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Effect of sample location on the reliability based design of pad foundations

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Site investigations that aim to sufficiently characterize a soil profile for foundation design, typically consist of a combination of *in situ* and laboratory tests. The number of tests and/or soil samples is generally determined by the budget and time considerations placed upon the investigation. Therefore, it is necessary to plan the locations of such tests to provide the most suitable information for use in design. This is considered the sampling strategy. However, the spatial variability of soil properties increases the complexity of this exercise. Results presented in this paper identify the errors associated with using soil properties from a single sample location on a pad foundation designed for settlement. Sample locations are distributed around the site to identify the most appropriate sample location and the relative benefits of taking soil samples closer to the center of the proposed footing. The variability of the underlying soil profile is also shown to a have a significant effect on the errors due to sampling location. Such effects have been shown in terms of the statistical properties of the soil profile. The performance of several common settlement relationships to design a foundation based on the results of a single sample location have also been examined.

Keywords: Foundation; Reliability; Site investigation; Sampling; Cost

1. Introduction

In practice, a foundation design utilizes information from a finite number of geotechnical tests to represent the underlying soil properties. Although there are inherent uncertainties associated with the test methods themselves, the more significant errors are due to limited sampling leading to inaccurate profile characterizations. Filippas *et al*. (1988) considered these errors to be statistical uncertainties. Such errors are a manifestation of the spatial variability of soil, where properties may vary considerably from one location to another (Vanmarcke 1983). As such, it is not possible to treat these errors in a simplified manner. Instead, this paper uses results from a simulation model to measure the effect of sampling location on the reliability of pad foundations designed for settlement. Although it is intuitive to suggest that the most appropriate sampling location to optimally characterize the soil profile below a footing is at the proposed footing location, it is not always practical to do so due to existing obstacles or other site restrictions. Results

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presented in this paper quantify the errors with a foundation design based on using soil profile information sampled at a distance from the proposed footing location.

Analyses are undertaken for foundations with a single pad footing, as well as a system of four and nine pad footings. Footings are designed by using several different, common settlement prediction models that make use of the elastic properties of the soil. Results are presented in the form of an average design error, which provides a measure of the degree of over- or under-design using the information at the sampled location with respect to an optimal design based on the complete knowledge of the soil profile. An expression for cost saving is also given in terms of average design error, whereby conclusions regarding the cost effectiveness of different sampling locations are also presented.

2. Methodology

Stochastic modelling has been used by a number of authors to investigate the reliability of foundation design with respect to settlement (Fenton *et al.* 2003), and to a lesser degree, bearing capacity (Griffiths *et al.* 2002). Jaksa *et al.* (2003) extended these models to investigate the effectiveness of site investigations on the design of a foundation. The framework involves simulating three-dimensional soil profiles by generating three-dimensional random fields to represent the spatial variability of soils. Using the complete and exact knowledge of these simulated soil properties, it is possible to obtain an optimal design, which is only attainable as a result of the soil properties being numerically generated. Although Jaksa *et al.* (2003) suggested the use of a three-dimensional finite element analysis (3DFEA) for this design, any traditional process suffices provided the complete soil properties are used. These will be explored in greater detail later in this paper.

In contrast to using the complete soil properties, the three-dimensional soil profile is sampled at only a relatively small number of locations, akin to the implementation of a site investigation. The sampled data are then used with a common settlement relationship to predict footing settlement. Comparing the results of this design process with the optimal design based on complete knowledge indicates whether the sampling strategy yields an over- or under-design. The added advantage of using simulated soil profiles lies with the ability to generate numerous similar, but different profiles, that all comply with the same underlying statistics, by means of Monte Carlo simulation. Therefore, by generating 1000 realizations, measures of the probability and degree of over- and under-design can be determined.

The soil profile is simulated by a three-dimensional random field, generated using the Local Average Subdivision (LAS) method (Fenton 1990, Fenton and Vanmarcke 1990). The LAS generates a random field having a normal distribution with a target mean and variance, which conforms to a prescribed correlation structure. The variability of the soil profile in this study is characterized by the coefficient of variation (COV), which is defined as the standard deviation divided by the mean (σ/μ). The correlation structure of the soil properties is characterized by a statistical parameter, the scale of fluctuation (SOF), which is roughly the distance within which properties are considered reasonably correlated (Vanmarcke 1983). A finite correlation structure with an exponentially decaying correlation for increasing distance is used in the simulation. Jaksa and Fenton (2002) found that although fractal models have their place in the modeling of soil variability, soils do not explicitly show fractal behavior. As such, a finite correlation structure has been adopted to generate the properties of the simulated soil profile. The Markov correlation function is commonly used to model the correlation of soil properties (Fenton 1990) and will be adopted here. The model is defined by:

$$\rho(\boldsymbol{\tau}) = \exp\left(\frac{-2|\tau|}{\theta}\right) \tag{1}$$

where $\rho(\boldsymbol{\tau})$ is the correlation between points separated by the distance vector $\boldsymbol{\tau} = \{\tau_x, \tau_y, \tau_z\}$ and θ is the SOF. Goldsworthy (2006) observed that soil profiles with an anisotropic correlation structure, where the SOF is different in the horizontal (x and y) and vertical (z) directions, has little impact on the errors associated with sampling in the manner described in this paper. Therefore, all analyses that follow consider only soil profiles with an isotropic correlation structure, where the SOF is the same in all directions. This allows the use of a single SOF value, θ, in equation (1). Second order statistics, including the mean, variance and SOF of soil properties, have been investigated by a number of authors (Kulhawy *et al.* 1991, Wickremesinghe and Campanella 1993, Cafaro *et al.* 1999). Properties in the simulated soil profile conform to a lognormal distribution to ensure non-negative values (Fenton and Griffiths 2005), and because the focus of the paper is on settlement, only the elastic properties of the soil are required. Research by others (Fenton and Griffiths 2005) has suggested that the variability of Poisson's ratio has minimal effect on foundation settlement, and, therefore, can be treated deterministically. Thus, only the elastic modulus is represented by a three-dimensional random field.

Nine different soil profiles, defined by the spatial statistics of the elastic modulus, have been investigated in this paper. Soils with a COV of 20, 50 and 100% are used to represent relatively uniform, variable and highly variable profiles, respectively. Fields with a SOF of 1, 4 and 16 m are used to represent soil profiles that vary in a rapid, moderately rapid, and slow fashion, respectively. Combinations of each of the

COV and SOF values are investigated forming the nine different soils.

