sigma_{c_u}/\mu_{c_u}$, and a normalized spatial correlation length $\Theta_{\ln c_u}=\theta_{\ln c_u}/B$, where B is the width of the footing.

Use of the lognormal distribution to characterize the variability of the undrained shear strength is preferred over the normal distribution because it avoids the generation of negative values of soil parameters that a normal distribution allows. Moreover, available field data indicate a lognormal distribution for some soil properties (Hoeksema and Kitanidis 1985; Sudicky 1986; Cherubini 2000). The lognormal distribution of the undrained shear strength c_u means that $\ln c_u$ is normally distributed and the standard deviation and mean of the underlying normal distribution of $\ln c_u$ are given by

$$\sigma_{\ln c_u}=\sqrt{\ln\{1+\mathrm{COV}_{c_u}^2\}} \tag{1}$$

$$\mu_{\ln c_u}=\ln \mu_{c_u}-\tfrac{1}{2}\sigma_{\ln c_u}^2 \tag{2}$$

Other properties of the lognormal distribution are

$$\mu_{c_u}=\exp(\mu_{\ln c_u}+\tfrac{1}{2}\sigma_{\ln c_u}^2) \tag{3}$$

$$\sigma_{c_u}=\mu_{c_u}\sqrt{\exp(\sigma_{\ln c_u}^2)-1} \tag{4}$$

$$\mathrm{Median}=\exp(\mu_{\ln c_u}) \tag{5}$$

$$\mathrm{Mode}=\exp(\mu_{\ln c_u}-\sigma_{\ln c_u}^2) \tag{6}$$

In this study, the random field is generated using the local average subdivision method (Fenton and Vanmarcke 1990; Fenton 1994). A lognormally distributed random field is obtained by first simulating a normally distributed random field $G(x)$, having zero mean, unit variance, and spatial correlation length $\theta_{\ln c_u}$. Then this underlying normally distributed random field is transformed to the desired cohesion field using the relationship

$$c_{u_i}=\exp\{\mu_{\ln c_u}+\sigma_{\ln c_u}G(x_i)\} \tag{7}$$

where x_i = vector containing the coordinates of the center of the ith element; and c_{u_i} = cohesion value assigned to that element.

An isotropic Markovian spatial correlation function, in which the correlation decays exponentially with distance, is used, and it can be expressed as

$$\rho(\tau)=\exp\left\{-\frac{2|\tau|}{\theta_{\ln c_u}}\right\} \tag{8}$$

where ρ = correlation coefficient between the underlying random field values at any two points separated by a distance τ. This correlation function governs the correlation structure of the underlying generated fields $G(x)$. The actual spatial correlation structure of soil deposits is usually not well known, especially in the horizontal direction (see, e.g., Asaoka and Grivas 1982; de Marsily 1985; DeGroot and Baecher 1993). Establishing the spatial correlation structure of a site having erratic variation in its soil properties would require an extensive amount of subsoil exploration, which may not be feasible in many projects due to the high cost. The various forms of commonly used spatial correlation functions and the procedure to estimate the correlation coefficient length are discussed in detail by Fenton (1999).

In the present study, for simplicity, the spatial correlation lengths in the vertical and horizontal directions are assumed to be equal, and the influence of $\Theta_{\ln c_u}$ is studied by a parametric approach. The assumption of isotropy in the correlation structure is sufficient to establish the basic stochastic behavior of the bearing capacity problem. Site specific refinements relating to anisotropy are left for future studies.

Finite Element Method

The bearing capacity analyses are carried out by the finite element method using a viscoplastic algorithm and the elastic–perfectly plastic Tresca yield criterion (Smith and Griffiths 1998). The soil

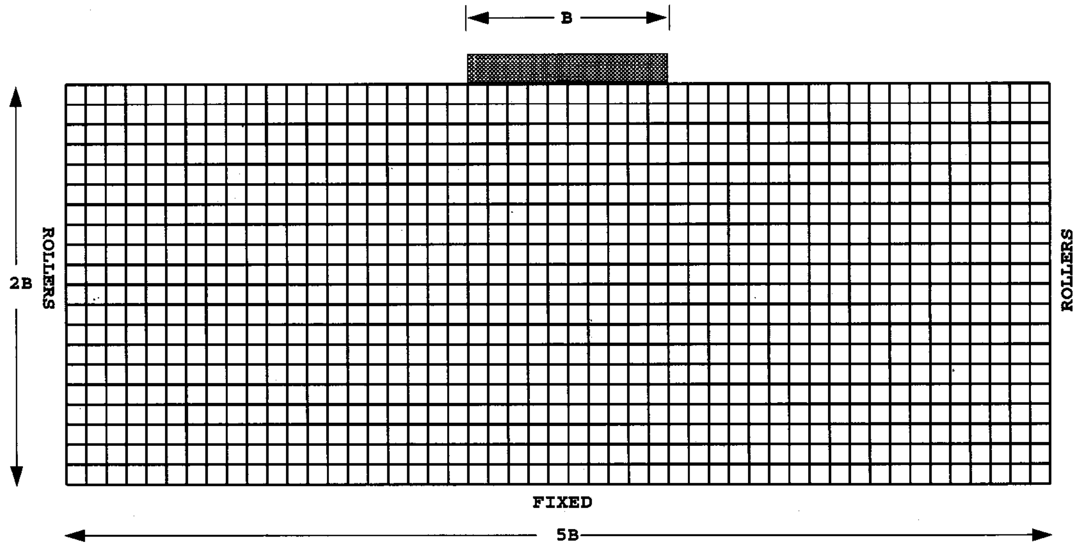

Fig. 1. Mesh used in probabilistic bearing capacity analyses

medium is discretized by isoparametric plane strain elements. A typical finite element mesh used is shown in Fig. 1. It consists of 1,000 eight-noded square elements, in 50 columns and 20 rows, of equal size with side length 0.1 m. The footing occupies ten elements, giving it a width of $B=1$ m. The nodes representing the footing width are incrementally displaced by an equal amount in the vertical direction, simulating a rigid footing condition with a uniform vertical settlement but without any rotation. In reality, the spatial variation of soil properties might cause a rotational movement of the footing (e.g., Nobahar and Popescu 2001), which is not considered in this study. Rough footing conditions are simulated by restraining horizontal movement of these nodes. The footing load for each increment is the summation of the nodal forces back-computed from the converged stress field after each increment. Bearing capacity failure of the footing was taken to have occurred when the back-computed footing load leveled out within quite strict tolerances.

The vertical stress distribution beneath a rigid footing is nonuniform, unlike that of a flexible footing where the load is applied equally at nodal points. High stress concentration is observed, in general, at the vicinity of the footing edge where soil undergoes high plastic strains. Since the footing load is back-computed from the stress field, a finer mesh is preferred in that vicinity to enhance the accuracy of the back-computed footing load. Use of a coarser mesh generally results in a higher value of back-computed footing load than the theoretical.

However, in the present study, a uniform finite element mesh consisting of square elements of equal size is used in order to simplify the random field generation and mapping of the cohesion value of each element. Although the complexity in the random field generation due to a finite element model with varying shapes and sizes of elements can be incorporated, it is not considered herein.

Monte Carlo Simulations

For each set of assumed statistical properties given by COV_{c_u} and $\Theta_{\ln c_u}$, Monte Carlo simulations are performed. These involve 1,000 realizations of the shear strength random field and the subsequent finite element analysis of bearing capacity. Each realization, while having the same underlying statistics, will have a quite different spatial pattern of shear strength values beneath the footing and hence a different value of bearing capacity. On completion of the bearing capacity analysis for 1,000 simulations, the bearing capacities are subjected to statistical analysis.

