# A PARAMETRIC STUDY ON RELIABILITY OF SPATIALLY RANDOM COHESIVE SLOPES

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## ABSTRACT

A parametric study on the reliability of a cohesive slope is carried out to investigate the influence of spatial variability of undrained shear strength ($c_u$). The random finite element method (RFEM), which uses random field theory and elasto-plastic finite element analysis, is adopted in this study. This study concentrates on the effect of soil variability, which is commonly measured by the coefficient of variation (*COV*) and scale of fluctuation ($\theta$), on the reliability of slopes with different geometries. Various slopes having combinations of slope angles ($\beta$) and depth factors ($D$) are considered. The numerical analyses are carried out using Monte Carlo simulations to enable the probabilities of failure ($P_f$) to be estimated. The deterministic factors of safety (*FOS*), based on the mean values of $c_u$, are also computed using the finite element method. The results of comparisons between the $P_f$ and the *FOS* values show that $\theta$ has a significant effect on $P_f$ for marginally stable slopes ($1 \leq FOS \leq 1.5$), even those slopes having low to intermediate values of *COV* (e.g. 0.1 – 0.3). Slopes having higher values of *COV* (e.g. 0.5 – 1), which have high *FOS* values (e.g. 1.5 – 5), are also vulnerable to failures depending on the values of $\theta$.

## 1 INTRODUCTION

The stability of a cohesive slope has traditionally been analysed by treating the soil material as uniform and homogenous. The factor of safety (*FOS*) is estimated using Taylor's (1937) charts or computed using a limit equilibrium method. These traditional (deterministic) stability analyses are normally based on the characteristic values of soil properties. However, it is well known that soil properties are spatially variable and hence homogeneity cannot be assumed (Vanmarcke 1977a). As a result, the stability of a slope cannot be defined by a factor of safety value, as the basis upon which such a value is determined assumes homogeneity. A more realistic approach to stability analysis considers the uncertainty and the variability in the characteristics of a soil and incorporates these characteristics into a probabilistic analysis. Probabilistic analyses utilise the full range of soil properties, which are randomly generated on the basis of their statistical characteristics. Doing so leads to a more realistic measure of safety called the probability of failure ($P_f$) or reliability ($1 – P_f$).

Probabilistic analyses have received considerable attention in the literature for the past three decades (e.g. Alonso, 1976; Tang *et al*., 1976; Vanmarcke, 1977b; D'Andrea and Sangrey, 1982; Li and Lumb, 1987; Mostyn and Li, 1993; Duncan, 2000; El-Ramly *et al*., 2002). More recently, Griffiths and Fenton (2000; 2004) introduced an approach called the random finite element method (RFEM) of analysis. This method combines random field theory (Vanmarcke 1977a; 1983) with non-linear elasto-plastic finite element analysis to explicitly account for the effect of the spatial variability of soil properties on the strength at a point within the soil mass. This paper uses the latter method to carry out a parametric study on the reliability of spatially random cohesive slopes. Doing so extends the studies of Griffiths and Fenton (2000), Fenton *et al*. (2003) and Griffiths and Fenton (2004) to investigate the effect of soil variability on the reliability of cohesive slopes having various slope angles ($\beta$) and depth factors ($D$). The general geometry of the slopes is shown in Figure 1. The slopes are assumed to be resting on a layer of sufficient thickness so that the base of the layer undergoes no strain. In this study, slope angles of 14° (4:1), 18.4° (3:1), 26.6° (2:1) and 45° (1:1) are considered and depth factors of 1, 2 and 3.

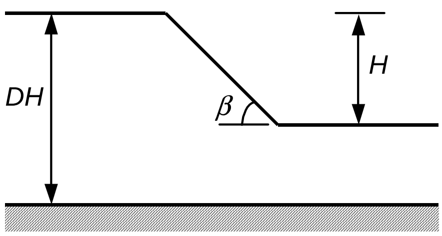


Figure 1: Geometry of cohesive slope problem.