In the following analyses, a site with a plan size of 50 × 50 m and a depth to bedrock of 30 m has been adopted. Three different foundation systems are investigated, consisting of a single pad footing and systems of four and nine pad footings. The single pad footing is located at the centre of the site and is subjected to a centrally applied 1500 kN point load. The footings in the multiple footing system are spaced evenly at a distance of 16 m in the four pad system and 8 m in the nine pad system (figure 1). Both systems are located centrally on the site. The magnitude of the centrally applied load to each of the footings in the multiple footing cases is representative of a 20 × 20 m, 3-storey structure, with a 3 kPa live load and a 5 kPa dead load. The individual footing loads are determined by proportioning the total structural load according to the tributary floor areas. This results in the footings having loads as shown in figure 1. No load factoring is used in the analysis.

The geotechnical data required by the settlement models is obtained by sampling soil properties at discrete locations around the site. In total, 625 individual sample locations are investigated. The sample locations are spaced at 2 m in each direction (figure 2), for the nine pad system. The grid of sample locations is centered about the footing system and arranged so that a sample location appears under each of the proposed footings. A vertical sample of properties is obtained at each plan location shown in figure 2, where properties are spaced at 0.5 m intervals due to computation time restrictions discussed later. Errors associated with specific testing methods have not been considered in this paper. Jaksa *et al.* (2005) investigated the influence of such errors on the effectiveness of site investigations. The soil property information obtained from the sample location is used in one of seven different settlement prediction models. Each prediction model is based on the assumption of linear-elastic behavior. The seven settlement prediction methods are listed and briefly discussed in table 1.

The following design criteria have been adopted: 25 mm maximum allowable settlement; and 0.0025 m/m maximum differential settlement (between footings). In order to obtain a footing design, an iterative procedure is followed: footings are initially set to a minimum size and the settlement of this footing is predicted using one of the seven settlement models. If the settlement exceeds the design criteria, the footing size is increased. After the size increase, the settlement is again predicted and compared to the design criteria. This process is repeated until the criteria are met. The footing size is increased in one direction at a time, which ensures that the load is applied at the center of the footing. This process is discussed in greater detail by Jaksa *et al.* (2005).

The 3DFEA implementation, which is based on the finite element code developed by Smith and Griffiths (2004) and uses a preconditioned iterative conjugate gradient solver, assumes that footings are rigid and unable to rotate. However, as several of the above-mentioned settlement models (table 1) estimate the settlement of a flexible load area, a method to estimate a rigid footing displacement

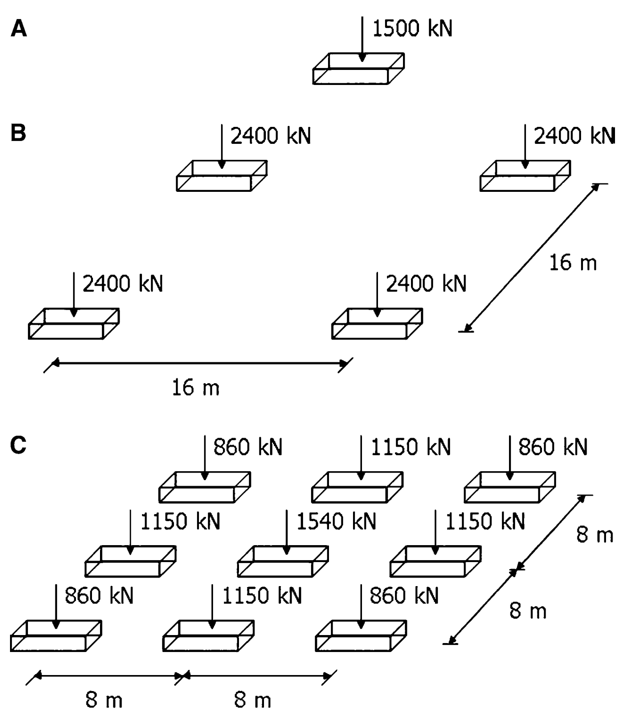

Figure 1. Design scenario of (a) a single pad footing and systems of (b) four and (c) nine pad footings.

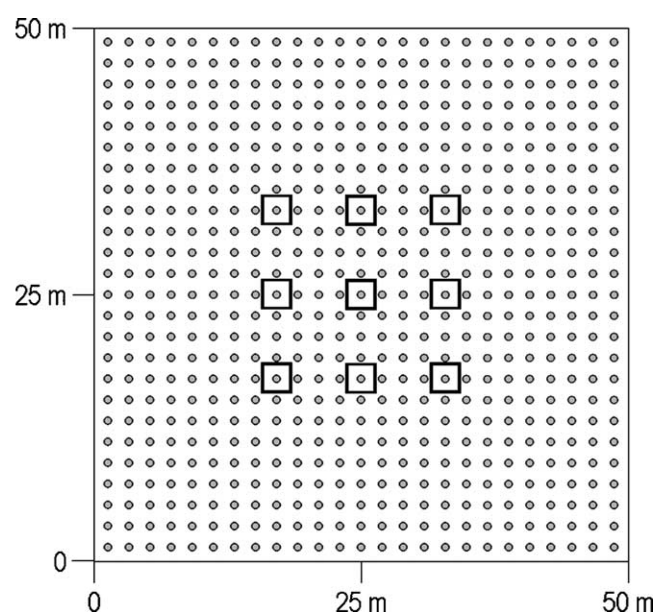

Figure 2. Sample locations with respect to a system of nine pad footings and site area.

Table 1. Settlement prediction models used to estimate the settlement of a pad footing.

No	Source	Description	Soil variability	Rigid settlement
1	Schmertmann (1970)	Idealized strain distribution – calibrated to CPT results and measured settlements	√[1]	√[2]
2	Timoshenko and Goodier (1951)	Theory of elasticity using correction factors based on footing and profile geometry	–	–
3	Newmark (1935)	Estimates the stress distribution in the profile by integrating Bousinessq stress equations	√[1]	–
4	Westergaard (1938)	As per Newmark (1935)	√[1]	–
5	Janbu *et al.* (1956)	Calibrated to 3DFEA	–	√[2]
6	Perloff and Baron (1975)	Uses influence factors for a rigid footing from Harr (1966)	–	√
7	Smith and Griffiths (2004)	3DFEA using preconditioned iterative conjugate gradient solver	√	√

Notes:
1. Accommodates vertical soil variability by treating profile in layers.
2. Based on measured settlements that are between rigid and flexible, but are assumed to be rigid in this paper.

from flexible footing settlements is adopted. This method is described in full by Goldsworthy (2006). The 3DFEA model also accommodates the settlement increase of a footing due to adjacent loaded footings. The other six settlement models do not incorporate this additional settlement, but have been modified here to accommodate such settlement. This is achieved using a method based on superposition, as discussed by Goldsworthy (2006).