The bearing capacity q_f is normalized by the mean undrained shear strength to give a bearing capacity factor

$$N_{c_i}=q_{f_i}/\mu_{c_u}, \quad i=1,2,\ldots,1{,}000 \tag{9}$$

In the present study, the mean undrained shear strength μ_{c_u} is held constant throughout the parametric study at a value of 100 kPa.

Deterministic Analysis

The results of deterministic analyses carried out with the mean undrained shear strength and $\mathrm{COV}_{c_u}=0$ are shown in Fig. 2. The estimated bearing capacity for a rough footing condition by the finite element method was 542.3 kPa, implying a bearing capacity factor of 5.423, which is 5.5% higher than the Prandtl closed form solution of $N_c=5.14$. This is due to the coarser and uniform finite element mesh used in the analysis. As discussed earlier, a better agreement between the computed result and the closed form solution can be obtained by using a finer mesh near the footing edge. Similarly, the estimated bearing capacity of a smooth footing by finite element analysis was 528.2 kPa, about 2.8% higher than the closed form solution. In the following discussion, and in order to take account of this discretization error, the mean bearing capacity from statistical analysis will be normalized by the finite element deterministic values mentioned above, and the effect of this marginally higher numerical estimation will be relatively insignificant. The deterministic bearing capacity will be referred to as q_{f_d}, i.e., $q_{f_d}=542.3$ kPa (or 528.2 kPa for the smooth footing case). Similarly, the deterministic bearing capacity factor will be referred to as N_{c_d}, i.e., $N_{c_d}=5.423$ (or 5.282 for the smooth footing case).

The displacement vector plot for a rough footing condition in Fig. 3 shows that the failure field is similar to Prandtl's failure

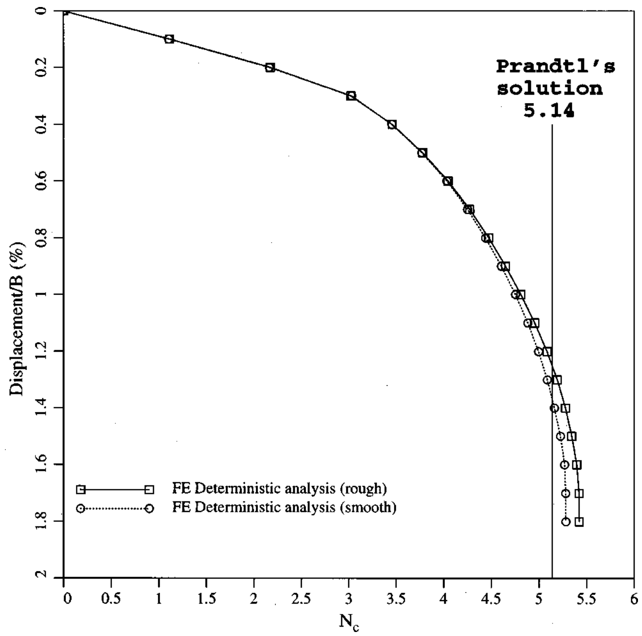

Fig. 2. Deterministic analysis of footings

mechanism, a triangular wedge moving downward as a rigid body with the displaced footing and zones of radial shear and passive Rankine triangular wedges symmetrical on either side.

Parametric Study

Analyses are performed with the mesh of Fig. 1 and with the input parameters taking the following values:

$$\Theta_{\ln c_u} = 0.125, 0.5, 1, 2, 4, 8$$

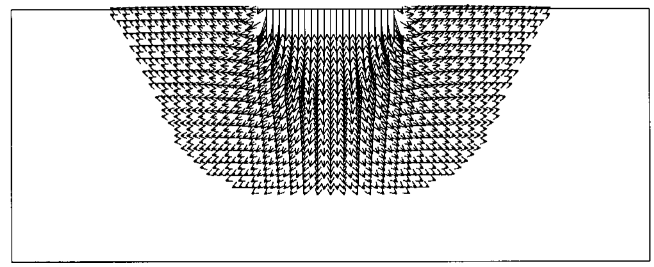

Fig. 3. Displacement vector at failure of rough footing in deterministic analysis

$$\mathrm{COV}_{c_u} = 0.125, 0.25, 0.5, 1, 2, 8.$$

Following 1,000 Monte Carlo simulations, the mean and standard deviation of the resulting 1,000 bearing capacities are computed.

The accuracy of the estimated bearing capacity statistics depends on the number of realizations carried out for each set of parameter values. An estimate that is based on only a few realizations will have a large standard error. In order to achieve stable (i.e., accurate) bearing capacity statistics, a quite high number of Monte Carlo realizations may be required. The influence of the number of realizations on the mean and standard deviation of bearing capacity of a rough footing is shown in Figs. 4(a and b) for two different sets of input values. It can be observed that 1,000 realizations, as used in this study, are generally adequate within the considered range of parameters. For further discussion on the optimal number of realizations, the interested reader is referred to Griffiths and Fenton (2001).

Fig. 5 shows two typical cases of generated random fields of the undrained shear strength for a fixed value of COV_{c_u} while $\Theta_{\ln c_u}$ is varied. In this figure, a gray scale is superimposed on the finite element mesh; the lighter regions indicate stronger soil and darker regions indicate weaker soil. It can be observed that for a

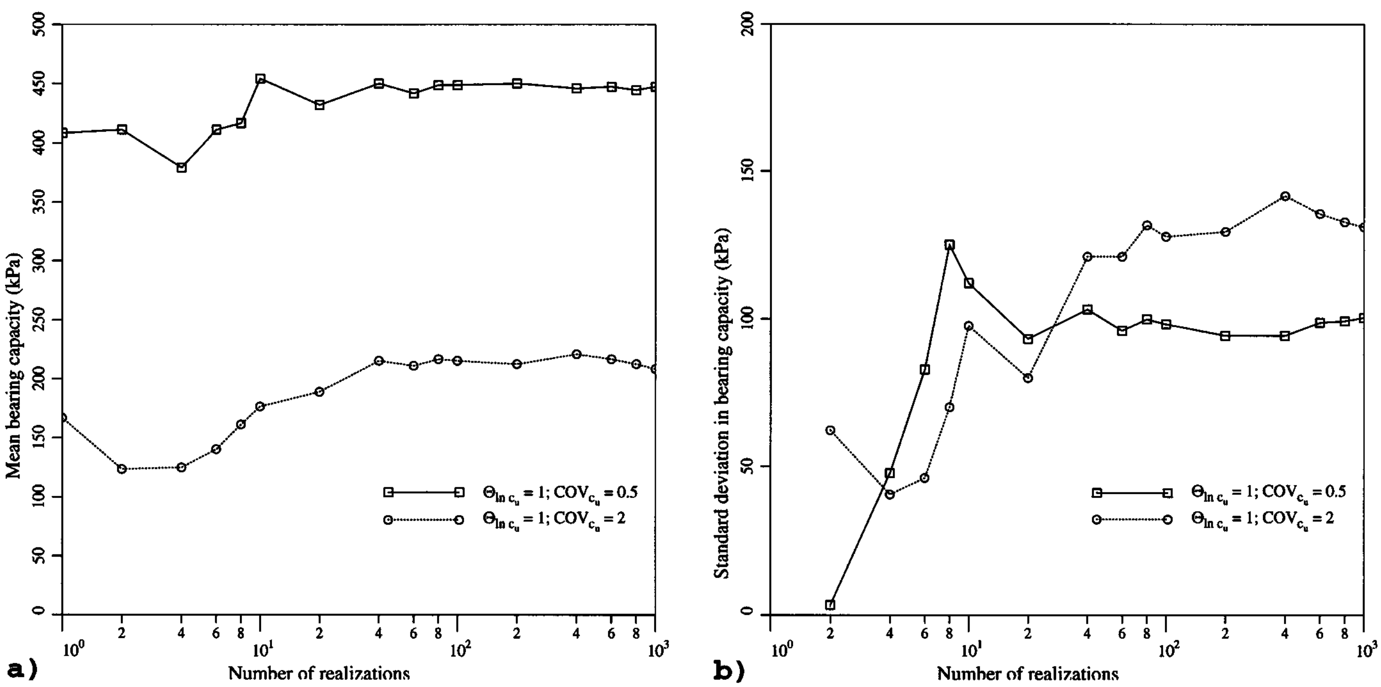

Fig. 4. Effect of number of realizations on bearing capacity statistics: (a) mean, (b) standard deviation

