Consequence factors in the ultimate limit state design of shallow foundations

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Abstract: The reliability-based design of shallow foundations is generally implemented via a load and resistance factor design methodology embedded in a limit state design framework. For any particular limit state, the design proceeds by ensuring that the factored resistance equals or exceeds the factored load effects. Load and resistance factors are determined to ensure that the resulting design is sufficiently safe. Load factors are typically prescribed in structural codes and take into account load uncertainty. Factors applied to resistance depend on both uncertainty in the resistance (accounted for by a resistance factor) and desired target reliability (accounted for by a newly introduced consequence factor). This paper concentrates on how the consequence factor can be defined and specified to adjust the target reliability of a shallow foundation designed to resist bearing capacity failure.

Key words: bearing capacity, reliability, consequence factors, load and resistance factor design, ultimate limit state, shallow foundation.

Résumé : La conception basée sur la fiabilité pour des fondations peu profondes est généralement appliquée à partir d'une méthodologie de conception basée sur des facteurs de chargement et résistance, celle méthodologie étant introduite dans un cadre de conception selon les états limites. Pour tout état limite particulier, la conception s'effectue par une vérification afin que la résistance prévue soit égale ou supérieure aux effets de chargement prévus. Les facteurs de chargement et de résistance sont déterminés de telle sorte que la conception finale est suffisamment sécuritaire. Les facteurs appliqués à la résistance dépendent de l'incertitude dans la résistance (considérée par le facteur de résistance) et de la fiabilité visée (considérée par le facteur de conséquence, nouvellement introduit). Cet article discute de la manière dont le facteur de conséquence peut être défini et spécifié afin d'ajuster la fiabilité visée d'une fondation peu profonde conçue pour résister à la rupture en capacité portante.

Mots-clés : capacité portante, fiabilité, facteurs de conséquence, conception basée sur des facteurs de chargement et résistance, état limite ultime, fondation peu profonde.

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Introduction

Geotechnical design codes and manuals worldwide are starting to migrate away from working stress design towards reliability-based design. In all codes reviewed by the authors, design reliability is achieved by making use of a load and resistance factor methodology embedded in a limit state design (LSD) framework. The codes–standard reviewed are as follows:

(1) *National building code of Canada* (NBCC), National Research Council of Canada (NRC 2005).
(2) *LRFD bridge design specifications*, American Association of State Highway and Transportation Officials (AASHTO 2007).
(3) *Canadian highway bridge design code* (CHBDC), Canadian Standards Association (CSA 2006).
(4) *Eurocode 7 geotechnical design – Part 1: General rules* (Eurocode 7), European Committee for Standardization (CEN 2003).
(5) *Bridge design, Part 3: Foundations and soil-supporting structures*, Australian standard 5100.3 (Standards Australia 2004).
(6) *Code of practice for foundation engineering*, Danish Geotechnical Institute (DGI 1985).

The LSD framework basically involves identifying possible failure modes (e.g., sliding, overturning, and bearing capacity failures) and then ensuring that the factored resistance to the failure mode is greater than or equal to the factored load effects that are trying to cause the failure. This paper will consider only the ultimate bearing capacity limit state for shallow foundations. Thus, for the bearing capacity ultimate limit state, the load and resistance factor design (LRFD) involves dimensioning the shallow foundation so that an equation of the following generalized form is satisfied:

$$\Psi_{\mathrm{u}}\varphi_{\mathrm{gu}}\hat{R}_{\mathrm{u}} \geq \sum_i I_i\alpha_i\hat{F}_i \qquad [1]$$

where $\hat{F}_i$ is the ith characteristic load effect, α_i is its corresponding load factor, I_i is an importance factor, $\hat{R}_{\mathrm{u}}$ is the ulti-

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mate geotechnical resistance obtained using characteristic geotechnical parameters, φ_{gu} is the ultimate geotechnical resistance factor, and Ψ_u is a consequence factor that is defined more clearly shortly. The goal of this paper is to investigate reliability-based design provisions required to ensure acceptable reliability. A minimum reliability level can also be expressed in terms of a maximum acceptable failure probability (reliability is one minus the failure probability), and it is the failure probability that will be considered in the following.

The ultimate geotechnical resistance factor, φ_{gu}, reflects uncertainty in the prediction models and in the geotechnical parameters used to estimate the characteristic resistance, $\hat{R}_u$, while both the consequence factor, Ψ_u, and the importance factor, I_i, are used to adjust the target maximum acceptable failure probability. The consequence factor is newly introduced here, for reasons discussed below. It serves the same basic purpose as the importance factor in adjusting the target failure probability to an acceptable level, which depends on the failure consequence level (lower acceptable failure probabilities for higher failure consequences).

The reasons a consequence factor is introduced into eq. [1] are as follows. On one hand, the importance factor is already well ensconced in structural engineering codes. It is largely aimed at adjusting the factored characteristic loads to account for failure consequence and is generally based on site-specific load distributions (usually snow, wind, and earthquake). It makes sense to apply the importance factor to the load side of eq. [1] because snow, wind, and earthquake loads, for example, are quite site-specific. On the resistance side, structural engineers typically deal with quality-controlled materials (e.g., steel, concrete, and wood), whose probability distributions are well known and relatively constant worldwide. Thus, for structural engineers, the resistance factor alone is generally adequate to account for resistance uncertainty.

On the other hand, geotechnical engineers are faced with large resistance uncertainties from site to site, and even within a site, and these uncertainties are generally quite unrelated to the loading type. There is a real desire in the geotechnical community to account for failure consequence even when the loading consists of just typical dead and live loads. For example, although the current CHBDC (CSA 2006) does account for failure consequences by adjusting seismic loads according to failure consequences, the code does not take failure consequences into account for any other type of loading. Nevertheless, under any loading scenario there is a huge difference between the consequences of failure of a multi-lane lifeline highway bridge in a major city, for example, and a bridge on a minor rural road. Thus, it makes sense to provide a factor on the resistance side of the LRFD equation that accounts for failure consequences independently of loading scenarios.

The consequence factor proposed in eq. [1] is aimed at adjusting the factored resistance to account for failure consequences in those cases not covered by the load side importance factor. The authors note that further research needs to be performed to establish the interaction between the importance and consequence factors and their combined effect on failure probability. Until such research has been carried out, the authors suggest that, if in doubt, the consequence factor be set to 1.0 whenever the importance factor is other than 1.0. It is important to avoid double-factoring via these two factors where not warranted. In this paper, the importance factor, I_i, will be assumed to have a value 1.0. The LRFD equation considered in this paper thus has the form

$$[2] \qquad \Psi_u \varphi_{gu} \hat{R}_u \geq \sum_i \alpha_i \hat{F}_i$$

In civil engineering, reliability-based design is typically couched in terms of a reliability index, β, which is defined as the distance, measured in number of standard deviations, between the mean of the total lifetime extreme load and the ultimate resistance. More specifically, if F is the actual lifetime extreme total load acting on a system and R is the actual minimum lifetime resistance, both of which are uncertain and thus modeled as being random, then failure occurs sometime in the system's lifetime if $R < F$. As these are both random variables, the probability of system failure is

$$[3] \qquad p_f = P(R < F)$$

A common code development assumption is that both F and R are lognormally distributed, because then the ratio R/F is also lognormally distributed. Regarding the loads, Corotis and Doshi (1977) found that the lognormal distribution was a good fit to the live load distribution, only marginally beaten by the Gamma distribution. However, Chalk and Corotis (1980) suggested that when loads are sums of random variables, as is commonly the case at the foundation, the normal distribution would be better due to the central limit theorem. Unfortunately, the normal distribution suffers from the fact that it admits negative loads. As the normal and lognormal distributions are quite similar for coefficients of variation less than about 0.3, the evidence seems to suggest that the lognormal distribution is quite a reasonable load model. Regarding the resistance, Fenton and Griffiths (2003) demonstrated that the lognormal distribution was appropriate for bearing capacity. Because geotechnical resistance is usually governed by the weakest path through the soil, which is often well modeled by a geometric average, the central limit theorem also supports the lognormal hypothesis for the resistance, R. On the basis of the above argument, both F and R will be taken to be lognormally distributed.

Another common assumption in code development is that load and resistance are independent random variables. This is, admittedly, a very questionable assumption in geotechnical engineering because the shear strength of a frictional soil increases linearly with the stress level it is subjected to. However, the assumption of independence is conservative as higher loads no longer generally lead to higher shear strengths (which they would if positively dependent) and so this assumption leads to higher failure probabilities.

If it can be conservatively assumed that R and F are independent random variables, according to the above argument, then eq. [3] can be written

$$[4] \qquad p_f = P\left(\frac{R}{F} < 1\right) = P(\ln R - \ln F < 0) = \Phi\left(-\frac{\mu_{\ln R} - \mu_{\ln F}}{\sqrt{\sigma^2_{\ln R} + \sigma^2_{\ln F}}}\right)$$

where Φ is the standard normal cumulative distribution function, which involves the distance between the means of the "resistance" and the "load," $\mu_{\ln R} - \mu_{\ln F}$, measured in-number of standard deviations, $\sqrt{\sigma^2_{\ln R} + \sigma^2_{\ln F}}$. It is this distance between the "resistance" and "load" that is refered to as the reliability index, as it directly reflects reliability

[5] $$\beta = \frac{\mu_{\ln R} - \mu_{\ln F}}{\sqrt{\sigma^2_{\ln R} + \sigma^2_{\ln F}}}$$

In terms of the reliability index, the failure probability is simply written as

[6] $$p_{\mathrm{f}} = \Phi(-\beta)$$

This paper will refer mostly to the failure probability, p_{f}, relative to an acceptable maximum lifetime failure probability, p_{m}. However, the reliability indices corresponding to various values of p_{m} will be stated to put the analysis into context with other reliability-based design efforts.

Three failure consequence levels will be considered — high, medium, and low — corresponding to important structures where failure has high consequences (e.g., hospitals, schools, and lifeline highway bridges), typical structures that constitute the majority of civil engineering projects, and low-failure consequence structures (e.g., low-use storage facilities, low-use bridges, etc.). Most designs will be aimed at the medium failure consequence level, which in this paper will be assumed to have a maximum acceptable lifetime failure probability, p_{m}, of about 1/5000. This probability corresponds to a reliability index of about β = 3.5, which is in basic agreement with Meyerhof's (1995) estimate of the typical reliability of foundations on land. The maximum acceptable failure probability for the low consequence case is assumed to be p_{m} = 1/1000 (β = 3.1) while p_{m} = 1/10 000 (β = 3.7) is assumed for the high consequence case.