## 2 RANDOM FINITE ELEMENT METHOD

The slope is assumed to consist of undrained clay having a shear strength defined in terms of a frictional coefficient ($\phi_u$) of zero (i.e. $\phi_u = 0$) and a cohesive strength ($c_u$). Variability in the latter parameter is assumed to be defined according to a lognormal distribution, which is characterised by a mean ($\mu$) and a standard deviation ($\sigma$). The latter parameters can be expressed in terms of the dimensionless coefficient of variation, ($COV$), defined as:

$$COV = \frac{\sigma}{\mu} \qquad \textbf{(1)}$$

The spatial variability of soil properties is modelled by a dimensionless parameter referred to as the scale of fluctuation ($\theta$) (Vanmarcke 1977a; 1983), which expresses the correlation of properties with distance. A large value of $\theta$ implies a more smoothly varying field, while a small value of $\theta$ indicates a field that varies more randomly. The correlation structure of soil properties is defined by an exponentially decaying (i.e. Markovian) correlation function defined as:

$$\rho(\tau) = \exp\left(-\frac{2|\tau|}{\theta}\right) \qquad \textbf{(2)}$$

where $\tau$ is the distance between two points in the field.

The random field of shear strength values is simulated using the local average subdivision (LAS) method (Fenton and Vanmarcke, 1990). The LAS algorithm generates random variables correlated according to Equation 2. These variables are mapped onto the finite element mesh. Therefore, each finite element is assigned with a random variable and neighboring elements are correlated to each other. Figure 2 shows a typical mesh used in the study. The size of the finite elements is fixed at 1 m by 1 m for all geometries.

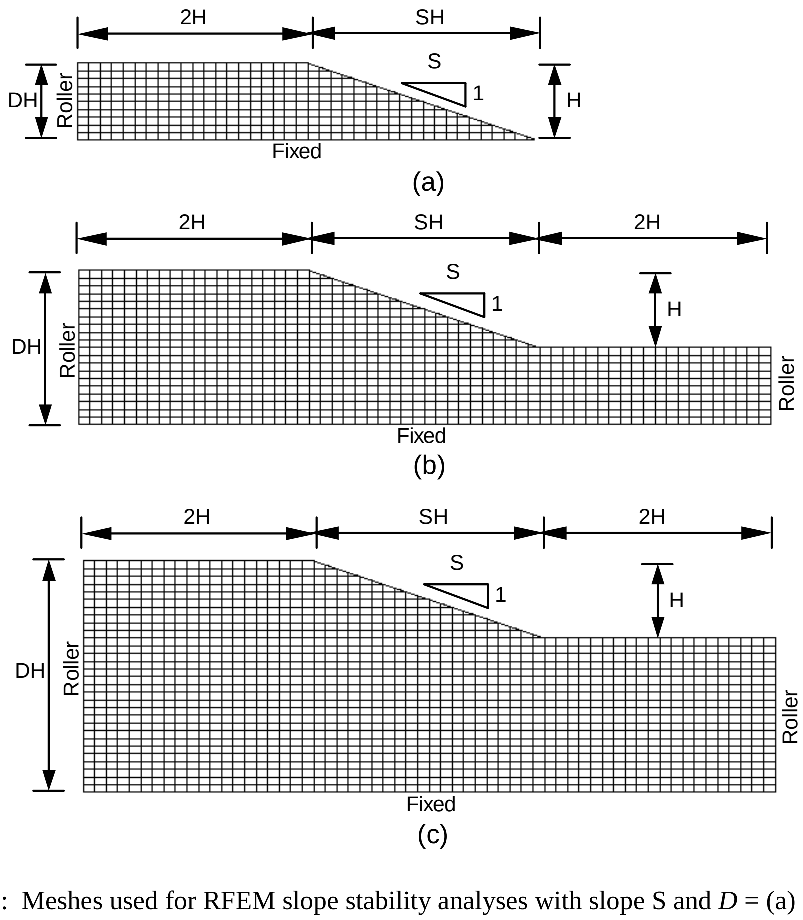


Figure 2: Meshes used for RFEM slope stability analyses with slope S and $D$ = (a) 1; (b) 2; (c) 3.