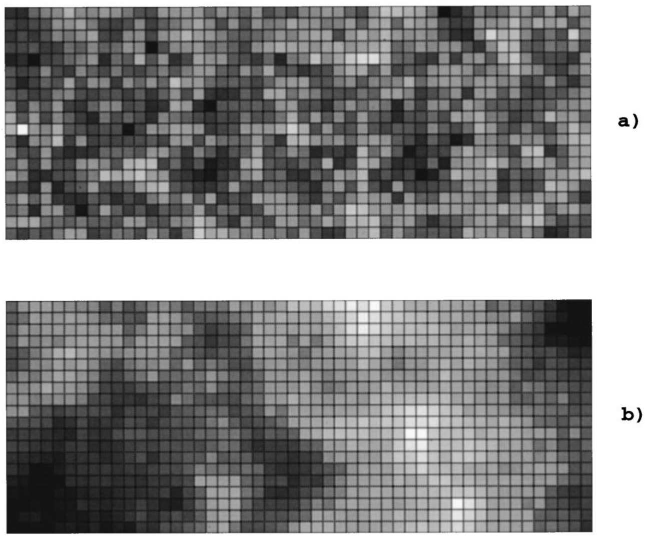

Fig. 5. Influence of $\Theta_{\ln c_u}$ on random field generation ($\mathrm{COV}_{c_u}=1$): $\Theta_{\ln c_u}$=(a) 0.125, (b) 8

small value of $\Theta_{\ln c_u}$ the shear strength changes rapidly from element to element creating a ragged field, and as $\Theta_{\ln c_u}$ increases the random field becomes smoother, or more slowly varying.

Figs. 6(a and b) show how the mean bearing capacity, normalized by the deterministic bearing capacity, varies with COV_{c_u} and $\Theta_{\ln c_u}$ for rough footing conditions. For low values of COV_{c_u}, the mean bearing capacity μ_{q_f} tends to the deterministic value. But for higher values of COV_{c_u} the mean bearing capacity falls steeply, especially for spatial correlation lengths of the order of $\Theta_{\ln c_u} \approx 0.5$. For example, in Fig. 6(a), the mean bearing capacity in a *highly* variable condition with $\Theta_{\ln c_u}=0.5$ and $\mathrm{COV}_{c_u}=8$ is only about 10% of the deterministic value. For the value of $\mathrm{COV}_{c_u}=0.5$, which Lee et al. (1983) suggest as an upper limit, with $\Theta_{\ln c_u}=1.0$, the μ_{q_f} is about 80% of the deterministic value. Fig. 6(b) indicates a minimum value of the mean bearing capacity is observed when $\Theta_{\ln c_u} \approx 0.5$, i.e., the correlation length is half of the footing width. At the next lower value of $\Theta_{\ln c_u}$, μ_{q_f} is seen in all cases to increase. It is speculated that in the limit of $\Theta_{\ln c_u} \rightarrow 0$, there are no "preferential" weak paths the failure mechanism can follow, and the mean bearing capacity will tend to the deterministic bearing capacity based on the geometric average of the shear strength, namely the median from Eq. (5). Thus, as $\Theta_{\ln c_u} \rightarrow 0$ $\mu_{q_f}/q_{f_d} \rightarrow (1+\mathrm{COV}_{c_u}^2)^{-1/2}$. The explanation lies in the fact that as the spatial correlation length decreases, the weakest path becomes increasingly tortuous, and its length correspondingly longer. As a result, the "lowest energy" path starts to look for shorter routes cutting through higher strength material. In the limit, as $\Theta_{\ln c_u} \rightarrow 0$, it is expected that the optimum failure path will be the same as in a uniform material at the median.

In principle, the $\Theta_{\ln c_u}=0$ case is somewhat delicate to investigate. Strictly speaking, any local average of a (finite variance) random $\ln c_u$ field having $\Theta_{\ln c_u}=0$ will have zero variance (since the local average will involve an infinite number of independent points). Thus, in the $\Theta_{\ln c_u}=0$ case the "local average" representation, i.e., the finite element method (as interpreted here), will necessarily return to a deterministic solution based on the median. The detailed investigation of this trend is also complicated by the fact that soil properties are never determined at the "point" level—they are based on a local average over the soil sample volume. While recognizing the apparent trend with small $\Theta_{\ln c_u}$ in this study, the theoretical explanation for the limiting trend is left for further research.

It is also hypothesized that $\Theta_{\ln c_u} \approx B$ (or $\Theta_{\ln c_u} \approx 1.0$) leads to the greatest reduction in μ_{q_f}, because it allows enough variability for a failure surface to develop which deviates significantly from the deterministic Prandtl mechanism (consisting of circular arcs

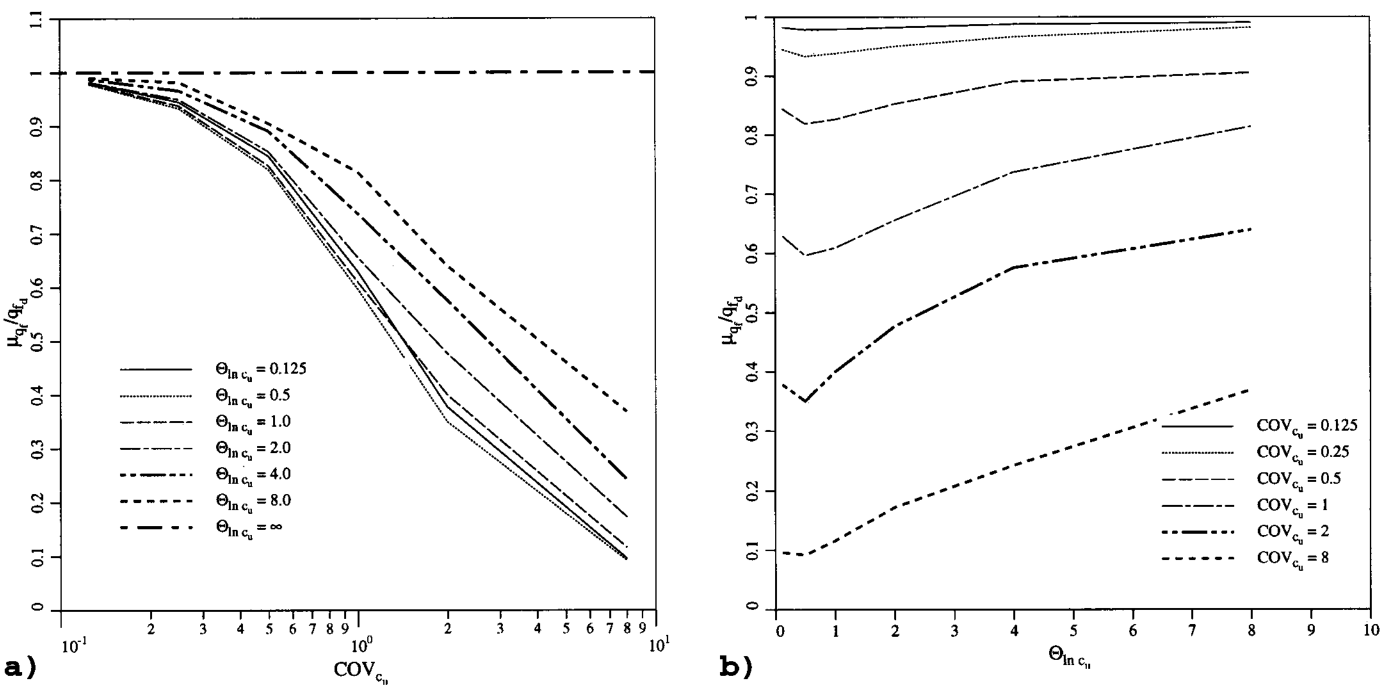

Fig. 6. Estimated mean bearing capacity as function of undrained shear strength statistics $\Theta_{\ln c_u}$ and COV_{c_u}