The comparison between the optimal design using complete knowledge of the soil profile and the design based on soil profile information from a sample location is generally presented as an averaged design error, evaluated from 1000 Monte Carlo realizations. This number of realizations was found to be statistically stable. The design error is a measure of the difference between the design using data at the sampled location and the optimal design given by:

$$DE_i = \frac{A_i - A_{\text{opt}}}{A_{\text{opt}}} \quad (2)$$

where DE_i is the average design error due to sample location i; A_i is the designed footing area using data at sample location i; and A_{opt} is the optimal footing design using complete knowledge of the soil profile. The design A_i is determined by sampling the soil properties in the vertical direction from the three-dimensional random field at plan location i. The sampled soil properties are used with one of the settlement prediction methods in the iterative design process discussed earlier to obtain a footing size, A_i. This is compared to a footing size obtained from the same process, but using complete knowledge of the soil, made possible by simulating the profile.

3. Effect of sample location on average design error using different settlement models

The analyses in this section compare the use of different settlement prediction techniques with respect to sample location and mean. This has required the use of a benchmark settlement prediction method, which in this case is 3DFEA, or Model 7 in table 1. The use of 3DFEA allows the effects of spatial variability of the soil profile to be more adequately incorporated into the analysis. However, there are several constraints and relative costs associated with using 3DFEA. The most significant cost is the computation time required to analyse a three-dimensional problem with 3DFEA. Large 3DFEA runs require hours, if not days, of computations using current computing resources. Therefore, a compromise with the number of elements has been necessary, leading to an element size of 0.5 m in each direction, resulting in a mesh size of $100 \times 100 \times 60$, representing a $50 \times 50 \times 30$ m site, as discussed previously. However, such a coarse element size raises another constraint of 3DFEA, where the footing size must be a multiple of the element size. This results in footings that are increased by 1 m in each direction to maintain a centrally applied load. To ensure consistency between the design using 3DFEA and a design using one of the other settlement prediction techniques, the same procedure to increase footings is used.

Figure 3 shows the respective relationships between average design error and the sample to footing separation distance for each settlement prediction model. The results shown in figure 3 are based on a moderately variable soil profile (COV = 50%) with moderately rapid fluctuations (SOF = 4 m). The different relationships shown in figure 3 for the various prediction techniques are a function of the simplifications and restrictions of each model, as well as the manner in which they use the soil properties from the sampled data. Earlier, we briefly mentioned that several of the settlement prediction models accommodate soil variability in the vertical direction by treating the profile as a series of layers (table 1). The settlement prediction models that do not treat the profile as a series of layers require the sampled data to be reduced into a single elastic modulus value. In this paper, the single elastic modulus value used in the relevant prediction models is determined using an arithmetic average of the soil properties in the sample. Fenton and Griffiths (2005) suggested that a geometric average maybe more suitable for averaging properties in a spatially random soil profile. However, the arithmetic

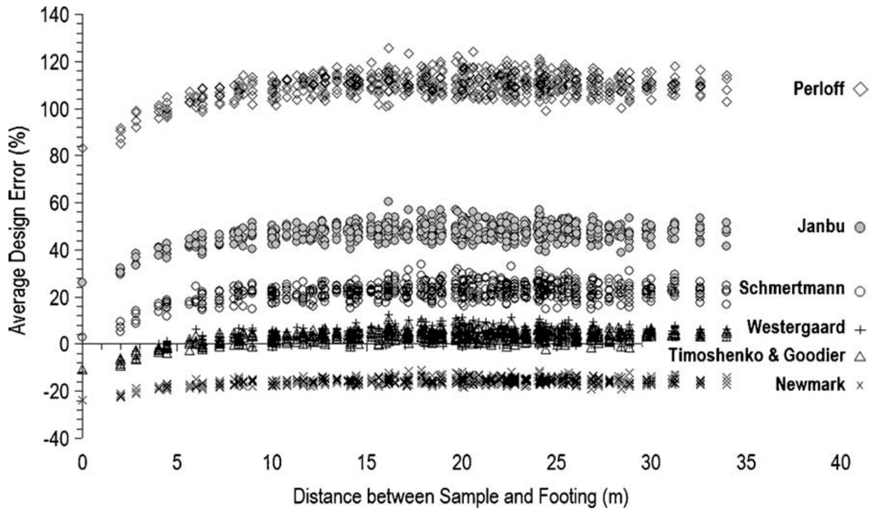

Figure 3. Relationship between average design error and sample-footing distance using different settlement predictors for a single pad foundation on a soil with COV = 50% and SOF = 4 m.

average has been used for the results in this paper, since it is commonly used in practice to average multiple properties. Additional discussion regarding the influence of different averaging methods on the design error has been treated by Goldsworthy (2006).

Three of the settlement predictors (Westergaard, Timoshenko and Goodier, Newmark) are shown to result in negative average design errors (figure 3). This suggests an under-design, where the designed footing size is typically smaller than the optimal design using 3DFEA and complete knowledge of the soil profile. This suggests that the use of these methods in the design situation presented footing designs that have a high probability of under-design or failure. On the other hand, the Perloff and Baron model provides the most conservative design. Even with the soil properties sampled from the centre of the footing, the average design error of the Perloff and Baron model is 80% larger than the 3DFEA design (figure 3), when the separation distance is zero.

By targeting the settlement model that provides the smallest positive average design error, the Schmertmann model appears to perform best. The Westergaard and Timoshenko and Goodier methods also show a small positive average design error when soil properties are used from a sample located further than 10 m from the center of the footing. However, a negative average design error results when these relationships are used with soil properties from a sample located within 10 m of the center of the footing. This situation is not recommended, as foundation failures are likely. Instead, increased conservatism should be included to guard against possible failures when the Westergaard or Timoshenko and Goodier relationships are used to design a footing based on the results of a single sample located within 10 m of the footing center. This is also the case when the Newmark relationship is used with results from any sample location.

4. Effect of sample location on average design error for different soil profiles

As the Schmertmann settlement prediction technique yielded the smallest positive average design error for all sampling locations (figure 3), and because of the limitations associated with 3DFEA mentioned earlier, this method will be used exclusively to investigate the effect of sampling location on the average design error for different soil profiles. In this section, the Schmertmann method is used to provide both the optimal design based on complete knowledge of the soil profile and the design using the soil properties at the sampled location. A major benefit of not using 3DFEA to determine the optimal design is the footing size is not constrained to the element size. Therefore, the footing design targets the design criteria with more precision than in the previous section. Although the Schmertmann relationship does not accommodate horizontal variability of soil properties, the footing design resulting from the use of this relationship with soil data from a vertical sample located at the centre of the footing is in close agreement with 3DFEA (figure 3). Therefore, the complete knowledge of the soil profile is now considered a vertical sample of soil properties at the centre of the footing, and the optimal design uses these data with the Schmertmann settlement relationship.