Most structural components in buildings are designed for reliability indices between about 3 and 4. However, most structural codes also strongly recommend that some level of redundancy exist, so that components that start to fail are able to shed load to adjacent components. The resulting system reliability is typically quite a bit higher than the component reliability. In other words, while a single component of a structure may have a failure probability of 1/5000 (β = 3.5), the probability of catastrophic failure of an entire properly designed redundant structure should be much lower.

In general, the target probability of failure of a foundation system should be matched to that of the structure it supports. In geotechnical engineering, this means that the issue of foundation redundancy also needs to be considered carefully. A structure resting on 100 footings has plenty of redundancy if the failure of one or a few footings over weak zones merely sheds load to those footings over stronger zones. In this case, the target "component" failure probability may be relaxed somewhat to, say, 1/1000. On the other hand, structures supported by a single pier will have no redundancy, and the target probability of failure of the single pier should be set equal to that of the supported structure. The effect of redundancy in geotechnical systems on reliability is still in need of much further research. At this time, engineering judgement is still a critical requirement in the assessment of required target failure probabilities of geotechnical components. Only component failure probabilities will be considered in this paper and effects of redundancy will be ignored. This is conservative in the sense that redundancy improves the system reliability.

The proposed "Foundations" clauses of the next edition of the *Canadian highway bridge design code* will include the three consequence levels considered in this paper. The proposal also includes three levels of site understanding (high, medium, and low), which reflects the degree to which the ground supporting the foundation being designed is understood and modeled. Both the resistance factor, φ_{gu}, and the consequence factor, Ψ_{u}, appearing in eq. [2] are aimed at achieving a probability of failure, p_{f}, which is less than the acceptable maximum failure probability, p_{m}, for the component. Conceptually, this could be achieved using just a single factor on the left-hand side of eq. [2], which would depend on both the level of understanding of the ground supporting a footing and on the failure consequence level. Although this would simplify the appearance of eq. [2], it would involve a table lookup to determine the factor itself — one axis of the table would be the level of site understanding and the other would be failure consequence level. In addition, a separate table would have to be provided for each limit state (e.g., bearing, sliding, overturning, settlement, etc.). The authors feel that it is much simpler to decompose the single factor into two separate factors — one for the level of site understanding (φ_{gu}) and the other for the level of failure consequence (Ψ_{u}) — whose product is the desired single factor.

Regarding site understanding, it is assumed in this paper that the ground conditions under the footing to be designed are estimated by a single soil sample (i.e., a standard penetration test (SPT) or cone penetration test (CPT) sounding), which would presumably be the one taken closest to the footing location. Three sampling locations are considered. The first is when the soil directly under the footing is sampled (r = 0 m), which would correspond to a high level of site understanding. The second is when the soil is sampled 4.5 m away from the footing centerline (r = 4.5 m), which may loosely correspond to a medium, or typical, level of site understanding (although this also depends on the correlation length, as will be discussed later). The third is when the soil is sampled at a distance of r = 9.0 m away from the footing centerline, which loosely corresponds to a low level of site understanding. The distances are somewhat arbitrary, and were chosen simply because they are the distances used in Fenton et al. (2008), which were based on the size of the simulated soil field used in that study.

Ideally, the two factors, Ψ_{u} and φ_{gu}, would be independent of one another, the consequence factor being dependent only on the desired failure probability and the resistance factor being dependent only on the level of site understanding. However, as will be shown below, the consequence factor does have some secondary dependence on site variability (higher variability corresponds to lower understanding). The dependence is relatively slight, therefore the assumption of independence is a reasonable approximation (so long as conservative values are selected) and code recommendations regarding the consequence factor can still be made considering only target failure probability.

The resistance factor, φ_{gu}, is aimed at the medium consequence level. That is, the value of φ_{gu} is set to achieve a failure probability of the designed footing (using $\Psi_u = 1.0$) of no more than $p_m = 1/5000$. The consequence factor, Ψ_u, is then used to adjust the target failure probability either up or down from the medium consequence level, as required by the importance of the supported structure. In this way, only three resistance factor values need be defined for each limit state (assuming three levels of site understanding) and only three consequence factors are required for the three consequence levels.

In this study, only one limit state will be considered, namely the bearing capacity ultimate limit state of a shallow foundation. The authors suspect that the consequence factor is largely independent of the limit state under consideration and is mostly only dependent on the desired change from the medium consequence level to either the high or low one. If this is true, and further research is required to formally show it, then only two sets of consequence factors (three values each) are likely to be required: one set for ultimate limit states and another for serviceability limit states (as ultimate and serviceability limit states have quite different maximum acceptable failure probabilities).

The load factors also have a direct impact on the probability of failure of the designed footing. In fact, for a fixed target probability of failure, the ratio of the factors on the resistance side to the total load factor (defined in such a way that the total load factor times the total load equals the sum of the factored loads) is a constant — as the total load factor increases, the resistance factor also increases to maintain the same failure probability. In other words, resistance factors are code specific — the resistance factors developed for use with load factors from the NBCC (NRC 2005) cannot be used with load factors from AASHTO (2007). The resistance factors must be used with the same load factors that were employed to develop them.

In particular, the load factors on the right-hand side of eq. [2] have been taken from the NBCC (NRC 2005). Using these load factors, the resistance factor, φ_{gu}, is derived using the theoretical framework presented by Fenton et al. (2008), as summarized in the next three sections, to achieve the medium target failure probability ($p_m \simeq 1/5000$) for which the consequence factor, Ψ_u, is set to 1.0. While the resistance factor is dependent on the choice of load factors, it is believed that the consequence factor is largely independent of the load factors, so that the results of this paper should be applicable to any code having similar maximum acceptable failure probabilities. This assertion, however, needs verification by further research.

The remainder of the paper concentrates on the consequence factor, Ψ_u, and how it varies, primarily with respect to the target failure probability. Other issues, such as the residual dependence of Ψ_u on site uncertainty, are also investigated to determine how best to specify Ψ_u in a design code. The theoretical results to be presented next are for a strip footing founded on a weightless soil and so, strictly speaking, are applicable only to the case considered. However, the weightless soil case is conservative with respect to foundation strength as it ignores the increased shear strength due to soil weight.

The restriction of attention to a strip footing simplifies the design process and allows the soil to be modeled as a two-dimensional (2-D) random field. The authors recognize that a fully three-dimensional (3-D) representation would be generally superior. However, a paper by Griffiths and Fenton (1997) found that there was little difference from a reliability point of view between the 2-D and 3-D models. In the problem studied here, the 2-D model does adequately handle the issue of correlation between soil samples and the ground under the footing, and so it is felt that a fully 3-D model would not significantly affect the results. The theoretical model presented next is relatively easily extended to the 3-D case, although this has yet to be done.

Random soil model

The soil cohesion, c, is assumed to be lognormally distributed with mean, μ_c, standard deviation, σ_c, and spatial correlation length θ. The correlation coefficient between the log-cohesion at a point $\boldsymbol{x}_1$ and a second point $\boldsymbol{x}_2$ is specified by a correlation function, $\rho(\boldsymbol{\tau})$, where $\boldsymbol{\tau} = \boldsymbol{x}_1 - \boldsymbol{x}_2$ is the vector between the two points. In this paper, a simple exponentially decaying (Markovian) correlation function will be assumed, having the form

$$[7] \qquad \rho(\boldsymbol{\tau}) = \exp\left(-\frac{2|\boldsymbol{\tau}|}{\theta}\right)$$

where $|\boldsymbol{\tau}| = \sqrt{\tau_1^2 + \tau_2^2}$ is the length of the vector $\boldsymbol{\tau}$ in two dimensions. The spatial correlation length, θ, is loosely defined as the separation distance within which two values of $\ln c$ are significantly correlated. Mathematically, θ is defined as the area under the correlation function, $\rho(\boldsymbol{\tau})$ (Vanmarcke 1984).

The correlation function, $\rho(\boldsymbol{\tau})$, has a corresponding variance reduction function, $\gamma(D)$, which specifies how the variance is reduced upon local averaging of $\ln c$ over some domain D. In the 2-D analysis considered here, $D = D_1 \times D_2$ is an area and the 2-D variance reduction function is defined by

$$[8] \qquad \gamma(D_1, D_2) = \frac{4}{(D_1 D_2)^2} \int_0^{D_1} \int_0^{D_2} (D_1 - \tau_1)(D_2 - \tau_2) \times \rho(\tau_1, \tau_2)\,\mathrm{d}\tau_1\,\mathrm{d}\tau_2$$

which can be evaluated using Gaussian quadrature (see Fenton and Griffiths (2008) for more details).

The friction angle, ϕ, is assumed to be bounded both above and below, so that neither normal nor lognormal distributions are appropriate. A beta distribution is often used for bounded random variables. Unfortunately, a beta-distributed random field has a complex joint distribution and simulation is cumbersome and numerically difficult. To keep things simple, a bounded distribution is selected that resembles a beta distribution, but which arises as a simple transformation of a standard normal random field, $G_\phi(\boldsymbol{x})$, according to

$$[9] \qquad \phi(\boldsymbol{x}) = \phi_{\min} + (1/2)(\phi_{\max} - \phi_{\min}) \times \left\{1 + \tanh\left[\frac{sG_\phi(\boldsymbol{x})}{2\pi}\right]\right\}$$

where $\phi_{\min}$ and $\phi_{\max}$ are the minimum and maximum friction angles in radians, respectively, and s is a dimensionless scale factor that governs the friction angle variability between its two bounds (see the tanh distribution in Fenton and Griffiths (2008), for more details). As the distribution of ϕ is assumed to be symmetric, its mean is the midpoint

$$[10] \qquad \mu_\phi = \frac{1}{2}(\phi_{\min} + \phi_{\max})$$

The standard deviation of ϕ is a function of the scale factor, s, which is closely approximated for $0 \leq s \leq 5$ by (Fenton and Griffiths 2008)

$$[11] \qquad \sigma_\phi \simeq \frac{0.46(\phi_{\max} - \phi_{\min})s}{\sqrt{4\pi^2 + s^2}}$$

The random friction angle field is assumed to have the same correlation function as the cohesion. If the spatial correlation structure of a soil is caused by changes in the constitutive nature of the soil over space, then both the cohesion and friction angle would have similar correlation lengths, making this assumption reasonable.