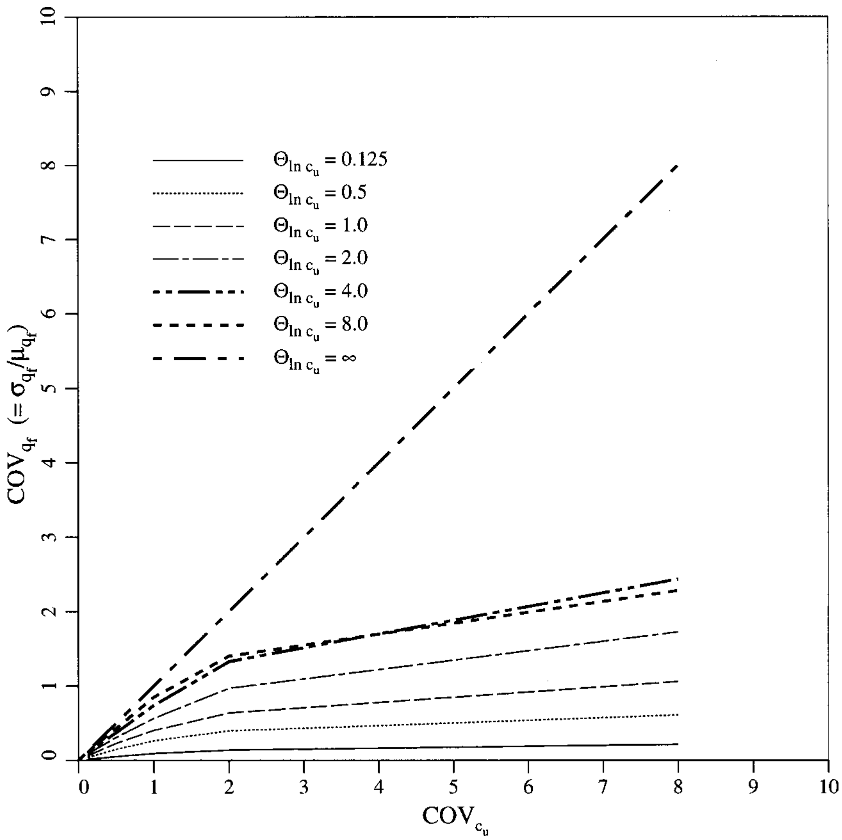

Fig. 7. Estimated coefficient of variation of bearing capacity (COV_{q_f}) as function of undrained shear strength statistics $\Theta_{\ln c_u}$ and COV_{c_u}

and straight lines). Too much spatial variability, where $\Theta_{\ln c_u} \rightarrow 0$ so that the weakest path would become too long, or too little, where $\Theta_{\ln c_u} \rightarrow \infty$ so that no weakest path exists, both tend back to the deterministic solution.

From these two figures, it can be seen that the bearing capacity of a footing on a heterogeneous soil with spatially varying undrained shear strength will generally be less than the deterministic bearing capacity computed with the mean shear strength μ_{c_u}. This important result shows that weak soil elements, rather than strong soil elements, tend to dominate the overall performance of the footing.

The horizontal line in Fig. 6(a) corresponds to the solution that would be obtained for the hypothetical case of $\Theta_{\ln c_u} = \infty$. This condition implies that each realization of the Monte Carlo process involves a uniform soil, albeit with properties varying from one realization to the next. In this case, the distribution of the bearing capacity q_f will be statistically similar to the underlying lognormal distribution of c_u but magnified by N_{c_d}, i.e., the mean bearing capacity will be equal to the deterministic bearing capacity for all values of COV_{c_u}.

The change in the coefficient of variation of computed bearing capacities, COV_{q_f}, with respect to COV_{c_u} for the rough footing is shown in Fig. 7. For this, the standard deviation in the estimated bearing capacity σ_{q_f} for each set of input parameters is normalized by the corresponding mean bearing capacity to obtain COV_{q_f}. At lower values of COV_{c_u}, the COV_{q_f} increases almost linearly with COV_{c_u}, and at higher values of COV_{c_u} the curves flatten and the rate of increase is reduced. It is noted that the influence of $\Theta_{\ln c_u}$ is also significant; COV_{q_f} increases with increasing $\Theta_{\ln c_u}$ and approaches the limiting value represented by the 45° inclined line for the hypothetical case of $\Theta_{\ln c_u} = \infty$ for which, as explained above, COV_{q_f} would be equal to COV_{c_u}.

Fig. 8 shows typical deformed meshes at failure under a rough footing for some of the realizations and the corresponding displacement vector plot. The deformed mesh is superimposed on a gray scale in which lighter regions indicate stronger soil and darker regions indicate weaker soil. As the variance of the shear strength for a spatially variable soil increases, the symmetry of the failure field is lost and the failure mechanism tends to go one way or the other. Failure of the footing in the very first load increment with negligible bearing capacity has also been observed during some realizations with $COV_{c_u} > 1$. In these cases, the random field simulation has generated a region of elements close to the footing with very low shear strength that is unable to provide any resistance to the footing load. Similarly, higher values of COV_{c_u} have also resulted in very high bearing capacities in some realizations, as high as 11 times the deterministic value.

Probabilistic Interpretation

The probability that the computed bearing capacity, for any set of input parameters, is less than the deterministic value, $q_f < q_{f_d}$, can be expressed as

$$P[q_f < q_{f_d}] = \Phi\left(\frac{\ln q_{f_d} - \mu_{\ln q_f}}{\sigma_{\ln q_f}}\right) \quad (10)$$

where Φ = cumulative normal function.