The finite element analysis is based on an elasto-plastic stress-strain law with a Tresca failure criterion. It uses 8-noded quadrilateral elements and reduced integration in both the stiffness and stress distribution parts of the algorithm. The plastic stress distribution is accomplished by using a visco-plastic algorithm. The theoretical basis of the finite element method is described by Smith and Griffiths (1998; 2004) and the application of the finite element method to slope stability analysis is described by Griffiths and Lane (1999). In summary, the analyses involve the application of gravity loading and the monitoring of stresses at all Gauss points. If the stresses at a point exceed the strength of the material at that point, as defined by the Tresca criterion, the program attempts to redistribute excess stress to neighboring elements that still have reserve strength. This iterative process continues until the Tresca failure criterion and global equilibrium are satisfied at all points within the mesh under strict tolerances. Slope failure is assumed to have occurred if the finite element algorithm has non-converged after 500 iterations (Griffiths and Fenton, 2004).

Based on a given set of statistics for a soil property (i.e. $\mu$, $\sigma$, $\theta$), multiple possible random fields can be generated. For each generated random field, a single finite element analysis is performed. The process is repeated $n_{sim}$ times as part of the Monte Carlo simulation process. The probability of failure ($P_f$) is then estimated by:

$$P_f = \frac{n_f}{n_{sim}} \tag{3}$$

where $n_{sim}$ is the total number of realisations in the simulation process, and $n_f$ is the number of realisations reaching failure. It was generally found that 2,000 iterations were adequate to give a reproducible estimate of the probability of failure.

In addition to the probabilistic analysis described above, a deterministic factor of safety (*FOS*) is also computed, using the finite element method, based on the mean value of $c_u$. In the finite element method, the *FOS* of a soil slope is defined as the factor by which the original shear strength parameters must be divided in order to bring the slope to the point of failure (Griffiths and Lane, 1999). The factor of safety is therefore defined as:

$$FOS = \frac{c_u}{c_{uf}} \tag{4}$$

where $c_u$ is the mean value of undrained shear strength and $c_{uf}$ is a factored value of $c_u$ that brings the slope to failure.

The advantage of the random finite element slope stability analysis method over alternative limit equilibrium methods, is that with the former method it is not necessary to define an initial slip surface. Failure within a slope is initiated by the development of a series of aligned elements, each of which has an undrained shear strength that is less than the shear stress being applied to the element. Figure 3 shows two typical random field realisations of undrained shear strength with different scale of fluctuation. The darker elements indicate stronger soils.

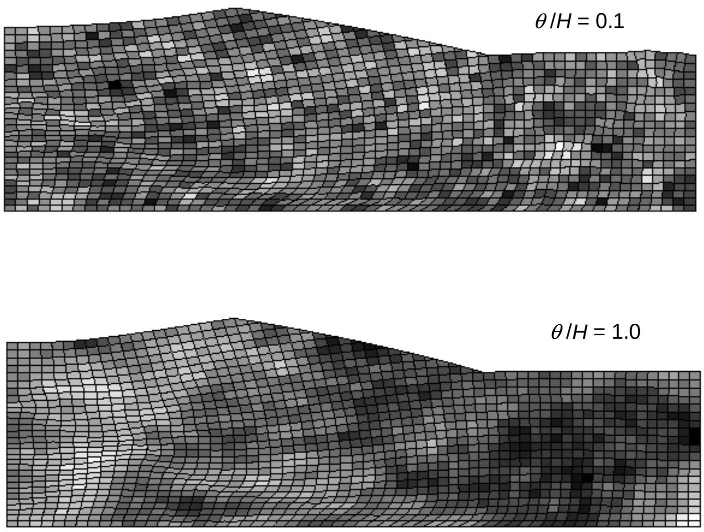


Figure 3: Typical random field realisations of undrained shear strength.