The two random fields, c and ϕ, are assumed to be independent. Nonzero correlations between c and ϕ were found by Fenton and Griffiths (2003) to have only a minor influence on the estimated probabilities of bearing capacity failure. As the general consensus is that c and ϕ are negatively correlated (Wolff 1985; Cherubini 2000) and the mean bearing capacity for independent c and ϕ was slightly lower than for the negatively correlated case (Fenton and Griffiths 2003), the assumption of independence between c and ϕ is slightly conservative.

Failure probability

To determine the required resistance and consequence factors, the probability of a shallow foundation reaching its bearing capacity ultimate limit state must be estimated. This probability will depend on the load distribution, load factors selected, and resistance distribution. The details of the following mathematical analysis can be found in Fenton et al. (2008) and will only be summarized here. Only dead and live loads have been considered, with load factors $\alpha_L = 1.5$ and $\alpha_D = 1.25$ (NRC 2005), and the analysis has been carried out using a simple example of a strip footing founded on a weightless $c - \phi$ soil. For this case, the characteristic ultimate bearing capacity, $\hat{q}_u$, is given by Terzaghi's (1943) relationship, which for a weightless soil simplifies to

$$[12] \qquad \hat{q}_u = \hat{c}\hat{N}_c$$

where $\hat{c}$ is the soil's characteristic cohesion and $\hat{N}_c$ is the characteristic bearing capacity factor, the latter being a function of the soil's characteristic friction angle, $\hat{\phi}$

$$[13] \qquad \hat{N}_c = \frac{e^{\pi \tan\hat{\phi}} \tan^2[(\pi/4) + (\hat{\phi}/2)] - 1}{\tan\hat{\phi}}$$

The characteristic soil parameters, $\hat{c}$ and $\hat{N}_c$, are obtained by sampling the soil at some distance, r, from the footing location, estimating the soil's cohesion and friction angle from the sample, and then using some sort of average as the characteristic design value. In particular, as it is assumed that cohesion is lognormally distributed, the characteristic cohesion value used is the geometric average of the observations (as this is also lognormally distributed). For example, suppose that m soil samples are taken at a distance r = 4.5 m from the footing centerline. Then the characteristic cohesion is computed from the observations, c_i, as

$$[14] \qquad \hat{c} = \left(\prod_{i=1}^{m} c_i\right)^{1/m} = \exp\left(\frac{1}{m}\sum_{i=1}^{m} \ln c_i\right)$$

which is somewhat low-value dominated (i.e., a somewhat conservative estimate of the mean — it is also an estimate of the *median* cohesion value). The distance that the sample is taken from the footing location affects how strongly the characteristic value is expected to match the actual cohesion under the footing. The farther away from the footing that the sample is taken, the less likely it is to accurately predict conditions under the footing.

The characteristic value of the friction angle is computed as an arithmetic average of the sample observations

$$[15] \qquad \hat{\phi} = \frac{1}{m}\sum_{i=1}^{m} \phi_i$$

The arithmetic average is used here because ϕ is assumed to follow a symmetric bounded distribution and the arithmetic average preserves the mean. That is, the mean of $\hat{\phi}$ is the same as the mean of ϕ. It is also assumed that the friction angle observations, ϕ_i, are taken at the same location as the cohesion observations.

The characteristic dead and live loads are defined in terms of the means of the dead and live load distributions according to

$$[16a] \qquad \hat{F}_D = k_D \mu_D$$

$$[16b] \qquad \hat{F}_L = k_L \mu_L$$

in which the mean loads have been scaled up to upper quantiles by applying bias factors, k_D and k_L. This is done to yield characteristic or "design" loads having a measure of safety associated with them, i.e., a relatively lower probability that these design loads will be exceeded in the design lifetime (Allen (1975) suggests an exceedance probability of about 8% to 9%). The bias factors, k_D and k_L, are estimated by Becker (1996) and Allen (1975) to be 1.18 and 1.41, respectively.

For a strip footing of width B, the characteristic ultimate geotechnical resistance is

$$[17] \qquad \hat{R}_u = B\hat{c}\hat{N}_c$$

Using these results in eq. [2] leads to a design relationship for the required footing width

$$[18] \qquad B = \frac{\alpha_L \hat{F}_L + \alpha_D \hat{F}_D}{\Psi_u \varphi_{gu} \hat{c}\hat{N}_c}$$

Once the footing width has been determined, the footing is constructed and loaded. The probability of failure involves determining the probability that the actual lifetime extreme load acting on the footing, F, exceeds the actual soil resistance, $R = B\bar{c}\bar{N}_c$, where the overbars indicate that these parameters are the equivalent soil parameters as "seen" by the footing. More specifically, if the spatially variable soil underlying the footing is replaced by a uniform soil (properties the same everywhere) having cohesion $\bar{c}$ and friction angle $\bar{\phi}$ and the bearing capacity of the uniform soil is exactly the same as the actual spatially varying soil, then $\bar{c}$ and $\bar{\phi}$ are the equivalent soil parameters. The equivalent bearing capacity coefficient is defined in terms of the equivalent friction angle according to the usual formula

$$[19] \qquad \bar{N}_c = \frac{e^{\pi \tan\bar{\phi}} \tan^2[(\pi/4) + (\bar{\phi}/2)] - 1}{\tan\bar{\phi}}$$

The probability of bearing capacity failure can now be computed as

$$[20] \qquad p_{\mathrm{f}} = P(F > B\bar{c}\bar{N}_c) = P\left(F\frac{\hat{c}\hat{N}_c}{\bar{c}\bar{N}_c} > \frac{\alpha_{\mathrm{L}}\hat{F}_{\mathrm{L}} + \alpha_{\mathrm{D}}\hat{F}_{\mathrm{D}}}{\Psi_{\mathrm{u}}\varphi_{\mathrm{gu}}}\right)$$

All five quantities on the left-hand side of the inequality, i.e., F, $\hat{c}$, $\bar{c}$, $\hat{N}_c$, and $\bar{N}_c$, are random. If these random quantities are combined into a single random variable

$$[21] \qquad Y = F\frac{\hat{c}\hat{N}_c}{\bar{c}\bar{N}_c}$$

then the desired probability can be re-expressed as

$$[22] \qquad p_{\mathrm{f}} = P\left(Y > \frac{\alpha_{\mathrm{L}}\hat{F}_{\mathrm{L}} + \alpha_{\mathrm{D}}\hat{F}_{\mathrm{D}}}{\Psi_{\mathrm{u}}\varphi_{\mathrm{gu}}}\right)$$

and the task is to find the distribution of Y. Assuming that Y is lognormally distributed (an assumption found to be reasonable by Fenton et al. (2007), and which is also supported to some extent by the central limit theorem), then

$$[23] \qquad \ln Y = \ln F + \ln\hat{c} + \ln\hat{N}_c - \ln\bar{c} - \ln\bar{N}_c$$

is normally distributed and p_{f} can be found once the mean and variance of $\ln Y$ are determined. The mean of $\ln Y$ is

$$[24] \qquad \mu_{\ln Y} = \mu_{\ln F} + \mu_{\ln\hat{c}} + \mu_{\ln\hat{N}_c} - \mu_{\ln\bar{c}} - \mu_{\ln\bar{N}_c}$$

and the variance of $\ln Y$ is

$$[25] \qquad \sigma^2_{\ln Y} = \sigma^2_{\ln F} + \sigma^2_{\ln\hat{c}} + \sigma^2_{\ln\bar{c}} + \sigma^2_{\ln\hat{N}_c} + \sigma^2_{\ln\bar{N}_c} - 2\mathrm{Cov}(\ln\bar{c}, \ln\hat{c}) - 2\mathrm{Cov}(\ln\bar{N}_c, \ln\hat{N}_c)$$

where the load, F, and soil properties, c and ϕ, have been assumed mutually independent.

To find the parameters in eqs. [24] and [25], the following two assumptions are made:

(1) The equivalent cohesion, $\bar{c}$, is the geometric average of the cohesion field over some zone of influence, D, under the footing

$$[26] \qquad \bar{c} = \exp\left(\frac{1}{D}\int_D \ln c(\boldsymbol{x})\,\mathrm{d}\boldsymbol{x}\right)$$

Note that in this two-dimensional analysis, D is an area and the above (eq. [26]) is a two-dimensional integration. If $c(\boldsymbol{x})$ is lognormally distributed, as assumed, then $\bar{c}$ is also lognormally distributed.

(2) The equivalent friction angle, $\bar{\phi}$, is the arithmetic average of the friction angle over the same zone of influence, D

$$[27] \qquad \bar{\phi} = \frac{1}{D}\int_D \phi(\boldsymbol{x})\,\mathrm{d}\boldsymbol{x}$$

This relationship also preserves the mean, i.e., $\mu_{\bar{\phi}} = \mu_{\phi}$.

The averaging domain was found by trial and error to be best approximated by $D = W \times W$, centered directly under the footing, where W is taken as 80% of the average mean depth of the wedge zone directly beneath the footing, as given by the classical Prandtl failure mechanism

$$[28] \qquad W = \frac{0.8}{2}\hat{\mu}_B \tan\left(\frac{\pi}{4} + \frac{\mu_\phi}{2}\right)$$

In eq. [28], μ_ϕ is the mean friction angle (in radians), within the zone of influence of the footing, and $\hat{\mu}_B$ is an estimate of the mean footing width obtained by using mean soil properties (μ_c and μ_ϕ) in eq. [18]

$$[29] \qquad \hat{\mu}_B = \frac{\alpha_{\mathrm{L}}\hat{F}_{\mathrm{L}} + \alpha_{\mathrm{D}}\hat{F}_{\mathrm{D}}}{\Psi_{\mathrm{u}}\varphi_{\mathrm{gu}}\mu_c\mu_{N_c}}$$

where, to first order, the mean of N_c is

$$[30] \qquad \mu_{N_c} \simeq \frac{e^{\pi \tan\mu_\phi}\tan^2[(\pi/4) + (\mu_\phi/2)] - 1}{\tan\mu_\phi}$$

Using the above information and assumptions, the components of eqs. [24] and [25] can be computed as follows (given the basic statistical parameters of the loads, c, ϕ, the number and locations of the soil samples, and the averaging domain size D):