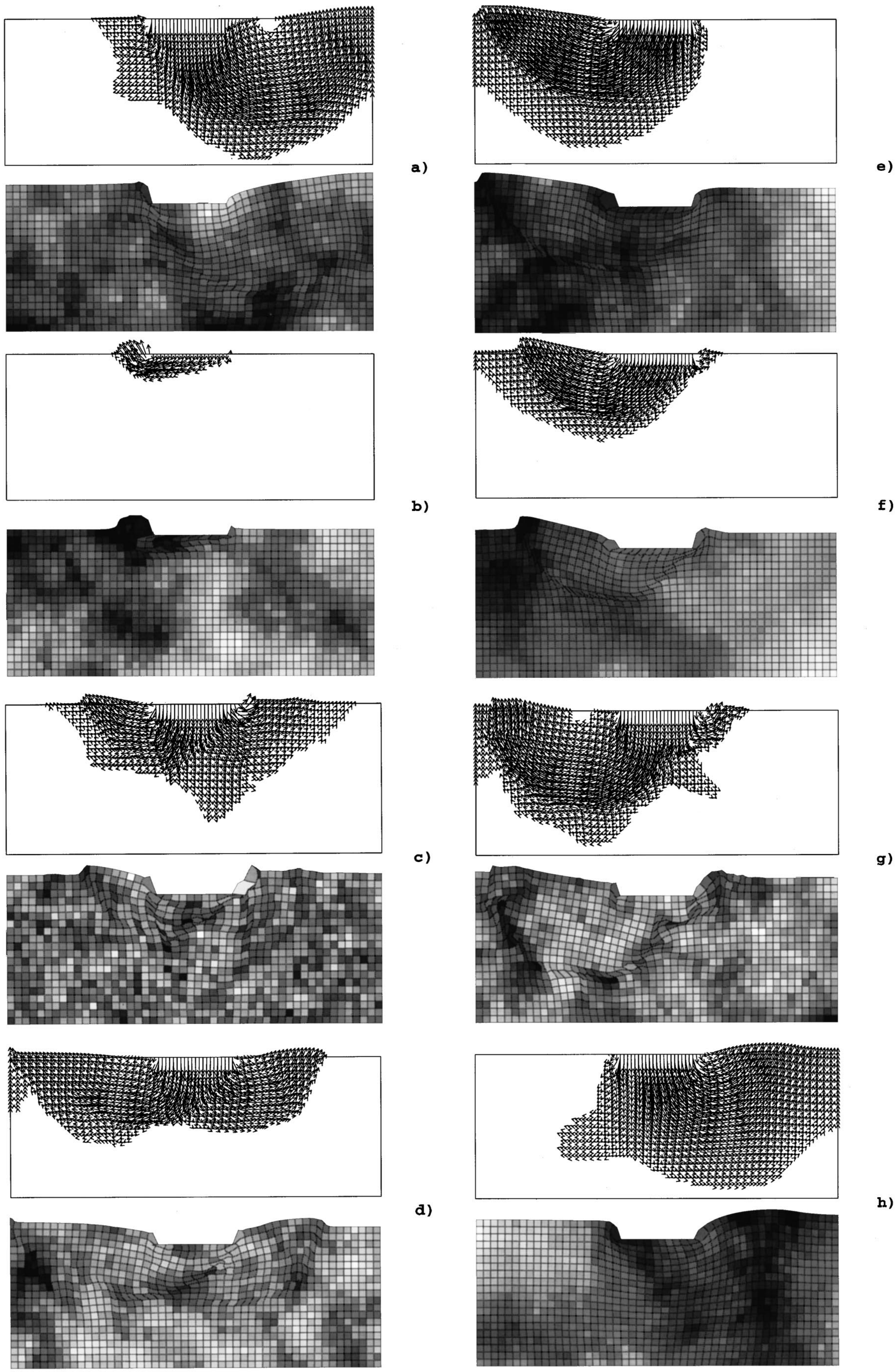

Fig. 8. Typical deformed meshes and corresponding plot of displacement vectors for some realizations (rough footing) (a–h)

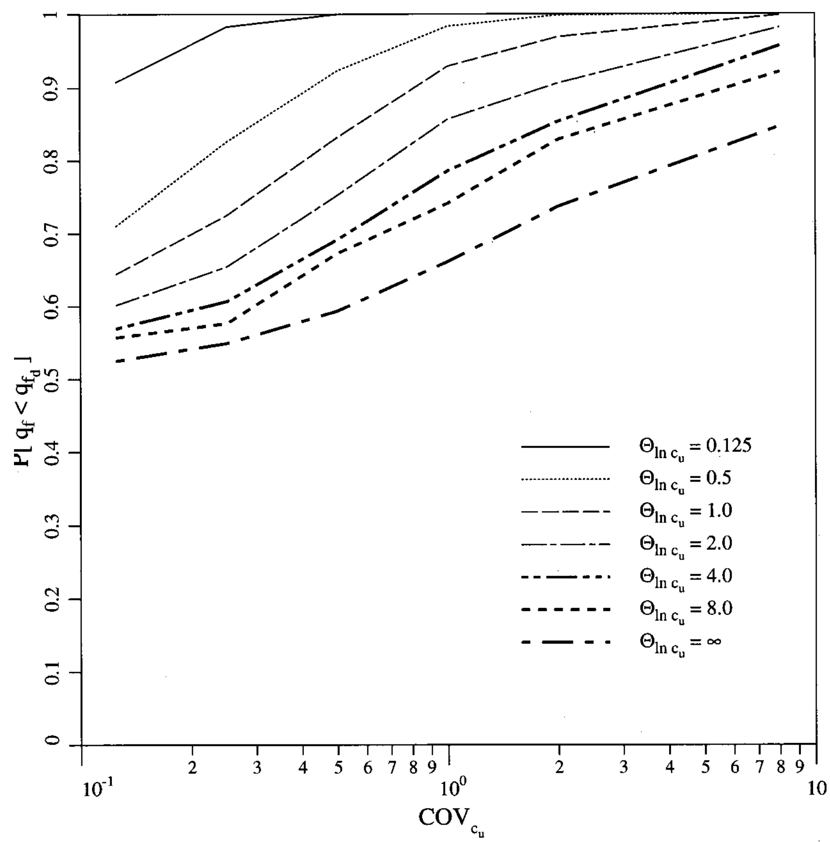

Fig. 9. Probability that bearing capacity will be lower than deterministic value

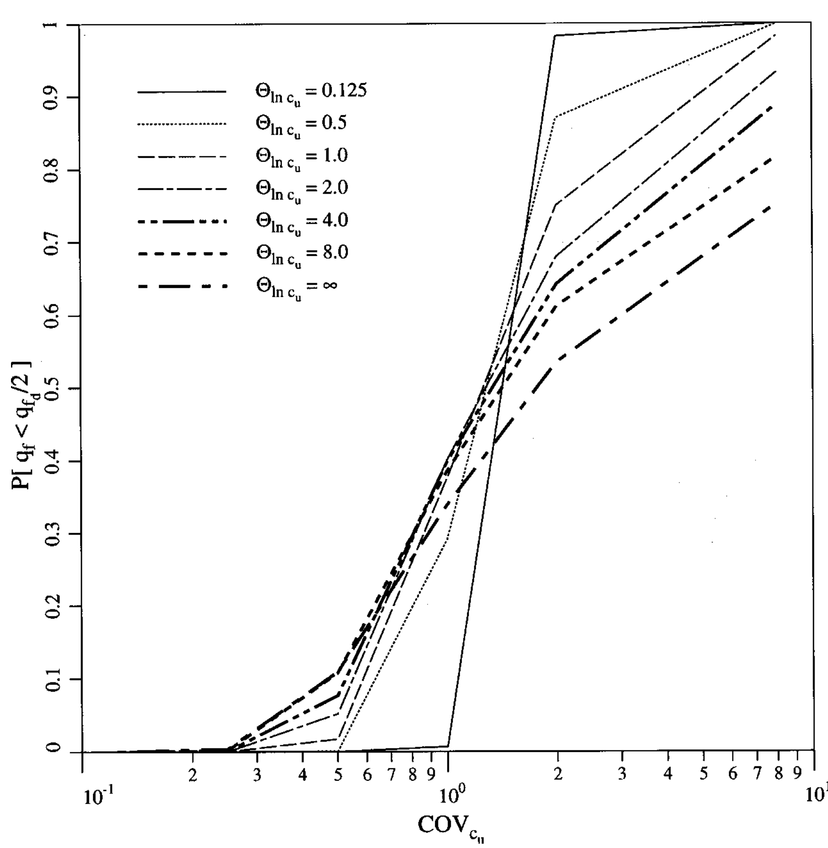

Fig. 10. Probability that bearing capacity will be lower than deterministic value with factor of safety $F = 2$