## 3 PARAMETRIC STUDIES

A parametric study was carried out in which only the undrained shear strength was defined according to a distribution (i.e. lognormal). Other parameters were based on their mean values and they remained constant for each finite element analysis. The undrained shear strength is expressed in the form of a dimensionless stability coefficient given by:

$$N_s = \frac{c_u}{\gamma H} \tag{5}$$

where $c_u$ is the mean undrained shear strength, $\gamma$ is the unit weight of the soil and $H$ is the height of the slope. Other parameters are based on their mean values, such as the unit weight $\gamma$ = 20 kN/m$^3$, slope height $H$ = 10 m, Young's modulus of the soil $E = 1 \times 10^5$ kPa and Poisson's ratio $\nu = 0.3$. The scale of fluctuation is made dimensionless by dividing it by the height of the slope (i.e. $\theta/H$). The values assumed for each parameter are summarised in Table 1.

Table 1: Input parameters for parametric studies

| Parameter | Input values |
|---|---|
| $\beta$ | 14°, 18.4°, 26.6°, 45° |
| **D** | 1, 2, 3 |
| $N_s$ | 0.1, 0.2, 0.3, 0.4, 0.5 |
| **COV** | 0.1, 0.3, 0.5, 1.0 |
| $\theta/H$ | 0.1, 0.5, 1, 5, 10 |

Figure 4 summarises the probability of failure for all slope geometries considered in this study. For each set of slope geometries and parameters, as described in Table 1, a deterministic *FOS* can be obtained based on the mean undrained shear strength. This value is plotted on the horizontal axis of Figure 4. By varying the soil variability parameters (i.e. *COV* and $\theta/H$), $P_f$ is obtained for each case via the Monte Carlo simulation process. This value is plotted on the vertical axis.