(1) Assuming that the total load F is equal to the sum of the maximum live load, F_{L}, acting over the lifetime of the structure and the static dead load, F_{D}, i.e., $F = F_{\mathrm{L}} + F_{\mathrm{D}}$, both of which are random, then

$$[31a] \qquad \mu_{\ln F} = \ln(\mu_F) - \frac{1}{2}\ln(1 + v_F^2)$$

$$[31b] \qquad \sigma^2_{\ln F} = \ln(1 + v_F^2)$$

where $\mu_F = \mu_{\mathrm{L}} + \mu_{\mathrm{D}}$ is the sum of the mean (maximum lifetime) live and (static) dead loads, and v_F is the coefficient of variation of the total load defined by

$$[32] \qquad v_F = \frac{\sqrt{\sigma^2_{\mathrm{L}} + \sigma^2_{\mathrm{D}}}}{\mu_{\mathrm{L}} + \mu_{\mathrm{D}}}$$

(2) With reference to eq. [14],

$$[33] \qquad \mu_{\ln\hat{c}} = E\left(\frac{1}{m}\sum_{i=1}^{m}\ln c_i\right) = \mu_{\ln c}$$

where E is the expectation operator, and

[34] $$\sigma^2_{\ln\hat{c}} \simeq \frac{\sigma^2_{\ln c}}{m^2}\sum_{i=1}^{m}\sum_{j=1}^{m}\rho(\boldsymbol{x}_i - \boldsymbol{x}_j)$$

where $\boldsymbol{x}_i$ is the spatial location of the center of the ith soil sample (i = 1, 2, …, m) and ρ is the correlation function defined by eq. [7]. Assuming that ln$\hat{c}$ actually represents a local average of ln$c$ over a domain of size $\Delta x \times H$, where Δx is the horizontal dimension of the soil sample, which, for example, can be thought of as the horizontal zone of influence of a CPT or SPT sounding, and H is the depth over which the samples are taken, then $\sigma^2_{\ln\hat{c}}$ is probably more accurately computed as

[35] $$\sigma^2_{\ln\hat{c}} = \sigma^2_{\ln c}\gamma(\Delta x, H)$$

(3) With reference to eq. [26]

[36] $$\mu_{\ln\bar{c}} = E\left[\frac{1}{D}\int_D \ln c(\boldsymbol{x})\,\mathrm{d}\boldsymbol{x}\right] = \mu_{\ln c}$$

[37] $$\sigma^2_{\ln\bar{c}} = \sigma^2_{\ln c}\gamma(W, W)$$

where $\gamma(W, W)$ is defined by eq. [8].

(4) As $\mu_{\hat{\phi}} = \mu_{\phi}$ (which can be seen by taking expectations of eq. [15]), the mean and variance of $\hat{N}_c$ can be obtained using first-order approximations to expectations of eq. [13] (Fenton and Griffiths 2003), as follows:

[38] $$\mu_{\ln\hat{N}_c} = \mu_{\ln N_c} \simeq \ln\frac{e^{\pi\tan\mu_\phi}\tan^2[(\pi/4) + (\mu_\phi/2)] - 1}{\tan\mu_\phi}$$

[39] $$\sigma^2_{\ln\hat{N}_c} \simeq \sigma^2_{\hat{\phi}}\left(\frac{\mathrm{d}\ln\hat{N}_c}{\mathrm{d}\hat{\phi}}\bigg|_{\mu_\phi}\right)^2 = \sigma^2_{\hat{\phi}}\left\{\frac{bd}{bd^2 - 1}f - \frac{1 + a^2}{a}\right\}^2$$

where $a = \tan(\mu_\phi)$, $b = e^{\pi a}$, $d = \tan[(\pi/4) + (\mu_\phi/2)]$, and $f = \pi(1 + a^2)d + 1 + d^2$. The variance of $\hat{\phi}$ is given by

[40] $$\sigma^2_{\hat{\phi}} \simeq \frac{\sigma^2_{\phi}}{m^2}\sum_{i=1}^{m}\sum_{j=1}^{m}\rho(\boldsymbol{x}_i\boldsymbol{x}_j) = \sigma^2_\phi\gamma(\Delta x, H)$$

where $\boldsymbol{x}_i$ is the spatial location of the center of the ith soil observation (i = 1, 2, …, m). See eq. [11] for the definition of σ_ϕ.

(5) As $\mu_{\bar{\phi}} = \mu_\phi$ (by taking expectations of eq. [27]), the mean and variance of $\bar{N}_c$ can be obtained in the same fashion as for $\hat{N}_c$ (in fact, they only differ due to differing local averaging in the variance calculation). With reference to eqs. [19] and [38]

[41] $$\mu_{\ln\bar{N}_c} = \mu_{\ln\hat{N}_c} = \mu_{\ln N_c}$$

[42] $$\sigma^2_{\ln\bar{N}_c} \simeq \sigma^2_{\bar{\phi}}\left(\frac{\mathrm{d}\ln\bar{N}_c}{\mathrm{d}\bar{\phi}}\bigg|_{\mu_\phi}\right)^2 = \sigma^2_{\bar{\phi}}\left\{\frac{bd}{bd^2 - 1}f - \frac{1 + a^2}{a}\right\}^2$$

[43] $$\sigma^2_{\bar{\phi}} = \sigma^2_\phi\gamma(W, W)$$

See previous item for definitions of a, b, d, and f. The variance reduction function, $\gamma(W, W)$, is defined for two dimensions by eq. [8] and eq. [11] defines σ_ϕ.

(6) The covariance between the observed cohesion values and the equivalent cohesion beneath the footing is obtained as follows for $D = W \times W$ and $Q = \Delta x \times H$:

[44] $$\mathrm{Cov}[\ln\bar{c}, \ln\hat{c}] \simeq \frac{\sigma^2_{\ln c}}{D^2Q^2}\int_D\int_Q \rho(\boldsymbol{x}_1 - \boldsymbol{x}_2)\,\mathrm{d}\boldsymbol{x}_1\,\mathrm{d}\boldsymbol{x}_2 = \sigma^2_{\ln c}\gamma_{DQ}$$

where γ_{DQ} is the average correlation coefficient between the two areas D and Q. The area D denotes the averaging region below the footing over which equivalent properties are defined and the area Q denotes the region over which soil samples are gathered. These areas are illustrated in Fig. 1. In detail, γ_{DQ} is defined by

[45] $$\gamma_{DQ} = \frac{1}{(W^2\Delta xH)^2}\int_{-W/2}^{W/2}\int_{H-W}^{H}\int_{r-\Delta x/2}^{r+\Delta x/2}\int_0^H \rho(\xi_1 - x_1, \xi_2 - x_2)\,\mathrm{d}\xi_2\,\mathrm{d}\xi_1\,\mathrm{d}x_2\,\mathrm{d}x_1$$

where r is the horizontal distance between the footing centerline and the centerline of the soil sample column. Equation [45] can be evaluated by Gaussian quadrature.

(7) The covariance between $\bar{N}_c$ and $\hat{N}_c$ is similarly approximated by

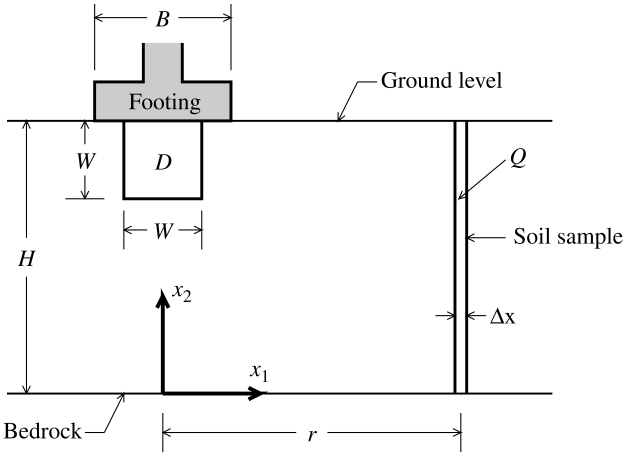
[46] $$\mathrm{Cov}[\ln\bar{N}_c, \ln\hat{N}_c] \simeq \sigma^2_{\ln N_c}\gamma_{DQ}$$

[47] $$\sigma^2_{\ln N_c} \simeq \sigma^2_{\phi}\left(\frac{\mathrm{d}\ln N_c}{\mathrm{d}\phi}\bigg|_{\mu_\phi}\right)^2 = \sigma^2_{\phi}\left\{\frac{bd}{bd^2 - 1}f - \frac{1 + a^2}{a}\right\}^2$$

Substituting these results into eqs. [24] and [25] gives

[48] $$\mu_{\ln Y} = \mu_{\ln F}$$

Fig. 1. Averaging regions used to predict probability of bearing capacity failure.



$$\sigma^2_{\ln Y} = \sigma^2_{\ln F} + (\sigma^2_{\ln c} + \sigma^2_{\ln N_c}) \times [\gamma(\Delta x, H) + \gamma(W, W) - 2\gamma_{DQ}] \quad [49]$$

which can now be used in eq. [22] to produce estimates of p_{f}. Letting

$$q = \alpha_{\mathrm{L}}\hat{F}_{\mathrm{L}} + \alpha_{\mathrm{D}}\hat{F}_{\mathrm{D}} \quad [50]$$

allows the probability of failure to be expressed as

$$\begin{aligned} p_{\mathrm{f}} &= P[Y > q/(\Psi_{\mathrm{u}}\varphi_{\mathrm{gu}})] \\ &= P\{\ln Y > \ln[q/(\Psi_{\mathrm{u}}\varphi_{\mathrm{gu}})]\} \\ &= 1 - \Phi\left\{\frac{\ln[q/(\Psi_{\mathrm{u}}\varphi_{\mathrm{gu}})] - \mu_{\ln Y}}{\sigma_{\ln Y}}\right\} \end{aligned} \quad [51]$$

Once the probability of failure is computed via eq. [51], it can be compared to the maximum acceptable failure probability, p_{m}. If p_{f} exceeds p_{m}, then the resistance factor and (or) the consequence factor need to be reduced (specifically, the product $\Psi_{\mathrm{u}}\varphi_{\mathrm{gu}}$ needs to be reduced). The determination of required consequence factors proceeds in two steps:

(1) Consider first the medium (typical) consequence level and set $\Psi_{\mathrm{u}} = 1$. For a variety of different levels of variability in soil properties, degrees of spatial correlation between soil properties, and distances between footing location and sample location, estimate the probability of footing failure using eq. [51]. For each case, adjust the resistance factor, φ_{gu}, until $p_{\mathrm{f}} = p_{\mathrm{m}}$. This, then, is the required resistance factor. Note that the value of φ_{gu} is needed to compute $\hat{\mu}_B$ in eq. [29], which is then used to compute the averaging dimension W in eq. [28]. As the resistance factor is not known ahead of time, the solution must proceed iteratively: use a starting value of φ_{gu} = 0.7, compute a new value and repeat using the latest resistance factor. The iteration has been found to converge very quickly, usually in about 3 to 5 cycles, under a relative error tolerance of 1/1000.