The average design error for the sampling locations shown in figure 2, for a soil profile with constant SOF (4 m) and increasing COV, is shown in figure 4. Note the change in scales for the average design error. The results

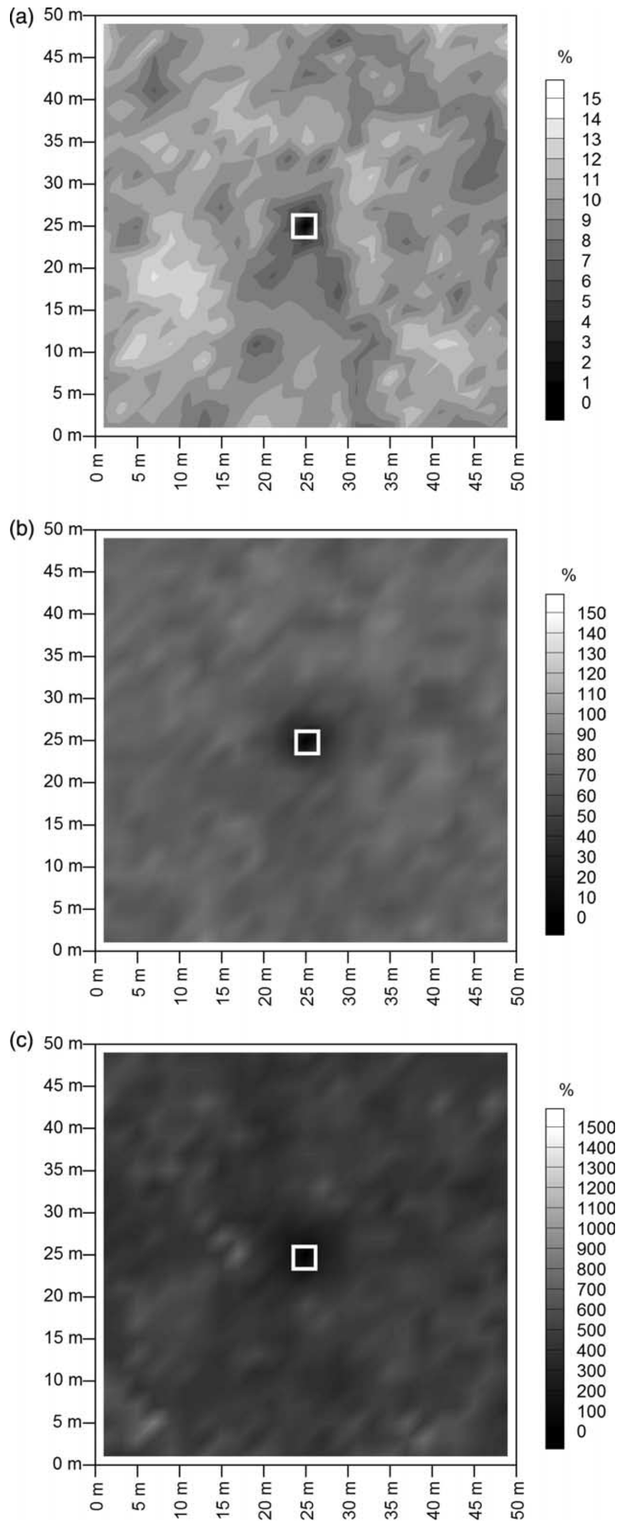

Figure 4. Average design error for a single pad footing with respect to sample location based on a soil with a SOF of 4 m and a COV of (a) 20%, (b) 50% and (c) 100% (note change in scales). [White square indicates footing location.]

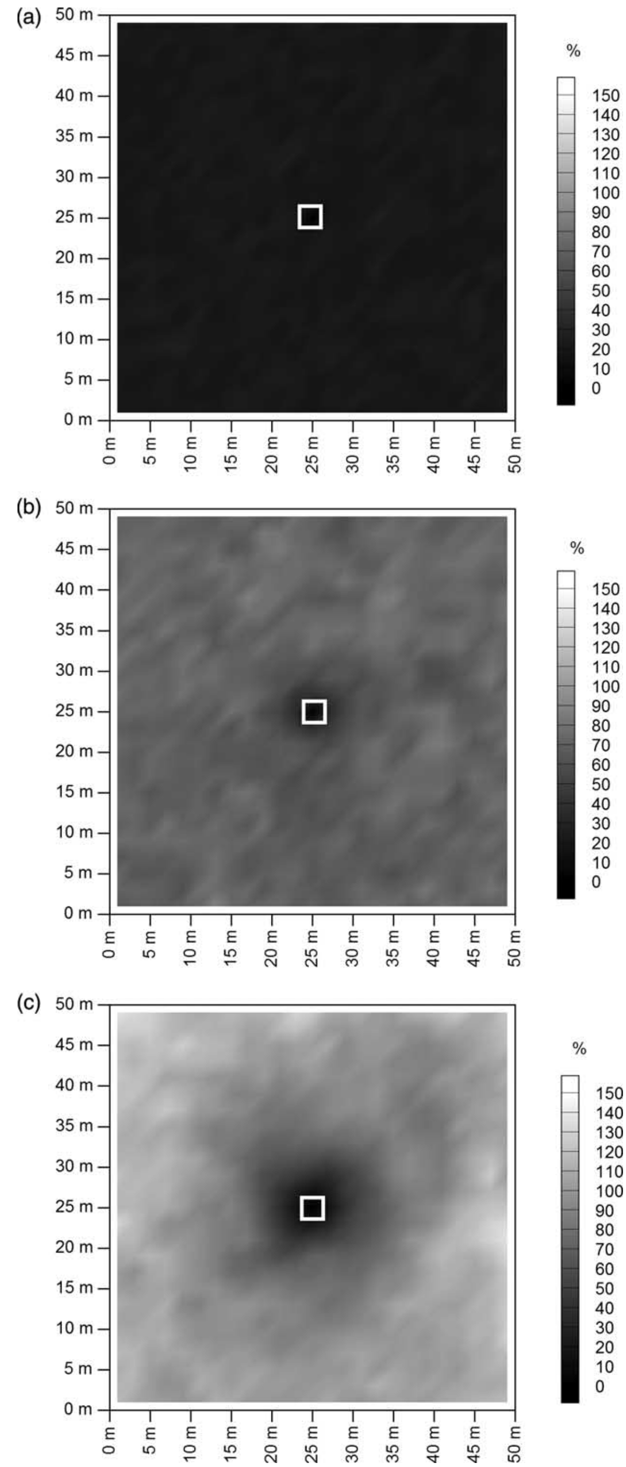

Figure 5. Average design error for a single pad footing with respect to sample location based on a soil with a COV of 50% and a SOF of (a) 1 m, (b) 4 m and (c) 16 m. [White square indicates footing location.]