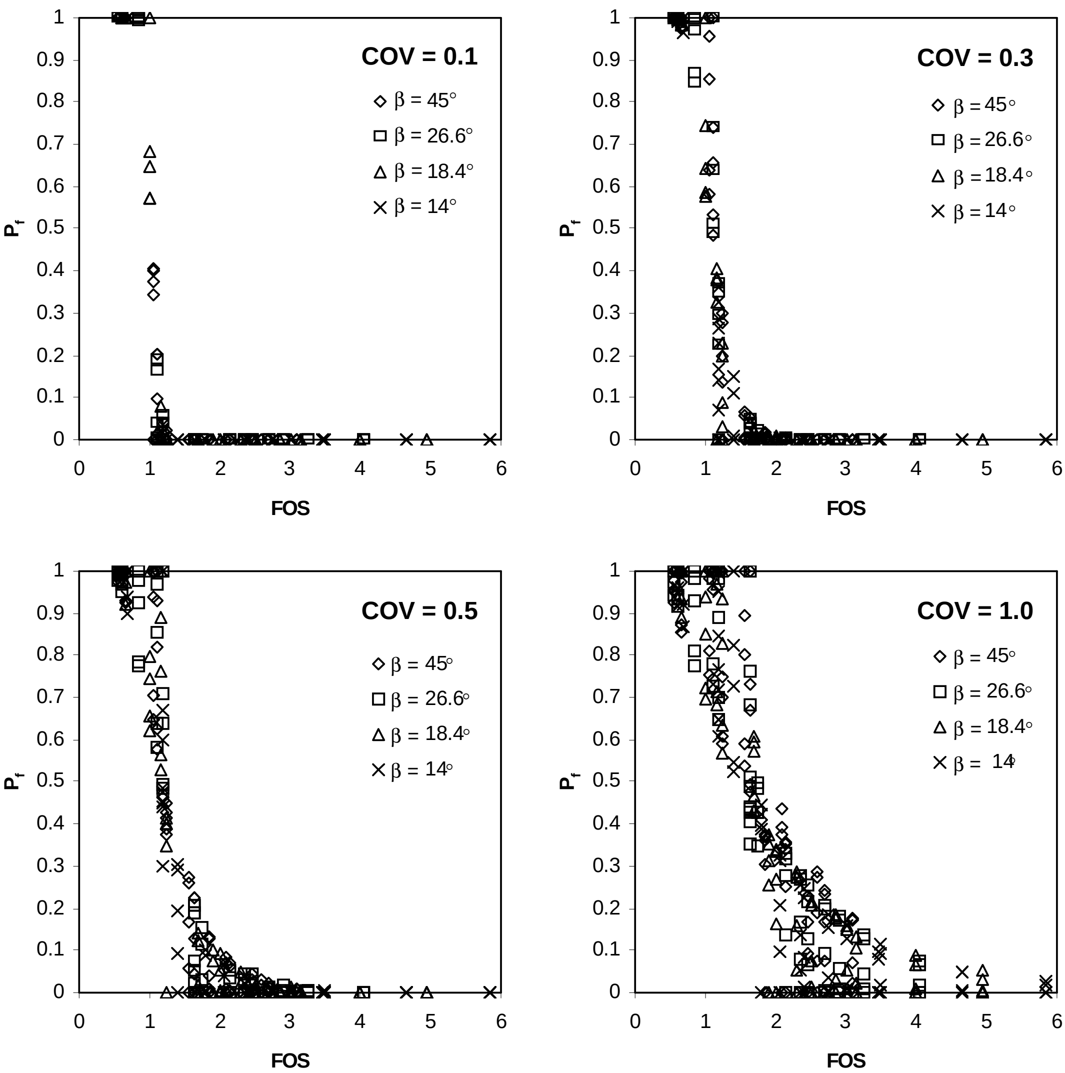


Figure 4: $P_f$ versus *FOS* for *COV* = 0.1, 0.3, 0.5, and 1.0.

To distinguish between the influences of *COV* and $\theta/H$, $P_f$ is plotted against *FOS* for four different *COV*s (i.e. 0.1, 0.3, 0.5 and 1.0). It can be observed from Figure 4 that, for a given *FOS*, the corresponding $P_f$ values vary between zero and unity. Generally, even for the case of low to intermediate values of *COV* (e.g. 0.1 and 0.3), slope failure is still likely to happen when the *FOS* > 1. For example, to achieve a zero $P_f$, the deterministic *FOS* must be greater than 1.3 and 1.5 for *COV*s of 0.1 and 0.3 respectively. This result suggests that soil variability should always be considered in the stability analysis of a marginally stable slope as a *COV* of 0.1 – 0.3 is commonly observed in practice (Lee et al. 1983; Kulhawy et al. 1991). For high values of *COV* (e.g. 0.5 and 1.0), the distribution of $P_f$ is more widely distributed. In this case, even slopes with high *FOSs* (e.g. 2 – 5) are vulnerable to failure. The results in Figure 4 show that the deterministic *FOS* is a poor indicator of the stability of a slope, as slopes with high *FOSs* could be associated with high $P_f$ values depending on the values of *COV* and $\theta/H$.

Figures 5 shows the influence of $\theta/H$ on the $P_f$ for $\beta$ = 45°, 26.6°, 18.4°, and 14° with the stability coefficient and depth factor fixed at $N_s$ = 2 and *D* = 2. The corresponding *FOS*s for each slope geometry were found to be 1.1, 1.2, 1.25 and 1.4 respectively. As previously mentioned, a large value of $\theta/H$ represents a more correlated field, while a small value indicates a more randomly varying field. It can be noted from these charts that $P_f$ increases as *COV* increases. This result is expected because a higher *COV* will result in lower strength values and failure is dominated by these low strength regions. Hence, the probability of failure will increase accordingly.