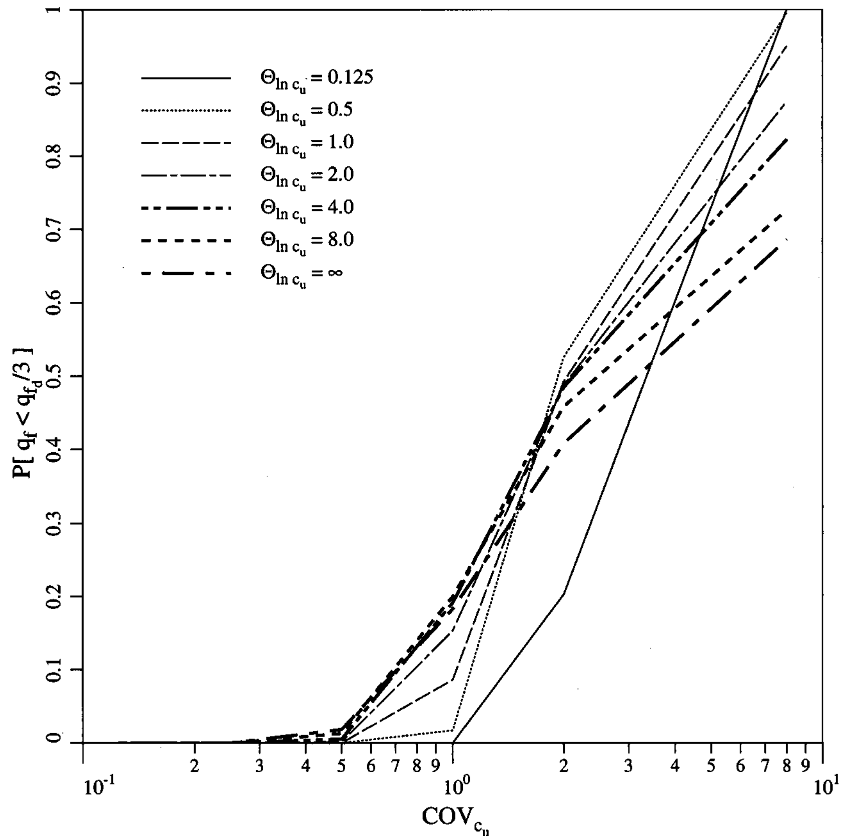

Fig. 11. Probability that bearing capacity will be lower than deterministic value with factor of safety $F=3$

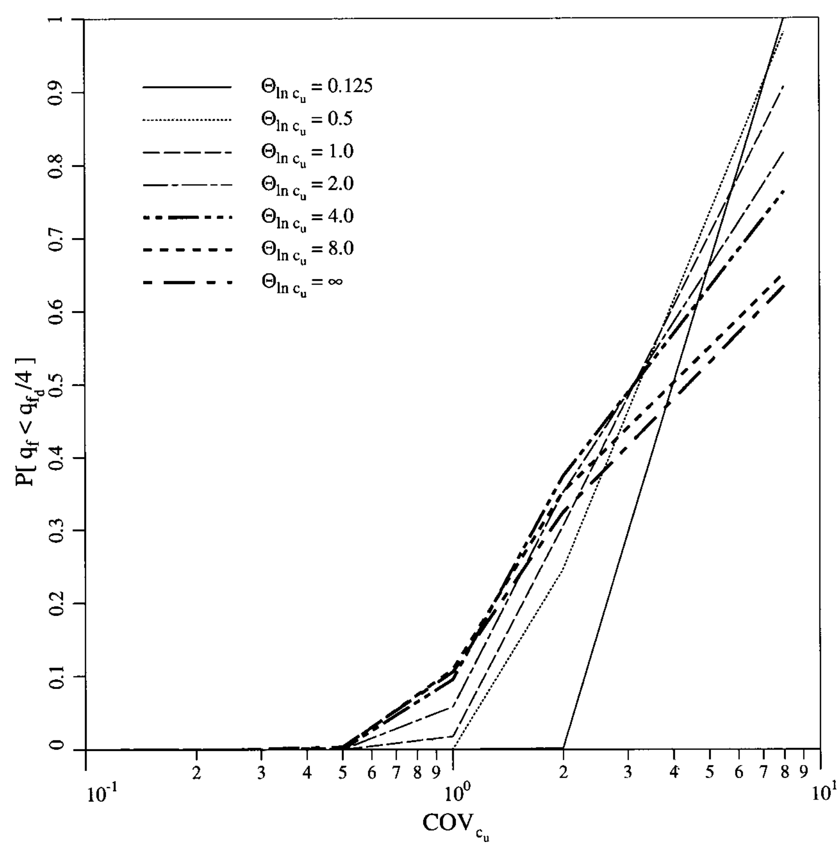

Fig. 12. Probability that bearing capacity will be lower than deterministic value with factor of safety $F=4$

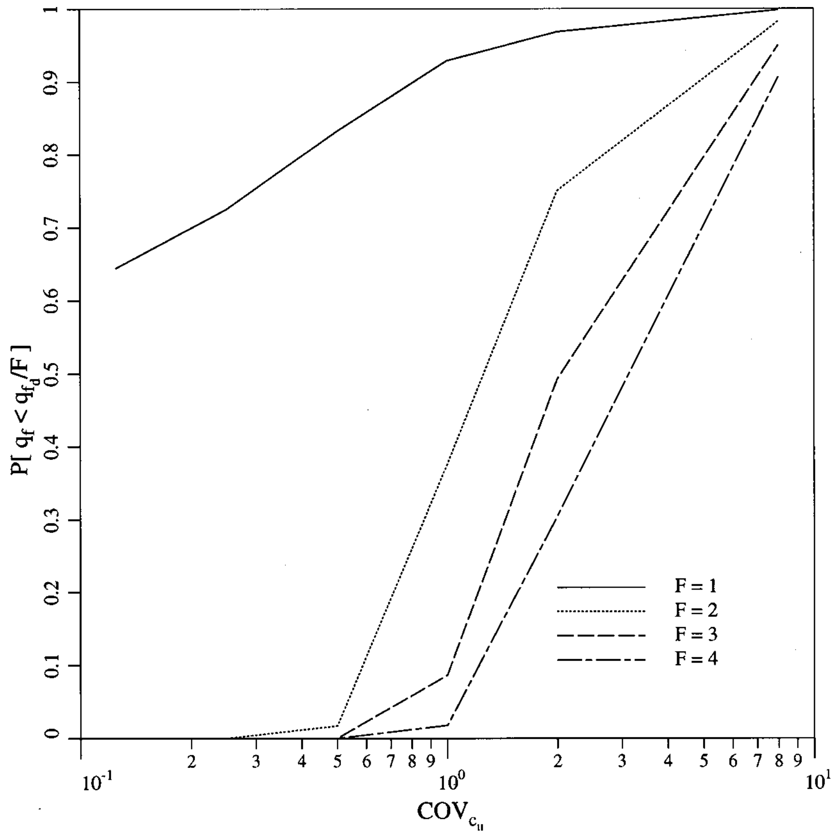

Fig. 13. Probability that bearing capacity will be lower than deterministic value for different factors of safety F for $\Theta_{\ln c_u}=1$