shown in figure 4 indicate that an increase in soil profile variability (COV) increases the magnitude of average design error, but does not noticeably affect the relationship between average design error and sample location. This is seen in figure 4 by the minimal variation in average design error for each soil profile. In other words, it is of little consequence if a sample is taken at a location that is 5 or 25 m from the footing location. However, the results shown in figure 5 suggest that the SOF of the underlying elastic modulus has a marked impact on the relationship between sample and footing location. Here, the COV is held constant at 50% and the SOF is increased from 1 m [figure 5(a)] to 16 m [figure 5(c)].

As the SOF increases, there is a growing region of similar average design error surrounding the footing. This suggests that the SOF affects the relationship between the average design error and the sample-to-footing separation distance. This effect is clearly shown in figure 6 as a scatter plot between the average design error and the sample-to-footing separation distance based on soil profiles with a constant COV of 50% and increasing SOF, that is, the same data as given in figure 5. Superimposed on figure 6 are general trend lines fitted by eye.

The contour plots shown in figure 5 suggest that at small SOFs, there appears no optimal location to sample data. Instead, the average design error is independent of the sampling location, as shown in figure 6. On the other hand, as the SOF increases, the average design error becomes increasingly dependent on the sampling location. It is apparent from the contour plot in figure 5(c) and the results in figure 6 that the average design error increases as the sample is located further from the footing for soil profiles with large SOFs, as one would intuitively expect. Furthermore, the average design error for a soil profile with a SOF = 16 m based on a sample taken at a location 10 m from the footing center, is approximately 300% greater than the average design error at the same sample location in a soil profile with a SOF = 1 m. This increased error, for profiles with larger SOFs, is clearly shown in figure 6. At first glance, a larger error in a soil profile with a larger SOF appears to contradict convention, as the correlation of soil properties within the profile is greater. However, when a sample is taken in a rapidly varying profile (i.e. a low SOF), the properties fluctuate evenly both above and below the mean [figure 7(a)]. Therefore, the sample mean is relatively close to the population mean. However, in a profile with a high SOF (slowly varying field), adjacent properties are similar and generally either above or below the mean value, which yields a sample mean further from the population mean [figure 7(b)]. The sample mean is also affected by the size of the sample. In this case, where the Schmertmann settlement relationship is used, only soil properties to a depth equal to twice the least plan dimension of the footing are considered. Therefore, the sample size is restricted.

The results in figure 7 show that the difference between the sample and population means in the profile with a high SOF is larger than the difference in the profile with a low SOF. The difference is even greater if the depth of analysis is restricted to <10 m, which is common in the Schmertmann settlement method. The increased difference between the sample and population means for the profiles with a higher SOF drives the increase in average design error, as shown in figures 5 and 6. However, as the SOF of the soil profile increases, properties within the site area become similar. Therefore, as the SOF approaches infinity, the soil variability within the site area approaches zero, which yields a

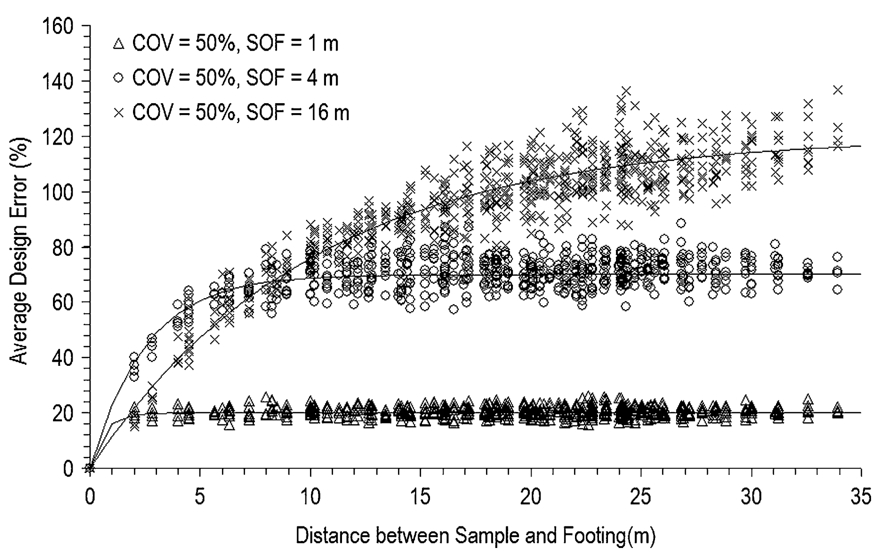

Figure 6. Relationship between average design error and sample to footing separation distance for a single pad footing based on a soil with a varying SOF.