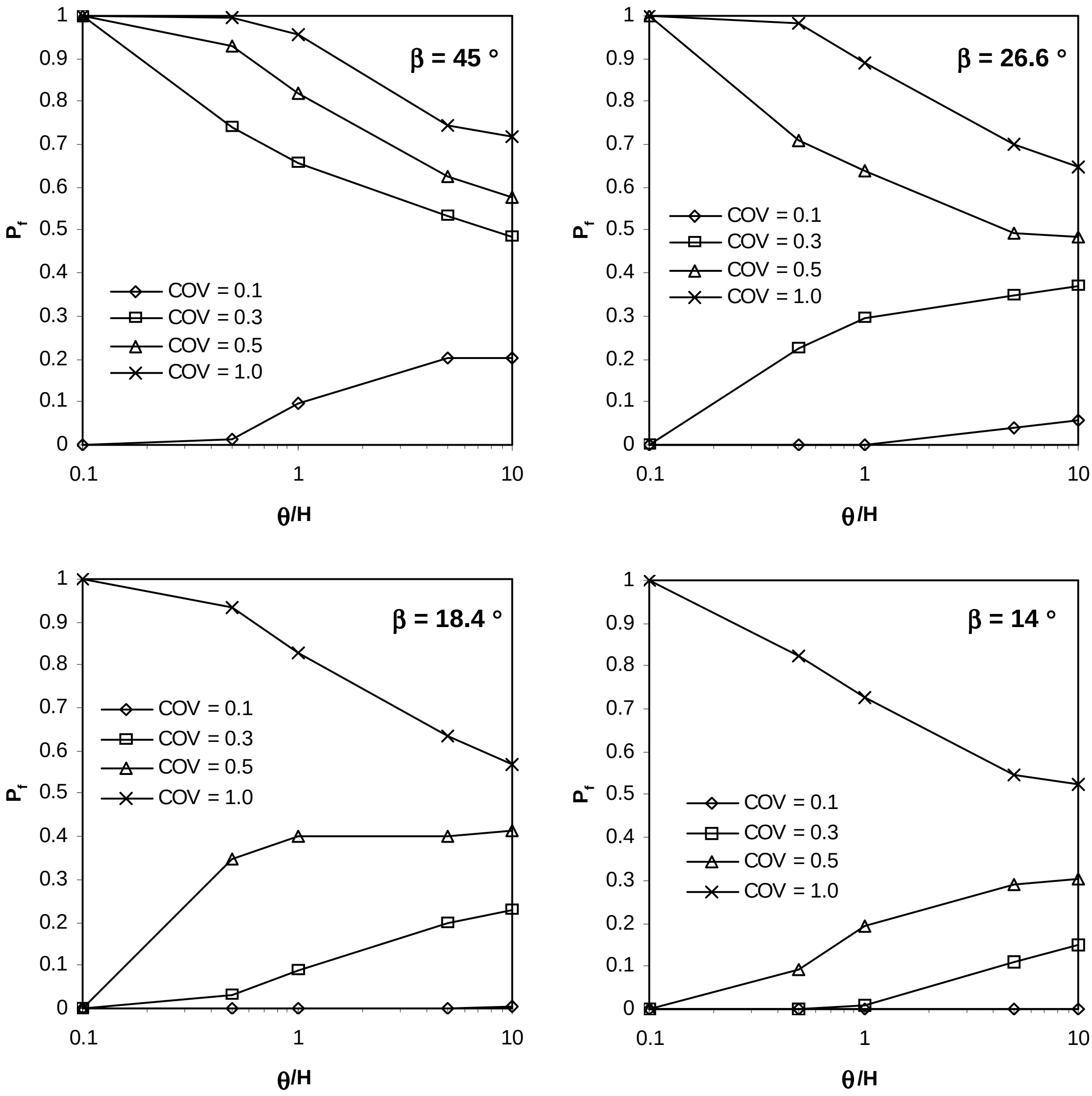


Figure 5: $P_f$ versus $\theta/H$ for $\beta$ = 45°, 26.6°, 18.4°, and 14°; $N_s$ = 0.2 and *D* = 2