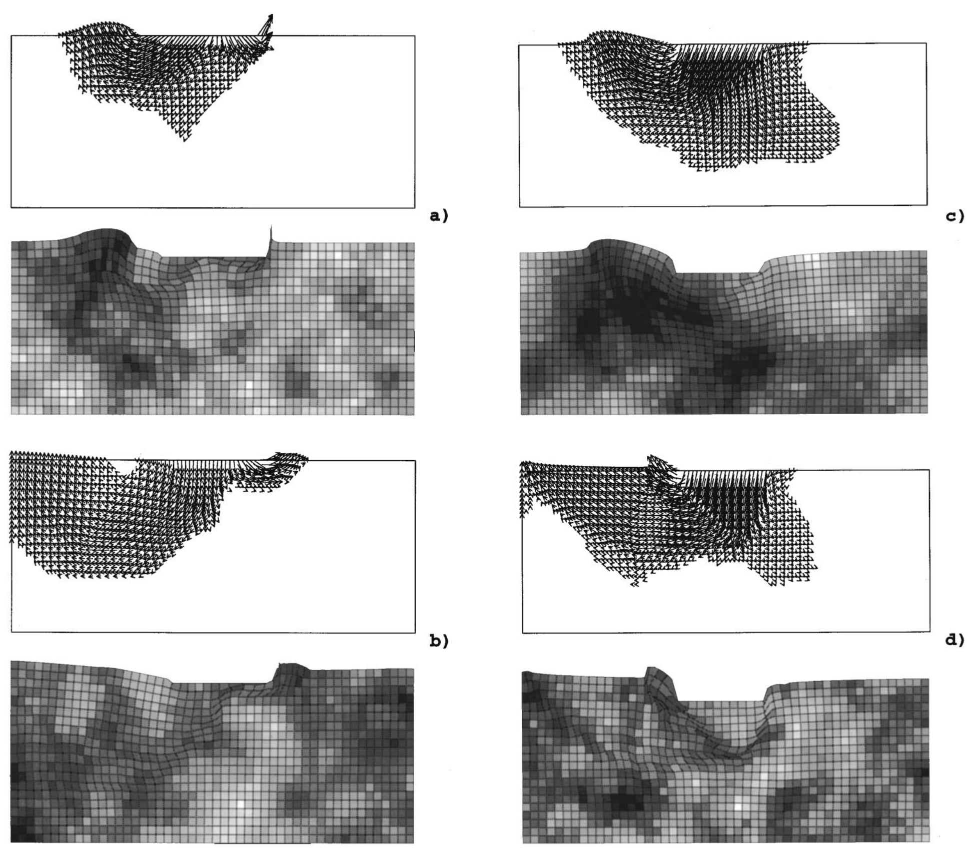

Fig. 14. Typical deformed meshes and corresponding plot of displacement vectors for some realizations (smooth footing) (a–d)

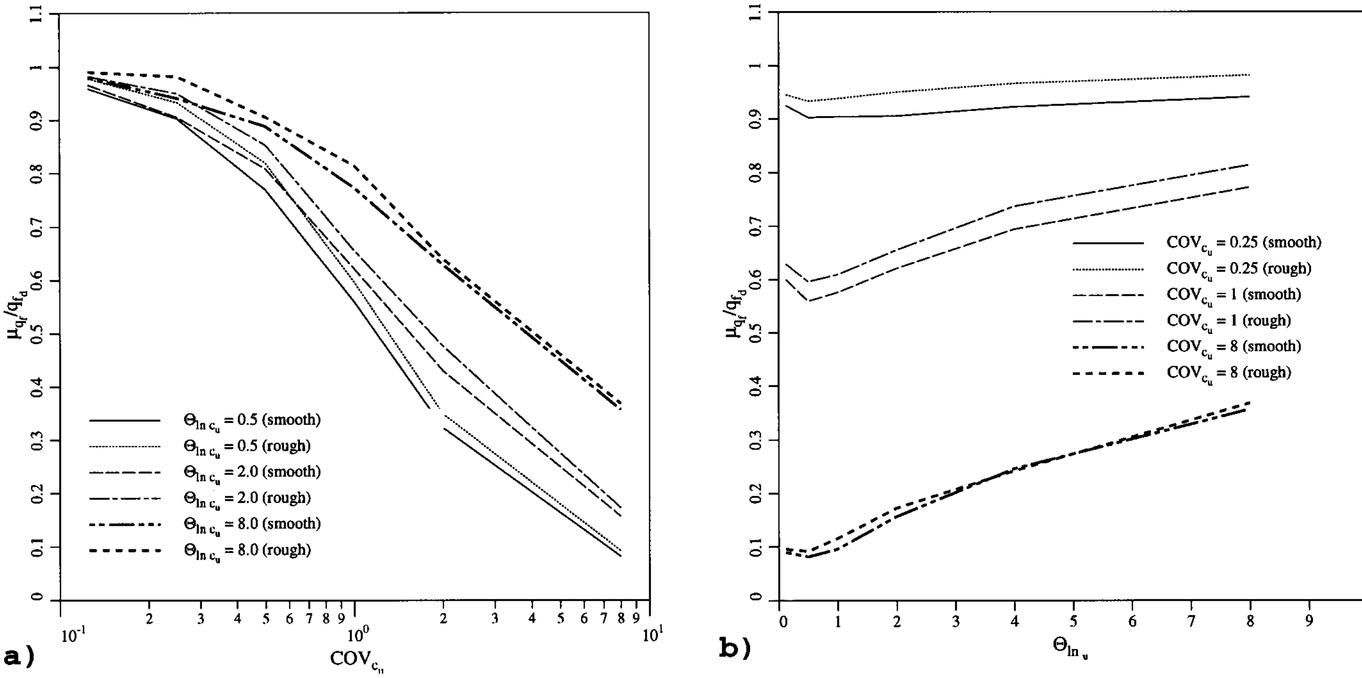

Fig. 15. Comparison of mean bearing capacity for smooth and rough footings

The summary of $P[q_f<q_{f_d}]$, shown in Fig. 9 for rough footing conditions, indicates that there is always a greater than 50% chance that the bearing capacity of the footing on a soil with spatially random shear strength is less than the deterministic bearing capacity based on the mean. Higher probabilities correspond to higher values of COV_{c_u} and lower values of $\Theta_{\ln c_u}$. The higher probabilities corresponding to low values of $\Theta_{\ln c_u}$ occur due to the reduced variance of the bearing capacity values. The ''bunching up'' of bearing capacity values, combined with a reduced mean (Figs. 6), means that the majority of the bearing capacity distribution lies below the deterministic bearing capacity based on the mean strength. Theoretically, as the COV_{c_u} approaches zero, all the probabilities in Fig. 9 tend to 0.5, irrespective of the value of $\Theta_{\ln c_u}$. It can also be observed from Fig. 9 that this convergence toward 0.5 occurs faster for higher values of $\Theta_{\ln c_u}$.

For a fixed mean, the lognormal distribution will become increasingly skewed to the right as the *COV* increases (which is to say, the median moves farther to the left from the mean), so that