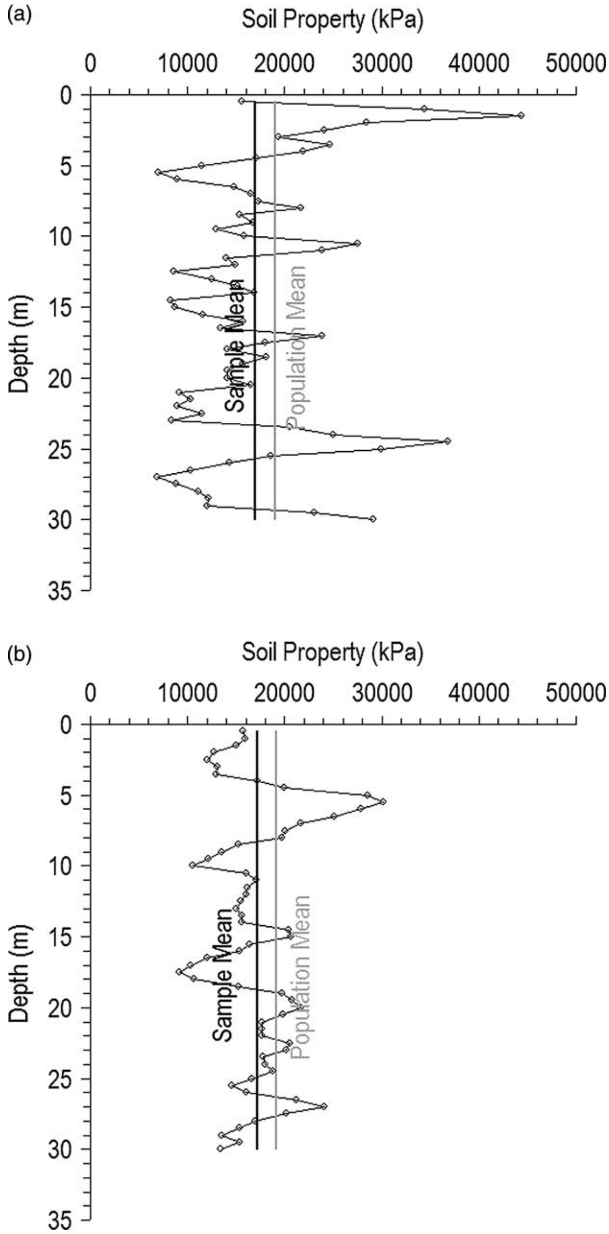

Figure 7. Difference between sampling in a soil profile with (a) low SOF and (b) high SOF.

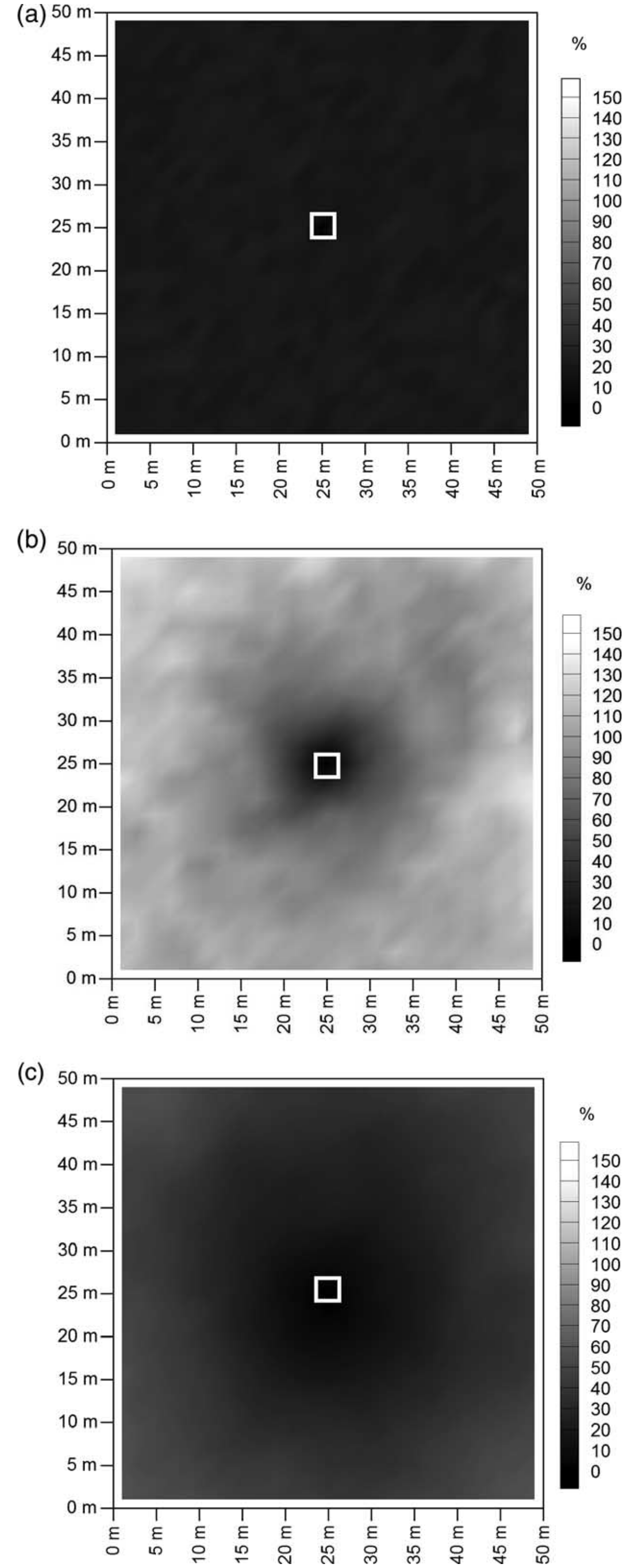

Figure 8. Average design error for a single pad footing with respect to sample location based on a soil with a COV of 50% and a SOF of (a) 1 m, (b) 16 m and (c) 100 m. [White square indicates footing location.]

similar condition to the results shown in figure 5(a) for a profile with low SOF. This is shown in figure 8, where the COV is held constant at 50% and the SOF is increased from 1 m [figure 8(a)] to 100 m [figure 8(c)].

The results presented in figure 8 indicate that an increase in SOF from 16 m [figure 8(b)] to 100 m [figure 8(c)] reduces the average design error for all sampling locations and returns to a condition similar to that shown in figure 8(a), where the SOF = 1 m. This suggests the presence of a 'worst-case' SOF, where the average design error is a maximum at the same sample location for different soil profiles. Similar worst-case SOF conditions have been observed by Fenton *et al*. (1996) when dealing with the reliability of settlement estimates using finite element analysis. Such a worst-case SOF can be used to ensure conservatism and estimate the upper bound of the average design error without needing to determine the SOF of the soil profile.

The effect of sample location on the average design error of a four and nine pad system is shown in figures 9 and 10, respectively, based on soil profiles with increasing SOFs and a constant COV of 50%. As intuition would suggest, the results given in figures 9 and 10 indicate that the centre of the foundation system provides the optimal sampling location and yields the smallest average design error. This is especially true for the nine pad system on a soil profile with high SOF, as shown in figure 10(c). However, the results given in figure 9(c) for the four pad system on the soil profile with high SOF suggest that a sample location anywhere between the footings provides a suitable design, yielding a low average design error. It is expected that the centre of the system would still provide the optimal sampling location; however, there appears to be little benefit in taking a sample at the system centre compared with anywhere between the footings. This is due to the absence of a centralized footing in the four pad system. It is also important to recognize that each of the footings in the four pad system resists the same applied load, whereas the footings in the nine pad system are designed to resist loads of varying magnitudes.