Generally, these charts indicate two general trends in the results; the $P_f$ generally converges to unity or zero for different *COV*s. Increases in the value of $\theta/H$ will either increase or decrease the $P_f$ depending on the values of the applicable

*COV*. For example, in Figure 5, for slope with $\beta$ = 45°, $P_f$ increases when $\theta/H$ increases for $COV$ = 0.1 but $P_f$ decreases when $\theta/H$ increases for $COV \geq 0.3$. It is also noted that the results in Figure 5 are very similar to those observed by Griffiths and Fenton (2004) for a slope with $\beta$ = 26.6°, $D$ = 2 and $N_s$ = 0.25. These results indicate that, assuming a perfectly correlated field (i.e. a large $\theta$) or ignoring spatial variability, a slope stability analysis could overestimate or underestimate the probability of failure. Similar trends can also be observed from the results for slopes with $\beta$ = 26.6°, 18.4° and 14° respectively, as shown in Figure 5.

Figure 6 shows the relationship between the $P_f$ and $\theta/H$ for $\beta$ = 45°, 26.6°, 18.4°, and 14° with $D$ = 1, 2 and 3. The stability coefficient and coefficient of variation are fixed at $N_s$ = 0.2 and $COV$ = 0.5 respectively. For $\beta$ = 45° and 26.6°, $P_f$ increases as $\theta/H$ increase when $D$ = 1, while $P_f$ decreases as $\theta/H$ decreases when $D$ = 2 and 3. For $\beta$ = 18.4° and 14°, $P_f$ increases as $\theta/H$ increase when $P_f$ increases as $\theta/H$ increase when $D$ = 1 and 2, while $P_f$ decreases as $\theta/H$ decreases when $D$ = 3.