(2) Using the required resistance factor(s) determined in step 1 in eq. [51], repeat the procedure of step 1 except now at the high (reduced p_{m}) and low (increased p_{m}) consequence levels and adjust the consequence factor, Ψ_{u}, until $p_{\mathrm{f}} = p_{\mathrm{m}}$. This, then, is the required consequence factor.

Resistance factors

Resistance factors were determined at the medium consequence level for the specific problem of a strip footing on a weightless soil with parameters as follows:

(1) The lifetime extreme live load along the strip footing is assumed to be lognormally distributed (conservative) with mean μ_{L} = 200 kN/m and a coefficient of variation $\nu_{\mathrm{L}} = \sigma_{\mathrm{L}}/\mu_{\mathrm{L}} = 0.3$. The dead load is assumed to be lognormally distributed with mean μ_{D} = 600 kN/m and a coefficient of variation $\nu_{\mathrm{D}} = 0.15$. The mean values assumed here are not particularly important, as the design equation (see eq. [18]) takes the distance between the load and resistance distributions into account through the load and resistance factors.

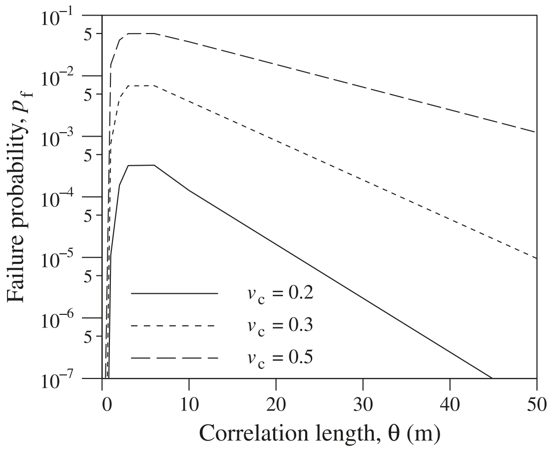
(2) The cohesion is assumed to have mean μ_c = 100 kN/m^2 and coefficient of variation varied as follows: ν_c = 0.1, 0.2, 0.3, and 0.5. As mentioned above, the mean value is expected to have minimum influence on the results, but the coefficient of variation definitely affects the resistance factor and has a slight influence on the consequence factor, as will be shown later.

(3) The friction angle is assumed to follow a bounded tanh distribution (Fenton and Griffiths 2008) over the range $\phi_{\min} = 10^\circ$ and $\phi_{\max} = 30^\circ$ with mean $\mu_\phi = 20^\circ$. The coefficient of variation of the friction angle is varied as ν_ϕ = 0.07, 0.14, 0.20, and 0.29, in step with the coefficient of variation of the cohesion. That is, it is assumed that as the cohesion variability increases, so does the friction angle variability.

(4) The correlation length, θ, is varied from a low of 0.1 m to a high of 50 m. Low values of θ lead to soil properties varying rapidly spatially, while high values mean that the soil properties vary only slowly with position. A large correlation length, of say θ = 50 m, means that soil samples taken well within 50 m from the footing location (e.g., at r = 10 m) will be quite representative of the soil properties under the footing. Lower failure probabilities are expected when the soil is sampled well within the distance θ from the footing. Interestingly, because the characteristic value derived from the soil sample is generally an average, when θ is very small (say, 0.01 m), then the sample will again accurately reflect the average conditions under the footing, regardless of the sampling location. The worst case correlation length occurs when θ is approximately equal to the distance from the footing to the sampling location.

Figure 2 illustrates how the probability of bearing capacity failure, computed using eq. [51], varies with correlation length and the figure clearly shows that a worst case correlation length exists. The figure presents failure probabilities when the soil is sampled at r = 4.5 m from the footing centerline and for the low consequence case (p_{m} = 1/1000) where a consequence factor Ψ_{u} = 1.15 was used (this will be derived later). The worst case (highest) failure probability occurs for values of correlation length near 4.5 m and it can be seen that the highest probability of failure, p_{f}, is slightly less than p_{m} = 1/1000 when the coefficient of variation of the soil properties is at a moderate level (ν_c = 0.2, ν_ϕ = 0.14). However, if the soil property variability reaches ν_c = 0.3 and ν_ϕ = 0.2, then the probability of fail-

Fig. 2. Failure probability versus correlation length for Ψ_u = 1.15 (low consequence), φ_{gu} = 0.50, and r = 4.5 m.



ure becomes unacceptable. See, for example, the ν_c = 0.5 curve (ν_ϕ = 0.29), which reaches a failure probability of 5% at the worst case correlation length. The unacceptable failure probabilities that occur for higher soil variabilities emphasizes the need to perform enough site investigations to reduce the residual variability to no more than moderate levels.

At the medium consequence level, p_m = 1/5000 and Ψ_u = 1.0, the steps outlined in the previous section can be followed to estimate target values of the resistance factor, φ_{gu}, for various sampling distances, r, correlation lengths, θ, and soil variabilities, ν_c and ν_ϕ. Figure 3 illustrates the results. As can be seen, the resistance factor varies significantly with all three parameters, r, θ, and ν_c. For example when r = 4.5 m in Fig. 3*b* and ν_c = 0.3 (ν_ϕ = 0.2), the resistance factor ranges from a high of φ_{gu} = 0.93 when θ = 0.1 m to a low of φ_{gu} = 0.38 at the worst-case correlation length, which is somewhere in the range, 3.0 ≤ θ ≤ 6 m.

Despite the significant variation in φ_{gu}, it is desired to select three design values for the resistance factor, one for each level of site understanding, which in this paper is associated with sampling distance, r. The decision was made to select a relatively conservative resistance factor corresponding to ν_c somewhere between 0.2 and 0.3 in the region of the worst-case correlation length. Thus, for the high understanding case, r = 0 m in Fig. 3*a*, a design resistance factor of φ_{gu} = 0.65 was selected. The design value is shown in the figure using a solid horizontal line. Similarly, for the medium understanding case, r = 4.5 in Fig. 3*b*, φ_{gu} = 0.50 was selected (which coincides with that recommended by the *Canadian foundation engineering manual* (Canadian Geotechnical Society 2006)), while for the low understanding case, r = 9.0 m in Fig. 3*c*, φ_{gu} = 0.4 was selected.

If the footing is designed using the design resistance factors, shown by solid lines in Fig. 3, then points on the curves lying below the solid lines are unconservative, having a failure probability in excess of the maximum acceptable failure probability of p_m = 1/5000. Conversely, all cases where ν_c ≤ 0.25 (approximately) are seen to be conservative, so it is felt that the selected design resistance factors are reasonable as long as site variability does not exceed a coefficient of variation of about 25% after site investigation.

Consequence factors

Having established the required resistance factors, attention can now focus on the consequence factor. Figure 4 illustrates how the probability of bearing capacity failure changes with the consequence factor for the medium site understanding case (r = 4.5 m), with a correlation length θ = 6 m, using design resistance factor φ_{gu} = 0.5. It can be seen that fairly small changes in the consequence factor, Ψ_u, can make large differences in the failure probability, p_f. As expected, the soil variability (ν_c) also has a very significant effect on p_f. The two horizontal lines in Fig. 4 bound the low to high failure consequence acceptable probabilities, p_m = 1/1000 to p_m = 1/10 000.

To illustrate how Fig. 4 works, one additional curve was produced for ν_c = 0.23. When Ψ_u = 1.0 (medium consequence), the ν_c = 0.23 case has failure probability $p_f \simeq 2 \times 10^{-4}$ = 1/5000, which is the maximum acceptable failure probability for medium consequences. To adjust this case to have failure probability $p_f = 1 \times 10^{-4}$ = 1/10 000 (high consequence), a consequence factor of about Ψ_u = 0.93 should be used — the required Ψ_u value occurs where the ν_c = 0.23 curve intersects the horizontal p_m = 1/10 000 line. The recommended consequence factor for this case will be rounded down to 0.90, as discussed shortly. Similarly, to adjust the ν_c = 0.23 case for a low consequence design (p_m = 1/1000), the consequence factor is obtained at the intersection of the ν_c = 0.23 curve and the upper horizontal line. This occurs at about Ψ_u = 1.13 (which was rounded to Ψ_u = 1.15 in the next section and for use in Fig. 2).

As mentioned in the Introduction, the consequence factor should ideally depend only on the target maximum acceptable failure probability, p_m, and not on soil variability, correlation length or sampling location. Variations in the latter three parameters should ideally be handled entirely by the resistance factor, φ_{gu}, which looks after the issue of site understanding. Figures 5 and 6 investigate the effect of site variability, correlation length, and sampling location on the consequence factor for high consequence level (Fig. 5) and for low consequence level (Fig. 6). Both figures use the *best estimates* of the resistance factors, specified by the curves in Fig. 3, for each value of r, θ, and ν_c, rather than on the proposed design resistance factors shown by the horizontal lines in Fig. 3 — the effect of using the proposed design resistance factors on the consequence factor is investigated in the next section. The horizontal lines shown in Figs. 5 and 6 are at the Ψ_u values recommended in the next section to illustrate how the best estimated consequence factors, based on best estimates of the resistance factors, compare to the consequence factors recommended in the next section.

For the high consequence level case, Fig. 5, the range in Ψ_u values, over the entire parameter set, is from 0.91 to 0.976, a relative change of only about 7%. When compared with the more than 200% relative change in resistance factors over the same parameter set (see Fig. 3), it can be safely concluded that the high consequence factor is largely independent of soil and sampling parameters (θ, ν_c, and r) and is primarily dependent on p_m. In any case, if the resistance factors are selected from the curves of Fig. 3, the

Fig. 3. Resistance factors versus correlation length for medium consequence level ($\Psi_u = 1.0$) at the three site understanding levels: (*a*) high (r = 0 m), (*b*) medium (r = 4.5 m), and (*c*) low (r = 9 m). Solid horizontal lines show selected design resistance factor values for each site understanding level.