Figures 9 and 10 also indicate different magnitudes of average design error for the four and nine pad systems, respectively. The results shown in figure 10 suggest that the average design error is larger for the same sample location in the nine pad system as for the four pad system (figure 9). This is a function of the number of footings in the system, the size of the applied loads and the mean elastic modulus of the soil profile. As the number of footings in the system increases, the size of the average design error is expected to increase due to the presence of additional footings to contribute to the average design error. As the size of the applied loads increases or, equivalently, the mean elastic modulus decreases, the average design error is expected to increase due to the requirement for a larger footing size to meet the same design criteria. The effects on the magnitude of average design error due to the number of footings in the system, size of the applied loads and the mean elastic modulus of the soil profile are beyond the scope of this paper.

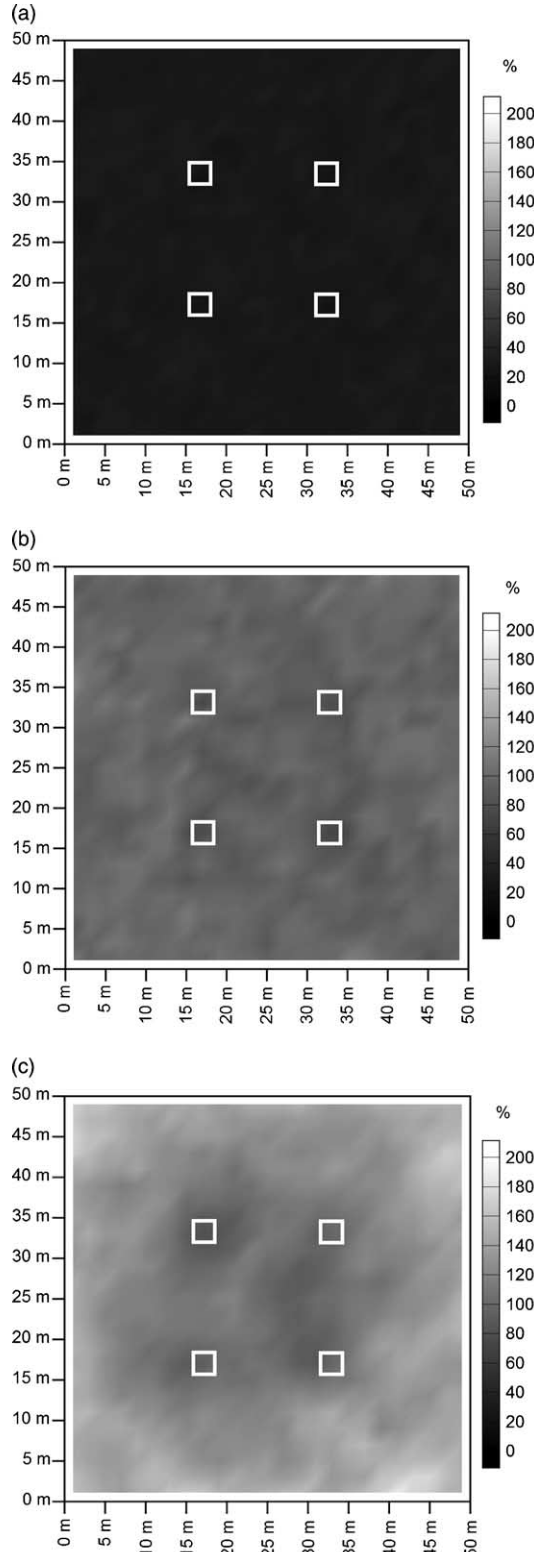

Figure 9. Average design error for a four pad footing system with respect to sample location based on a soil with a COV of 50% and a SOF of (a) 1 m, (b) 4 m and (c) 16 m. [White square indicates footing location.]

5. Application of average design error

There are additional benefits in expressing the results of these analyses in terms of an average design error rather

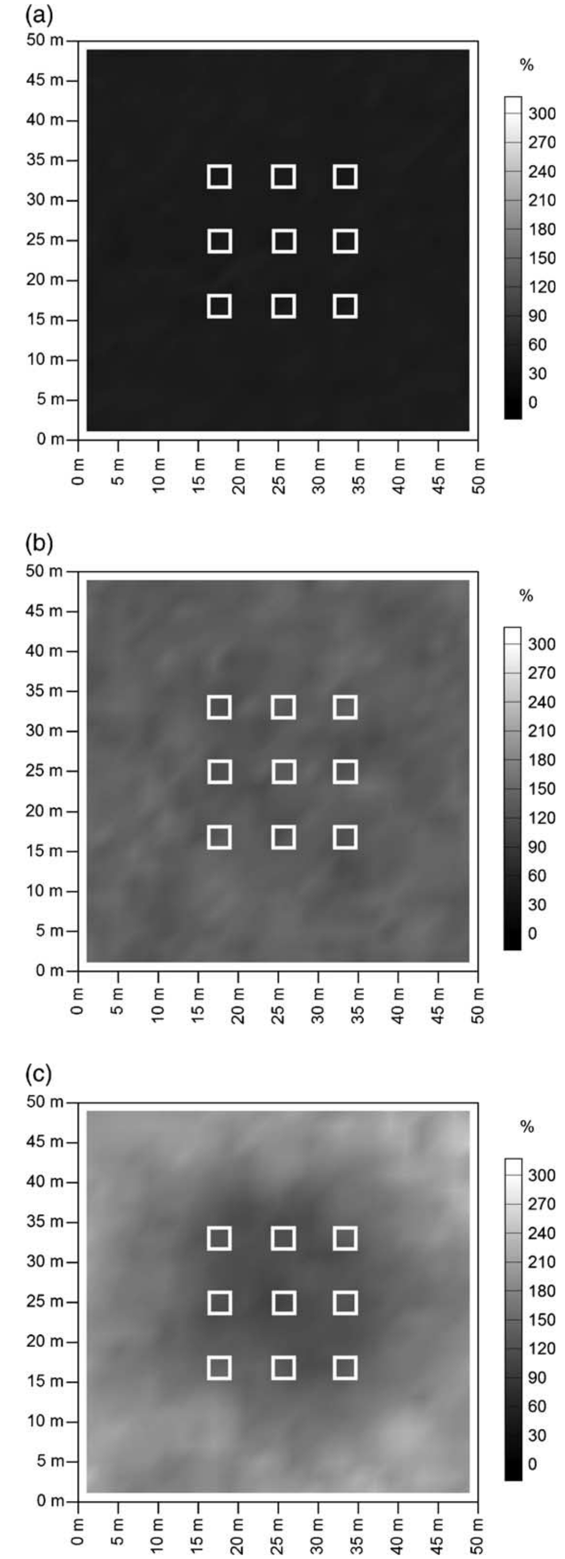

Figure 10. Average design error for a nine pad footing system with respect to sample location based on a soil with a COV of 50% and a SOF (a) 1 m, (b) 4 m and (c) 16 m. [White square indicates footing location.]