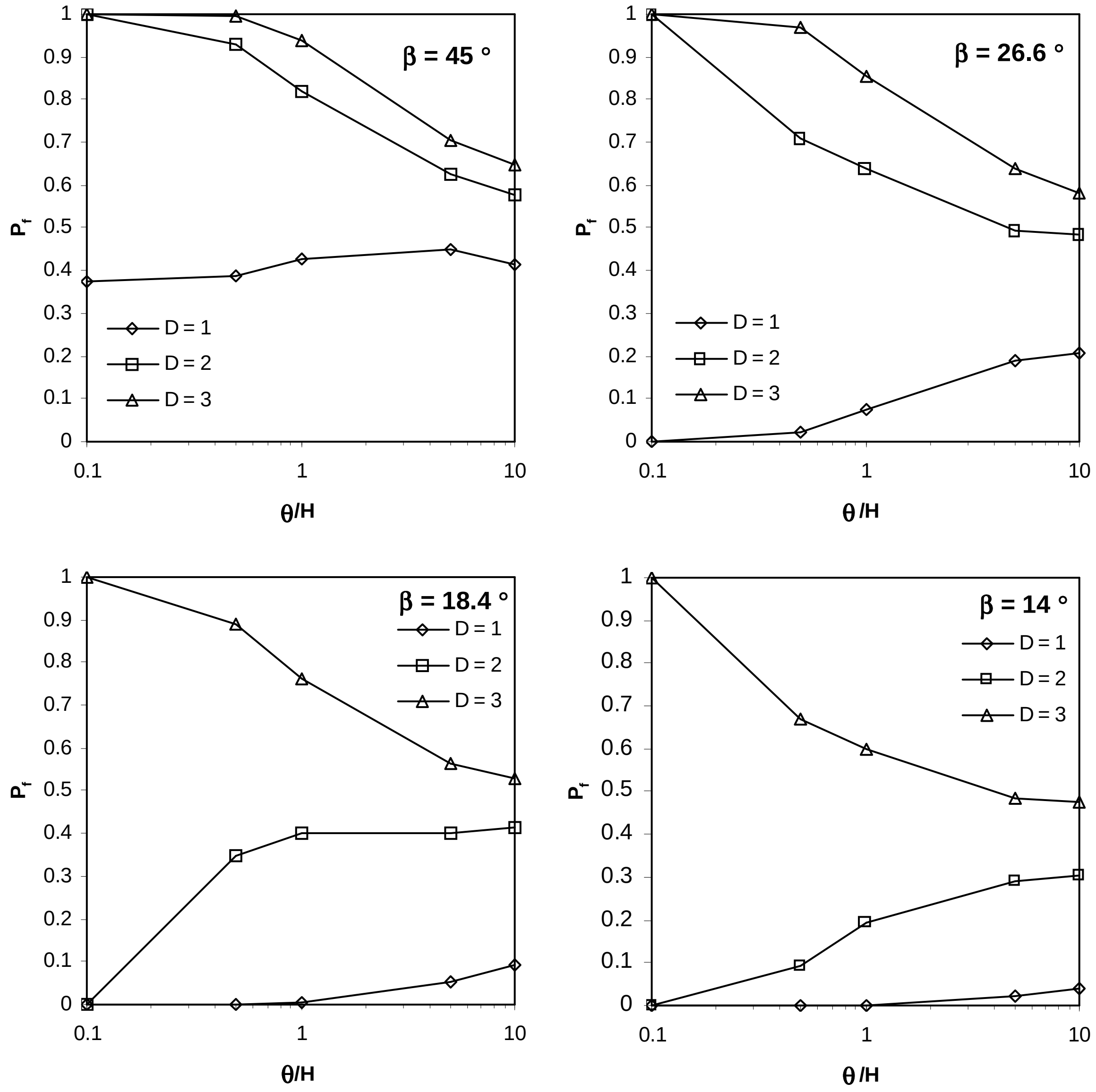


Figure 6: $P_f$ versus $\theta/H$ for $\beta$ = 45°, 26.6°, 18.4°, and 14°; $N_s$ = 0.2 and $COV$ = 0.5

Overall, the results show that the influence of $\theta/H$ on $P_f$ is more significant for slopes with *FOS*s close to unity (i.e. marginally stable). In the most critical cases, $P_f$ can either increase or decrease by approximately 50% as $\theta/H$ increases from 0.1 to 10. The results in Figure 5 and 6 suggest that spatial correlation of soil strength is an important factor to be considered in the stability analysis of a marginally stable slope.

## 4 SUMMARY AND CONCLUSIONS

In this paper, a parametric study involving the reliability of spatially cohesive slopes has been carried out to investigate the influence of the spatial variability of undrained shear strength ($c_u$). The random finite element method (RFEM), which uses random field theory and elasto-plastic finite element analysis, is adopted in this study. The undrained shear strength is treated as a spatially random variable, which is described in terms of a lognomal distribution. The slope angles ($\beta$) of 45°, 26.6°, 18.4° and 14° and depth factors ($D$) of 1, 2 and 3 are considered in the parametric studies. The probability of failure ($P_f$) of a slope is computed via the Monte Carlo simulation process. A deterministic factor of safety (*FOS*) is also computed based on the mean undrained shear strength values.

The results of a comparison between the $P_f$ and *FOS* values indicate that the scale of fluctuation ($\theta$) has a significant effect on the probability of failure of marginally stable slopes ($1 \leq FOS \leq 1.5$), even for low to intermediate values of the *COV* (e.g. 0.1 – 0.3). For higher values of *COV* (e.g. 0.5 – 1), slopes with high *FOSs* (e.g. 1.5 – 5) are also vulnerable to failure depending on the values of $\theta$. The results also indicate that $\theta$ has a significant influence on $P_f$. $P_f$ can either increase or decrease as $\theta/H$ increases from a low to a high value. If a perfectly correlated field (i.e large $\theta$) is assumed or spatial variability is ignored, a slope stability analysis could overestimate or underestimate the probability of failure. Therefore, the spatial correlation of soil strength should be considered in any stability analysis.

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