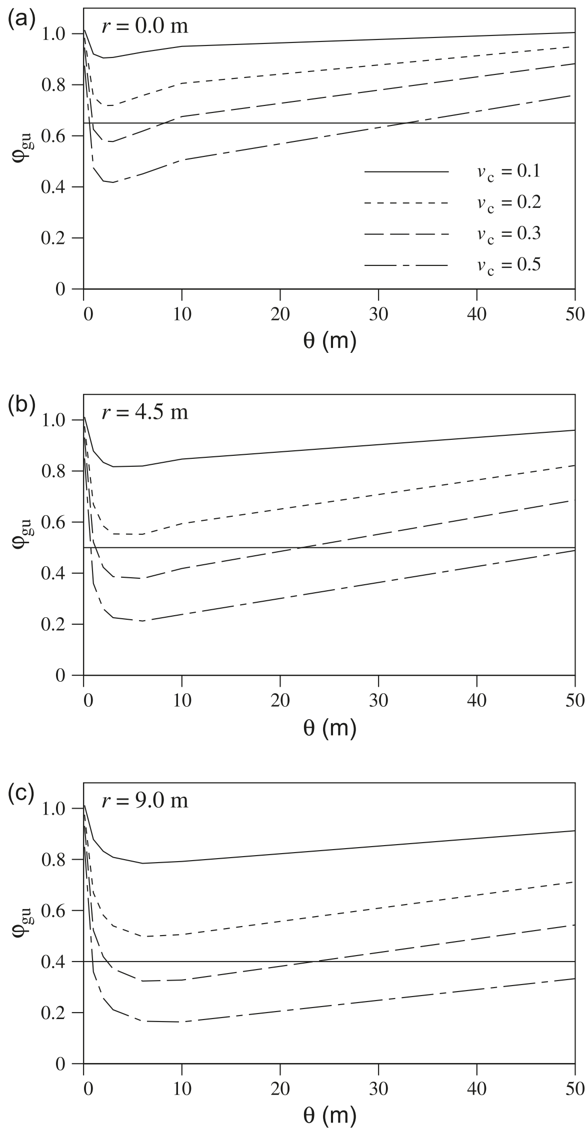
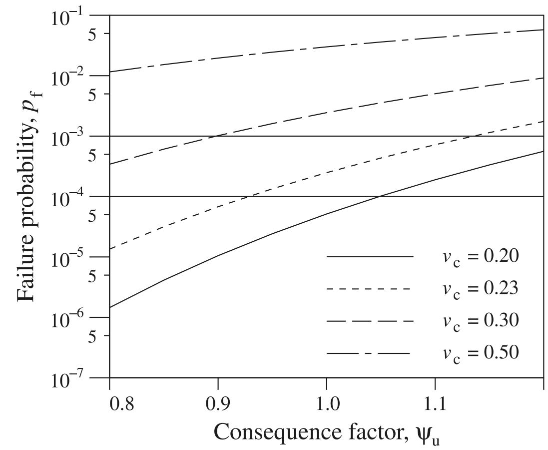


Fig. 4. Failure probability versus consequence factor for θ = 6 m, r = 4.5 m, and φ_{gu} = 0.50.



choice of $\Psi_u = 0.90$ is seen to be always conservative for the high consequence level.

For the low consequence level case, shown in Fig. 6, the range in Ψ_u is from 1.06 to 1.28, a relative change of about 19%. If the ν_c = 0.5 case is ignored, the relative change drops to about 13%. This is a wider range than achieved for the high consequence level, but still a small range when compared with the changes in the resistance factor. Again, it appears reasonable to conclude that the low consequence factor is largely independent of site understanding and primarily dependent on p_m when the resistance factor is selected from the curves in Fig. 3.

In both Figs. 5 and 6, it is important to emphasize that the resistance factors used in the determination of the consequence factors are *best estimates* taken from Fig. 3 and this leads to some rather counterintuitive results. For example, the consequence factors required for low consequence levels (Fig. 6) are higher for higher soil variability — normally the "resistance factor" becomes smaller as variability increases. However, Figs. 5 and 6 are intimately connected to Fig. 3. As variability increases, the resistance factors in Fig. 3 drop significantly, undercompensating when the consequence level increases and overcompensating when the consequence level decreases from the medium level. Figures 5 and 6 are presented primarily as evidence of the reduced dependence between site understanding and the consequence factor. The consequence factors required using fixed "code"-specified resistance factors are investigated in the next section.

Recommended consequence factors

In practice, the designer is unlikely to know the true soil variability (ν_c and ν_ϕ) and will almost certainly not know the true correlation length. In other words, the designer is most likely not going to be able to pick the optimum resistance factor from Fig. 3 and so must rely on design or code specified values, assuming some knowledge of the degree of site understanding. When the resistance factor is fixed at a certain value for a level of site understanding, the consequence factor shows much more variability with soil parameters. This is entirely understandable, as the effort of adjusting the design reliability is now thrown almost entirely onto the consequence factor, but complicates the selection of recommended design values for the consequence factor.

Figures 7 and 8 show how the consequence factor must vary when the resistance factor is fixed for each level of site understanding: φ_{gu} = 0.65 for high understanding (r = 0 m), φ_{gu} = 0.50 for medium understanding (r = 4.5 m), and φ_{gu} = 0.40 for low understanding. Note that the vertical scale on these plots has increased significantly from Figs. 5 and 6. For the high consequence case, Fig. 7, the consequence factor varies over a much larger range — the widest range of 0.38 to 2.47 occurs in the low understanding case (r = 9 m, Fig. 7*c*). For the low consequence case, Fig. 8, the range is from 0.52 to 2.69.

Considering Fig. 7, the task is to choose a factor for the high consequence case that is sufficiently conservative and yet not excessively so. Reducing the consequence factor re-

Fig. 5. Consequence factor versus correlation length for various sampling locations at high consequence level (p_m = 1/10 000) using best estimates of φ_{gu} from Fig. 3: (*a*) r = 0.0 m; (*b*) r = 4.5 m; (*c*) r = 9.0 m.

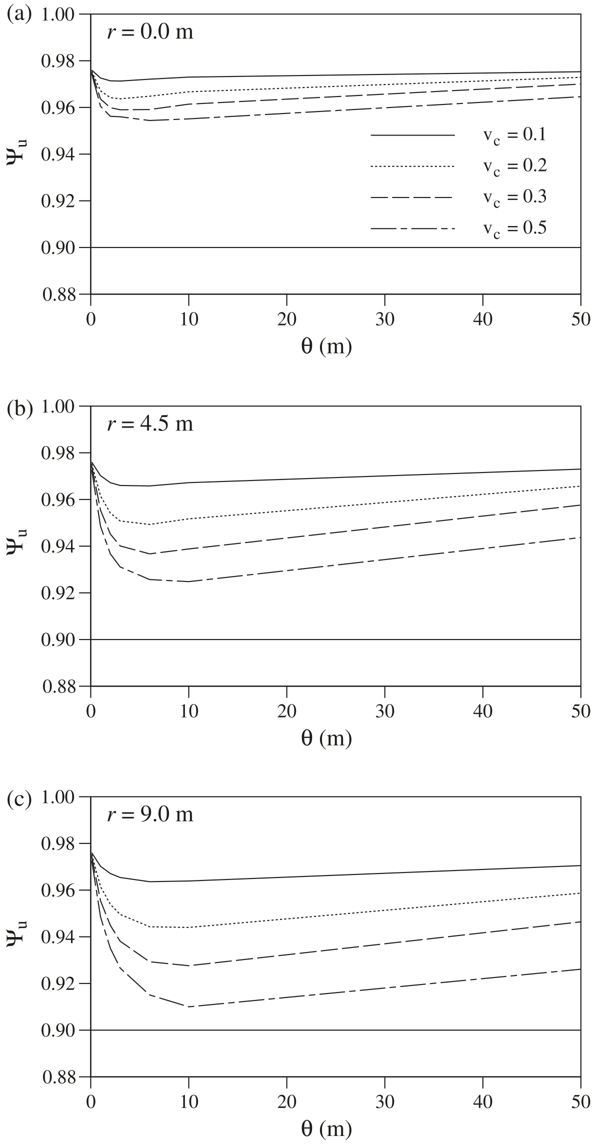
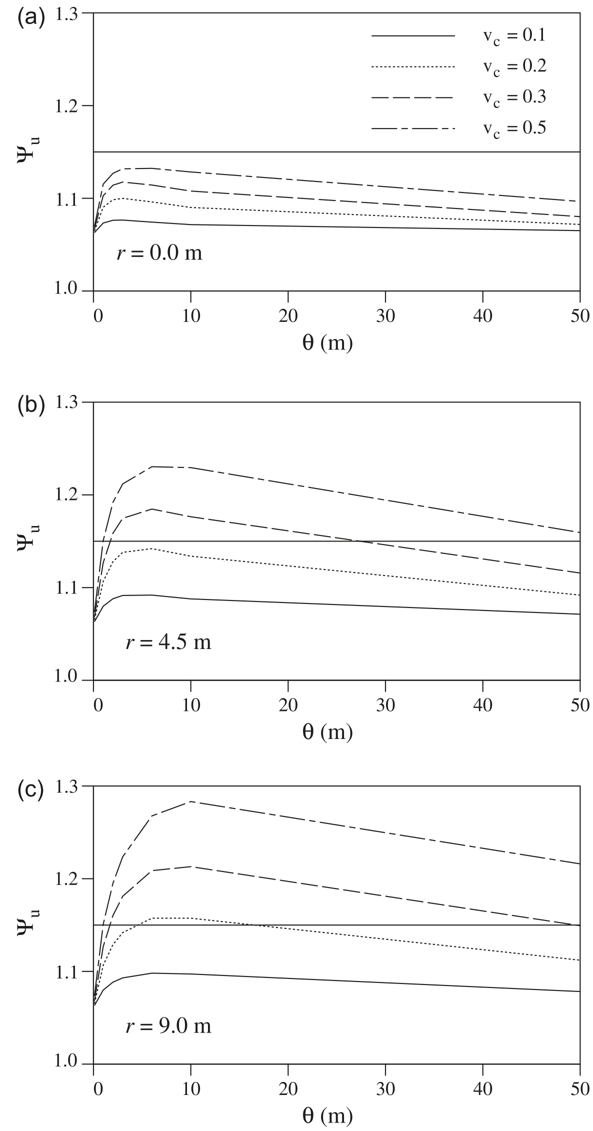


Fig. 6. Consequence factor versus correlation length for various sampling locations at low consequence level (p_m = 1/1000) using best estimates of φ_{gu} from Fig. 3: (*a*) r = 0.0 m; (*b*) r = 4.5 m; (*c*) r = 9.0 m.



sults in more conservative designs (lower failure probability). A solid horizontal line has been drawn across each plot at Ψ_u = 0.9 and it can be seen that this value is conservative for all $\nu_c \leq 0.25$ (approximately), in that the curves for ν_c = 0.1 and 0.2 lie above Ψ_u = 0.9. What this means is that if ν_c is known to be 0.1, for example, then using Ψ_u = 0.9 in the design would result in a failure probability well below the target of p_m = 1/10 000. On the other hand, if ν_c is not clearly known, then Ψ_u = 0.9 is reasonably conservative for all sites except those with large soil variability (e.g., $\nu_c \geq$ 0.3). If site investigation is sufficient to keep the residual variability below this level, then Ψ_u = 0.9 is a reasonable design value for the high failure consequence case, which will almost always lead to a failure probability well below p_m = 1/10 000 (β = 3.7).