than a probability of failure. The average design error, as defined in equation (2), quantifies the magnitude of over- or under-design, whereas a probability of failure does not express the size of the footing. As such, the use of the average design error enables conclusions regarding the cost-effectiveness of the design with respect to the sample location. The design error, as shown in equation (2), is a function of the designed footing area based on the information obtained at a sample location. Therefore:

$$C_i = f(A_i) \tag{3}$$

where C_i is the cost of the footing and can be expressed as a function of the footing area, A_i, designed using the information at sample location i. The cost can also be expressed in terms of a cost saving:

$$CS_i = \frac{C_{\max} - C_i}{C_{\text{opt}}} \tag{4}$$

where CS_i is the cost saving, $C_{\max}$ is the cost of the foundation based on property information at the worst sampling location (in terms of this paper this is considered the furthest distance from the center of the footing or foundation system), C_i is the cost of the footing design using property information at sample location i, and C_{opt} is the cost of the optimal footing design using complete knowledge of the profile. Hence, the cost saving is a measure of the financial benefits of using the data at sample location i, over the worst sample location and relative to the optimal footing. Using the formulation for design error as shown in equation (2), the cost saving can be expressed as:

$$CS_i = f(DE_{\max} - DE_i) \tag{5}$$

where DE_i is the design error at sampling location i and $DE_{\max}$ is the maximum design error. The results shown in figure 11 present the reduction of cost saving for an increasing distance between the sample and the center of the footing or foundation system. Conversely, the results shown in figure 11 also show the increase in cost saving as the sample is located closer to the center of the footing or foundation system. The curves shown in figure 11 are a function of the trend line fitted by eye to the average design error to sample-to-footing distance relationship, as shown in figure 6 for the single pad footing case.

The results shown in figure 11 suggest that the cost savings for locating a sample closer to the footing are greater for the single pad footing than for the four pad and nine pad systems. By halving the distance from 10 to 5 m between the sample and the centre of the footing or foundation system, there is an expected cost saving of 26% for the single footing case, 4% for the four pad system and 16% for the nine pad system. These cost savings are over the worst-case design, where the sample is taken at a

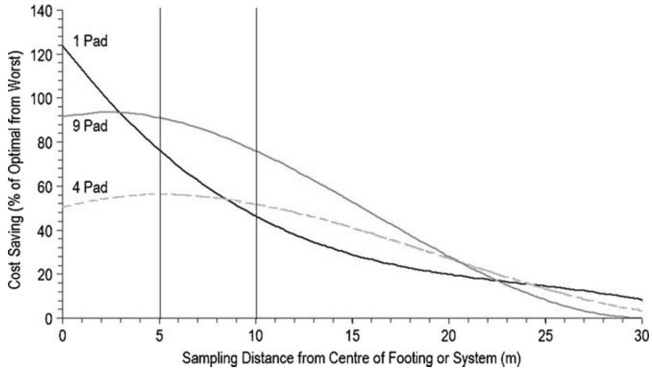

Figure 11. Relationship between cost savings and distance from center of foundation system on a soil profile with COV = 50% and SOF = 16 m.

distance of > 30 m from the footing center and are relative to the optimal footing design, as shown in equation (4). The percentage cost savings shown in figure 11 also suggest that there is little benefit in locating a sample closer than 10 m to the center of the four pad system. This distance is associated with the layout of the foundation system, where the footings are located 11.3 m from the centre of the foundation system, as shown in figure 1(b). This confirms the conclusions made earlier that a sample located anywhere between the footings in the four pad system provides a suitable design.

6. Conclusions and recommendations

The methodology and results presented in this paper enable conclusions to be drawn regarding the effect of sampling location on the design of a pad footing or a pad foundation system. Results have been presented in the form of an average design error, which although is not a probabilistic measure, does yield information regarding the magnitude of over- or under-design. Results show that as a sample is located further from the footing or foundation centre, a larger footing design results, and, therefore, a large positive average design error. As the sample is located increasingly further away from the footing centre, the average design error increases at a rate that is influenced by the SOF of the elastic modulus field. The maximum average design error is a function of both the COV and SOF of the elastic modulus field. A worst-case SOF has also been identified, where the average design error for the same relative position on the site gives the largest average design error. For the situations investigated, this worst-case appears to occur when the SOF is approximately 16 m. Therefore, it is plausible to make conservative estimates of the average design without quantifying the SOF of the soil profile.

As one would expect, the optimal sampling location is at the centre of the footing or foundation system. However, when no centralized footing exists, there appears to be little benefit in sampling at the center of the system compared to anywhere between the footings. The use of different settlement prediction models to provide footing designs based on sampled data was shown to have minimal effect on the relationship between average design error and sample location. However, varying degrees of conservatism were shown for the settlement prediction models investigated. The Schmertmann model was shown to provide the most suitable design for all sampling locations, whereas the Westergaard and Timoshenko and Goodier relationships yielded designs that were close to the optimal design when sampling occurred further than 10 m from the centre of the footing.

By using a simple linear relationship between expected cost savings and the average design error, a measure of the savings due to sample location has been estimated. This has shown that a significant saving is expected if the distance between the sample and the center of the footing or foundation system is reduced by as little as 5 m. It is anticipated that these results can be used to plan site investigations with knowledge of the effect of sampling location. Although it is not expected that the spatial statistics of a soil deposit, such as the SOF, will need to be accurately quantified, basic knowledge of the variability of relevant soil parameters will allow conclusions regarding sampling location and the associated errors. Instead, a worst-case SOF allows conservative estimates of the expected errors associated with sampling.

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Notation

α	proportional angle of adjacent footing with respect to loaded annulus
δ_{rgd}	settlement of rigid footing
δ_{cnt}	settlement of center of flexible footing
δ_{crn}	settlement of corner of flexible footing
θ	scale of fluctuation (isotropic)
σ	applied footing stress
$\boldsymbol{\tau}$	vector containing lags or separation distances
τ_x, τ_y, τ_z	lag or separation distance in each direction
A_i	footing design area using data at sample location i

A_{opt} footing design area using complete knowledge of the soil profile
C_i cost of footing using data at sample location i
C_{opt} cost of footing using complete knowledge of the soil profile
C_{max} cost of footing using data from the worst sample location
CS_i cost saving using data at sample location i
DE_i design error based on results using data at sample location i
DE_{max} design error based on results using data from the worst sample location

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