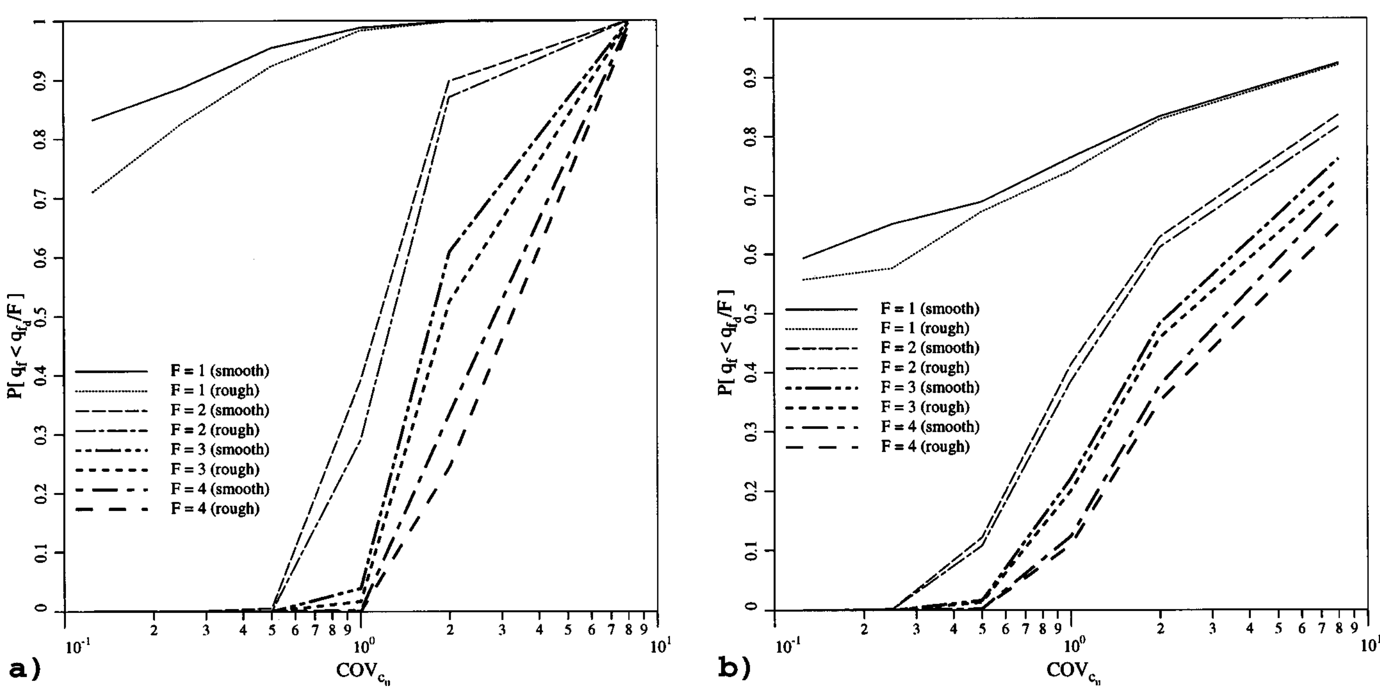

Fig. 16. Comparison of probability of design failure for smooth and rough footings: $\Theta_{\ln c_u}=$ (a) 0.5, (b) 8

there is an increasing area to the left of the mean. Thus, even a known lognormal distribution (as represented by the $\Theta_{\ln c_u}=\infty$ case) will show an increasing probability that $q_f<q_{f_d}$ with increasing COV_{c_u}. However, the fact that lesser values of $\Theta_{\ln c_u}$ show even higher values of $P[q_f<q_{f_d}]$ is indicative of the "weakest path" effect arising from spatial variability in soil properties. In effect, for $\Theta_{\ln c_u}<\infty$, the weaker soil regions in the resulting random field tend to dominate the failure mode.

The usual design practice for footings is deterministic. This involves the estimation of ultimate bearing capacity using average values of design parameters and application of a suitable factor of safety (F) to arrive at an allowable bearing capacity. The factor of safety used for shallow footings is generally between 3 and 4. The results in Fig. 9 imply a factor of safety of unity, so the probability results have been reinterpreted in Figs. 10–12 to indicate the estimated probability of design failure after the deterministic solution has been reduced by a factor of safety $F>1$. The probability of design failure $P[q_f<q_{f_d}/F]$ is now greatly reduced. For example, for the recommended maximum value of $\mathrm{COV}_{c_u}=0.5$, the chance of failure has been reduced to 11% for $F=2$, 1.3% for $F=3$, and 0.2% for $F=4$. Fig. 13 shows directly how F affects the probability of design failure for a range of COV_{c_u} where the correlation length is held constant.

The above results confirm that a factor of safety of 3–4 is generally able to reduce the probability design failure to negligible level for all soils in the recommended range of $\mathrm{COV}_{c_u} \leqslant 0.5$. Higher factor of safety values would be required for exceptionally variable soils.

Comparison with Smooth Footing Results

A parametric study for smooth footing conditions (e.g., Griffiths and Fenton 2001) has been carried out and compared with that for rough conditions. Some typical deformed meshes and the corresponding displacement vectors for realizations under smooth conditions are shown in Fig. 14. In a deterministic analysis, the failure mechanism of a smooth footing is similar to Hill's mechanism of failure with two smaller symmetrical triangular wedges underneath the footing, one sliding to the left and the other sliding to the right. As the COV_{c_u} increases, the symmetry of the failure field is lost; the slip lines on one side are shallower than on the other, or the failure mechanism tends to go one way or the other. The slip lines of smooth footing are generally shallower than that of rough footing.

Fig. 15 compares the mean bearing capacities under smooth and rough conditions after being normalized by their respective deterministic bearing capacity values. The general trend in the variation of the mean bearing capacities with respect to COV_{c_u} and $\Theta_{\ln c_u}$ for both footing conditions is similar; but the normalized mean bearing capacity of the smooth footing is somewhat lower than that of the rough footing. Fig. 16 compares the probability of design failure $P[q_f<q_{f_d}/F]$ for smooth and rough footings for $\Theta_{\ln c_u}=0.5$ and 8. Although the general trend in the variation of the probability of failure is similar for both footing conditions, the smooth footing generally has somewhat higher probabilities of design failure than in the rough case.

Since the slip lines at failure for a rough footing are generally deeper than for a smooth footing, a greater volume of soil mass is being sheared, resulting in a marginally higher bearing capacity and lower probability of failure than for the smooth case.

Concluding Remarks

A probabilistic study on the bearing capacity of a rough rigid strip footing on a soil with randomly varying shear strength has been carried out. Random field theory has been combined with a conventional nonlinear finite element algorithm, in conjunction with a Monte Carlo method. The parametric study carried out involves 1,000 realizations for each set of parameters. From this probabilistic study the following conclusions can be made.

The mean bearing capacity of a footing on a soil with spatially varying shear strength is always lower than the deterministic bearing capacity based on the mean value. This important observation is due to the linking up of weak elements beneath the footing, and shows that weak elements rather than strong elements tend to dominate the expected bearing capacity of a footing on spatially random soil.

The reduction in the expected bearing capacity was greatest for higher values of COV_{c_u} and values of the spatial correlation length $\Theta_{\ln c_u}$ on the order of the footing width.

The results confirm that a factor of safety of 3–4 would generally be adequate to reduce the probability of design failure to negligible levels for soils with $\mathrm{COV}_{c_u}\leqslant 0.5$.

The results of rough and smooth footings were compared. The trend in the variation of the mean bearing capacity with respect to COV_{c_u} and $\Theta_{\ln c_u}$ in both cases is generally similar. Due to the greater volume of soil involved in the failure mechanism beneath a rough footing, however, the bearing capacities were marginally higher and hence the probabilities of design failure marginally lower than in the smooth case.

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Notation

The following symbols are used in this paper:

B = footing width;
COV_{c_u} = coefficient of variation of undrained shear strength;
COV_{q_f} = estimated coefficient of variation of bearing capacity;
c_u = undrained shear strength;
c_{u_i} = undrained shear strength assigned to ith element;
E = Young's modulus;
F = factor of safety;
$f(\cdots)$ = probability density function;
$G(x)$ = standard Gaussian field with zero mean and unit variance;
$G(x_i)$ = local average of Gaussian field over ith element;
i = integers that count realizations or elements;
N_c = bearing capacity factor;
N_{c_d} = bearing capacity factor for deterministic solution;
N_{c_i} = bearing capacity factor for ith realization;
$P[\cdots]$ = probability;
q_f = bearing capacity;

q_{f_d} = deterministic bearing capacity;
q_{f_i} = bearing capacity for ith realization;
x_i = vector containing coordinates of center of ith element;
$\Theta_{\ln c_u}$ = dimensionless spatial correlation of log undrained shear strength;
$\theta_{\ln c_u}$ = spatial correlation length of log undrained shear strength;
μ_{c_u} = mean of undrained shear strength;
$\mu_{\ln c_u}$ = mean of log undrained shear strength;
$\mu_{\ln q_f}$ = estimated mean of log bearing capacity;
μ_{q_f} = estimated mean bearing capacity;
ν = Poisson's ratio;
ρ = correlation coefficient;
σ_{c_u} = standard deviation of undrained shear strength;
$\sigma_{\ln c_u}$ = standard deviation of log undrained shear strength;
$\sigma_{\ln q_f}$ = estimated standard deviation of log bearing capacity;
σ_{q_f} = estimated standard deviation of bearing capacity;
τ = distance between two points in random field; and
$\Phi(\cdots)$ = cumulative normal function.

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