A similar argument can be applied to Fig. 8 for the low consequence case, where a solid line at Ψ_u = 1.15 has been drawn across each plot. It can be seen that this value is not quite as conservative as the high consequence factor selected above in that the ν_c = 0.2 curve comes somewhat closer to Ψ_u = 1.15. The authors feel, however, that conservatism is not quite as important for the low failure consequence case, and therefore selected a somewhat higher value.

It is instructive to consider the values used by other codes to handle failure consequences. Most codes include an importance factor, *I*, which is the inverse of the consequence factor as it is applied to the load side of the LRFD equation (see eq. [1]). Table 1 compares the conservatively recommended consequence factors mentioned in the recommendations above (0.9 for high consequence and 1.15 for low consequence levels) to a variety of other codes.

At the high consequence level, the recommended conse-

Fig. 7. Consequence factor versus correlation length for various sampling locations at high consequence level (p_m = 1/10 000) using proposed design values of φ_{gu}: (*a*) r = 0.0 m, φ_{gu} = 0.65; (*b*) r = 4.5 m, φ_{gu} = 0.50; (*c*) r = 9.0 m, φ_{gu} = 0.40.

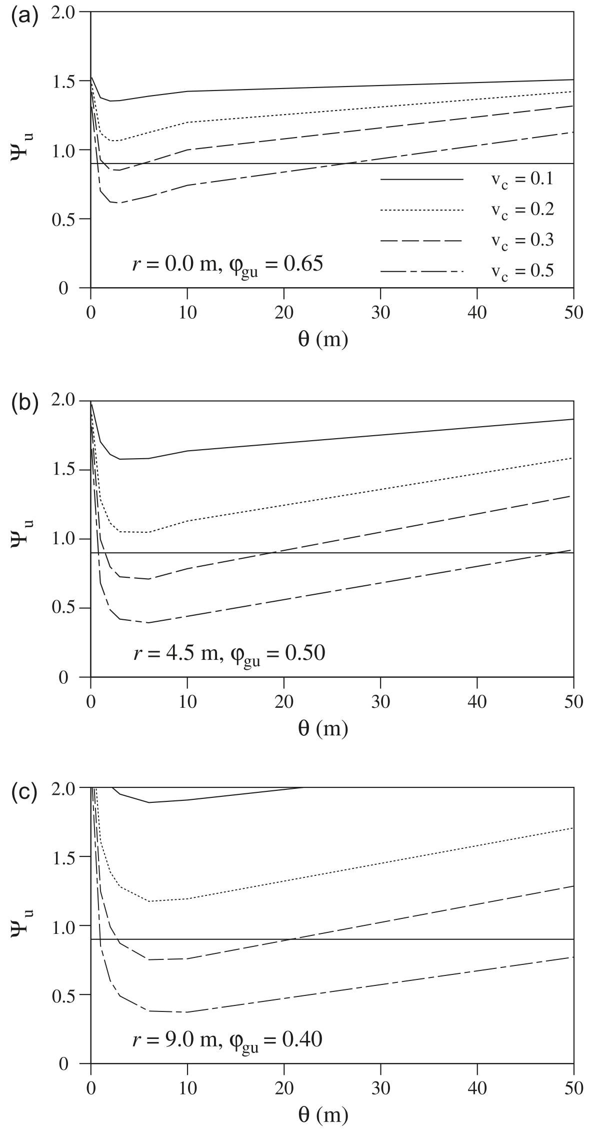
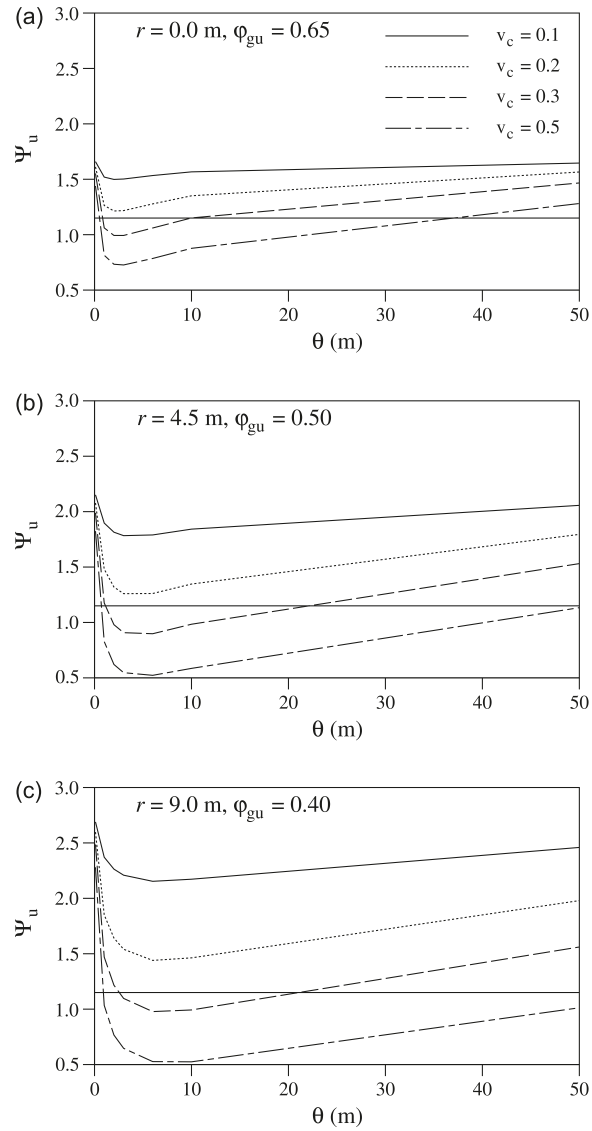


Fig. 8. Consequence factor versus correlation length for various sampling locations at low consequence level (p_m = 1/1000) using proposed design values of φ_{gu}: (*a*) r = 0.0 m, φ_{gu} = 0.65; (*b*) r = 4.5 m, φ_{gu} = 0.50; (*c*) r = 9.0 m, φ_{gu} = 0.40.



quence factor is in basic agreement with the inverse of the importance factors given by AASHTO(2007), Eurocode 1 (Gulvanessian et al. 2002), and NBCC (NRC 2005) for snow and wind. Both the Australian standard AS5100 (Standards Australia 2004) and the earthquake provision of the NBCC are more conservative than suggested here — this may be reasonable for earthquake loading given its significant uncertainty. The fact that the NBCC earthquake importance factor is quite a bit more conservative (0.77 versus 0.90) also suggests that both a consequence factor, on the resistance side, and an importance factor, on the load side, need to appear in the code to take into account the two disparate sources of uncertainty. This topic needs further study, but care needs to be taken to avoid double factoring. In particular, codes that include an importance factor on the load side need to be carefully calibrated with the consequence factor proposed here. In the case of seismic design, it probably makes sense to use Ψ_u = 1.0 to avoid double factoring, at least until further research suggests otherwise.

At the low consequence level, the consequence factor recommended here is in good agreement with that of Eurocode 1, while the AASHTO and NBCC values are less conservative (corresponding to a higher probability of failure). The authors are uncertain about what specific acceptable failure probability was being sought by AASHTO and NBCC — the value recommended here (Ψ_u = 1.15) may be overly conservative in Canada under snow, wind, and (or) earthquake loading, suggesting again that perhaps some level of double factoring (i.e., both a consequence factor on the resistance side and an importance factor on the load side) should be applied for the low consequence case when any of snow, wind, and (or) earthquake are being designed for.

Table 1. Comparison of consequence factors recommended in this paper to equivalent (1/*I*) values recommended in various codes.

	Consequence level		
Source	Low	Medium	High
Recommended in this paper	1.15	1.00	0.90
AASHTO (2007)	1.25	1.00	0.91
AS5100 (Standards Australia 2004)	—	1.00	0.83
Eurocode 1 (Gulvanessian et al. 2002)	1.11	1.00	0.91
NBCC ((NRC 2005), snow and wind)	1.25	1.00	0.87
NBCC ((NRC 2005), earthquake)	1.25	1.00	0.77

Conclusions

This paper introduces a new consequence factor into the limit state design of geotechnical systems and presents the theory required to estimate its value. For design purposes, the consequence factors recommended in this paper are 0.9 for high failure consequence, 1.0 for medium failure consequence, and 1.15 for low failure consequence levels. These values are in reasonable agreement with the importance factors employed by other codes worldwide, being possibly more conservative in the low consequence case when environmental loads are included in the design process and unconservative under earthquake loading (at least without an additional importance factor for earthquake loading).

Although the results presented here are mathematically rigorous and the theory is validated through simulation in a previous study, a number of simplifying assumptions were made in the model. These are as follows:

(1) The analysis considered only a strip footing. This allowed use of a simpler 2-D model. It is not expected that a full 3-D model would make much difference to the probabilistic results presented here.
(2) To restrict attention to the most important random soil properties (i.e., cohesion and friction angle), the soil was assumed to be weightless. This is a conservative assumption as soil weight adds to its strength.
(3) Only dead and live loads were considered. This is a typical code development assumption. Additional research is required to determine how the factors for extreme loads (e.g., earthquake) should be implemented alongside the consequence factor introduced here.
(4) The random soil properties were assumed to be isotropic (i.e., not layered) and stationary (same mean and variance everywhere). Soil layering tends to be a site-specific phenomenon. For code development, this simplifying assumption was deemed appropriate and not expected to signicantly affect the results.
(5) The load factors used were from NBCC (NRC 2005). It is expected that different load factors will primarily result in different resistance factors (changing linearly) and will not have a significant effect on the consequence factor. Preliminary investigations into this statement suggest that it is true and that the consequence factor changes by only one or two percent when, say, the load factors change from $\alpha_L = 1.5$ and $\alpha_D = 1.5$ to $\alpha_L = 1.7$ and $\alpha_D = 1.2$ (the latter as specified in CHBDC (CSA 2006)).
(6) Measurement and model errors were ignored, which strictly speaking means that the resistance and consequence factors presented here are upper bounds. However, the factors were developed assuming reasonable levels of soil variability ($\nu_c \simeq 0.25$) under worst case correlation lengths. It is likely that most sites will not have correlation lengths equal to the worst case. Also, in practice, the actual residual soil coefficient of variation, after a good site investigation, will likely be less than 25%. The sum of soil variability and variabilty due to measurement and modeling errors could bring the total variability back up towards 25%, which is probably realistic. Thus, the results presented here are deemed to be reasonable for practical use in code development.

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References

AASHTO. 2007. LRFD bridge design specifications. American Association of State Highway and Transportation Officials (AASHTO), Washington, D.C.

Allen, D.E. 1975. Limit states design – a probabilistic study. Canadian Journal of Civil Engineering, **2**(1): 36–49. doi:10.1139/l75-004.

Becker, D.E.. 1996. Eighteenth Canadian Geotechnical Colloquium: Limit States Design for Foundations. Part II. Development for the National Building Code of Canada. Canadian Geotechnical Journal, **33**(6): 984–1007. doi:10.1139/t96-125.

Canadian Geotechnical Society. 2006. Canadian foundation engineering manual. 4th ed. BiTech Publishers Ltd., Richmond, B.C.

CEN. 2003. Eurocode 7 Geotechnical design – Part 1: General rules. Standard EN 1997–1. European Committee for Standardization (CEN), Brussels, Belguim.

Chalk, P.L., and Corotis, R.B. 1980. Probability model for design live loads. Journal of the Structural Division, **106**(ST10): 2017–2033.

Cherubini, C. 2000. Reliability evaluation of shallow foundation bearing capacity on c', ϕ' soils. Canadian Geotechnical Journal, **37**(1): 264–269. doi:10.1139/cgj-37-1-264.

Corotis, R.B., and Doshi, V.A. 1977. Probability models for live-load survey results. Journal of the Structural Division, ASCE, **103**(ST6): 1257–1274.

CSA. 2006. Canadian highway bridge design code. CSA standard CAN/CSA-S6-06. Canadian Standards Association (CSA) Mississauga, Ont.

DGI. 1985. Code of practice for foundation engineering. DGI-Bulletin No. 36. Danish Geotechnical Institute (DGI), Copenhagen, Denmark.

Fenton, G.A., and Griffiths, D.V. 2003. Bearing-capacity prediction of spatially random $c - \phi$ soils. Canadian Geotechnical Journal, **40**(1): 54–65. doi:10.1139/t02-086.

Fenton, G.A., and Griffiths, D.V. 2008. Risk assessment in geotechnical engineering. John Wiley and Sons, New York.

Fenton, G.A., Zhang, X.Y., and Griffiths, D.V. 2007. Reliability of shallow foundations designed against bearing failure using LRFD. Georisk, **1**(4): 202–215.

Fenton, G.A., Griffiths, D.V., and Zhang, X. 2008. Load and resistance factor design of shallow foundations against bearing failure. Canadian Geotechnical Journal, **45**(11): 1556–1571. doi:10.1139/T08-061.

Griffiths, D.V., and Fenton, G.A. 1997. Three-dimensional seepage through spatially random soil. Journal of Geotechnical and Geoenvironmental Engineering, **123**(2): 153–160. doi:10.1061/(ASCE)1090-0241(1997)123:2(153).

Gulvanessian, H., Calgaro, J., and Holicky, M. 2002. Designer's guide to EN 1990 Eurocode: Basis of structural design. Thomas Telford Ltd., London.

Meyerhof, G.G. 1995. Development of geotechnical limit state design. Canadian Geotechnical Journal, **32**(1): 128–136. doi:10.1139/t95-010.

NRC. 2005. National building code of Canada. National Research Council of Canada (NRC), Ottawa, Ont.

Standards Australia. 2004. Bridge design, part 3: foundations and soil-supporting structures. Standard AS 5100.3–2004. Standards Australia, Sydney, Australia.

Terzaghi, K. 1943. Theoretical soil mechanics. John Wiley and Sons, New York.

Vanmarcke, E.H. 1984. Random fields: analysis and synthesis. MIT Press, Cambridge, Mass.

Wolff, T.H. 1985. Analysis and design of embankment dam slopes: a probabilistic approach. Ph.D. thesis, Purdue University, Lafayette, Ind.

List of symbols

a $\tan\mu_\phi$
B strip footing width
b $e^{\pi a}$
c cohesion
$\bar{c}$ geometric average of cohesion field over domain D
$\hat{c}$ geometric average of observed (sampled) cohesion values
c_i observed (sampled) cohesion value
D effective soil property averaging domain centered under footing (= $W \times W$)
D_1 x_1 dimension of the averaging domain D
D_2 x_2 dimension of the averaging domain D
d $\tan(0.25\pi + 0.5\mu_\phi)$
E expectation operator
F total true (random) footing load (kN/m)
$\hat{F}_i$ ith characteristic load effect
F_D true (random) dead load (kN/m)
$\hat{F}_\mathrm{D}$ characteristic dead load (= $k_\mathrm{D}\mu_\mathrm{D}$) (kN/m)
F_L true (random) maximum live load over design life (kN/m)
$\hat{F}_\mathrm{L}$ characteristic live load (= $k_\mathrm{L}\mu_\mathrm{L}$) (kN/m)
f $\pi(1 + a^2)d + 1 + d^2$
G_ϕ standard normal random field underlying friction angle
H depth of soil sample
I importance factor
k_L extreme lifetime live load bias factor
k_D dead load bias factor
m number of soil observations
N_c N-factor associated with cohesion, which is a function of ϕ
$\bar{N}_c$ equivalent N-factor associated with cohesion, which is based on an arithmetic average of the friction angle over domain D
$\hat{N}_c$ characteristic N-factor associated with cohesion, which is based on an arithmetic average of the observed friction angles over domain Q (m soil sample observations)
p_f probability of bearing capacity failure
p_m maximum acceptable probability of bearing capacity failure
Q characteristic soil property averaging domain (= $\Delta x \times H$)
q factored design load ($= \alpha_\mathrm{L}\hat{F}_\mathrm{L} + \alpha_\mathrm{D}\hat{F}_\mathrm{D}$)
$\hat{q}_\mathrm{u}$ ultimate bearing stress estimated from characteristic soil properties
R actual minimum lifetime resistance
R_u true ultimate resistance (random)
$\hat{R}_\mathrm{u}$ ultimate geotechnical resistance based on characteristic soil properties
r distance between soil sample and footing center, m
s scale factor used in distribution of ϕ
W side dimension of effective averaging domain D
$\boldsymbol{x}$ spatial coordinate, (x_1, x_2) in 2-D
x_i spatial direction (x_1 and x_2)
Y true load times the ratio of estimated to equivalent bearing capacity
α load factor
α_D dead load factor
α_i load factor corresponding to the ith load effect
α_L live load factor
β reliability index corresponding to maximum acceptable failure probability, p_m
γ_{DQ} average correlation coefficient between domains D and Q
$\gamma(D)$ variance function giving variance reduction due to averaging over domain D
Δx horizontal dimension of soil samples
θ correlation length of the random fields
μ_c cohesion mean
μ_F sum of the mean (maximum lifetime) live and (static) dead loads
$\mu_{\ln c}$ log-cohesion mean
$\mu_{\ln \hat{c}}$ mean of the estimate of log-cohesion based on a geometric average of cohesion observations
$\mu_{\ln \bar{c}}$ mean of the equivalent log-cohesion based on a geometric average of cohesion over domain D
μ_{N_c} mean of N_c
$\mu_{\ln N_c}$ mean of $\ln N_c$
$\mu_{\ln \hat{N}_c}$ mean of $\ln\hat{N}_c$
$\mu_{\ln \bar{N}_c}$ mean of $\ln\bar{N}_c$
μ_D mean dead load
μ_L mean extreme live load over design life
$\mu_{\ln F}$ mean total log-load on strip footing
$\mu_{\ln R}$ mean log-resistance of ground under strip footing
μ_ϕ mean friction angle
$\mu_{\hat{\phi}}$ mean of estimated friction angle
$\mu_{\bar{\phi}}$ mean of equivalent friction angle in zone of influence under footing
$\mu_{\ln Y}$ mean of $\ln Y$
$\hat{\mu}_B$ estimated mean footing width
ν_c coefficient of variation of cohesion
ν_D coefficient of variation of dead load
ν_F coefficient of variation of total load

ν_L coefficient of variation of extreme lifetime load
ν_ϕ coefficient of variation of friction angle
ξ_1, ξ_2 spatial coordinates in 2-D
$\rho(\boldsymbol{\tau})$ common correlation function
σ_c cohesion standard deviation
σ_D dead load standard deviation
σ_L standard deviation of extreme lifetime live load
$\sigma_{\ln F}$ standard deviation of total log-load
$\sigma_{\ln R}$ standard deviation of total log-resistance
$\sigma_{\ln c}$ log-cohesion standard deviation
$\sigma_{\ln \bar{c}}$ standard deviation of $\ln\bar{c}$
$\sigma_{\ln \hat{c}}$ standard deviation of $\ln\hat{c}$
σ_ϕ standard deviation of ϕ
$\sigma_{\hat{\phi}}$ standard deviation of $\hat{\phi}$
$\sigma_{\bar{\phi}}$ standard deviation of $\bar{\phi}$
$\sigma_{\ln N_c}$ standard deviation of $\ln N_c$
$\sigma_{\ln \bar{N}_c}$ standard deviation of $\ln\bar{N}_c$
$\sigma_{\ln \hat{N}_c}$ standard deviation of $\ln\hat{N}_c$
$\sigma_{\ln Y}$ standard deviation of $\ln Y$
$\boldsymbol{\tau}$ vector between two points in the soil domain
τ_1, τ_2 horizontal and vertical component, respectively, of the distance between two points in the soil domain
Φ standard normal cumulative distribution function
ϕ friction angle (radians unless otherwise stated)
$\bar{\phi}$ arithmetic average of ϕ over domain D
$\hat{\phi}$ arithmetic average of the m observed friction angles
$\phi_{\min}$, $\phi_{\max}$ minimum and maximum friction angle, respectively
φ_{gu} ultimate geotechnical resistance factor
Ψ_u consequence factor for ultimate limit state design