Reliability-Based Transmission Line Design

Gordon A. Fenton and Nancy Sutherland

***Abstract*—It is well known that environmental loads, e.g., wind and ice, acting on power transmission lines are highly uncertain, as are the structural strengths of the towers supporting the lines. The design of such systems must take uncertainty into account in order to achieve acceptable reliability at a reasonable cost. The paper presents a simulation-based methodology for the optimal design of a transmission line which considers uncertainties in both environmental loads and structural resistance. The methodology is developed and illustrated for the simple problem of determining the optimal span length required for designing against tower failure. Wind, ice, and tower resistances are simulated over the extent of the transmission line and over the design life of the transmission system. Total expected system cost, along with the estimated probability of lifetime failure, are produced for a range of possible span lengths, allowing an informed decision regarding the optimum span length for the tower strength limit state.**

***Index Terms*—Climatic loads, design methodology, poles and towers, reliability modeling, risk analysis, transmission lines.**

I. Introduction

THE goal of a reliability-based design is to produce a system which is both robust and economical. In order to achieve this goal, design decisions must be optimally made in the face of uncertainty. Although uncertainty can be reduced to some extent through experimentation and statistical studies, it remains significant due to variability in loads, construction quality, model/design error, and system degradation. In particular, uncertainty in environmental loading due to extreme climatic events has lead to a re-evaluation of current transmission line design and upgrading practices [1].

This paper presents a reliability-based methodology for the design of an electrical power transmission system consisting of conductors and transmission towers spanning between a generating plant and a destination group of customers (e.g., a small city, or a segment of a large city). To develop and illustrate the methodology, a single design decision will be examined—optimization of the conductor span between towers with respect to the tower failure limit state. Deciding on a span length depends on many possible limit states (e.g., tower capacity, sag, vibration, and tension). In this paper, only the limit state of tower capacity will be considered, although it is recognized that this limit state may not govern the design and a fully reliability-based design must apply the same ideas expressed in this paper to all other limit states and include all other design variables. As such, it is emphasized that the proposed methodology is to be viewed as a supplement to current design methods (since the latter consider all limit states and design variables) and as an increment to the pioneering work of other reliability-based power transmission researchers (e.g., Ghannoum and Phoon).

Regarding the tower capacity limit state, as the span between towers increases, the conductor, ice and wind loads acting upon each tower increases, resulting in an increased probability of tower failure for a given tower design. On the other hand, increasing the span reduces the required number of towers and thus the initial system cost. The best design span with respect to the tower capacity limit state will involve a tradeoff between the cost of failure over the design life of the transmission system and the initial system cost.

A reliability-based design considers both the probability of an adverse event occurring and the consequence of that event should it occur [2] to determine an optimal design decision. Consequence is generally expressed in monetary terms and typically based on the cost of repairs, remediation, human safety, inconvenience, or other losses. The structural failure of a transmission tower is an event with multiple adverse consequences including possible power outages, tower replacement costs, incremental generation costs (running a generator or purchase from another source), and other incidental costs which are difficult to quantify, such as unfavorable consumer perception. Power outages, in turn, can lead to losses associated with homes (e.g., refrigerator contents), businesses, and even personal safety, particularly during the winter.

When new transmission lines are designed, or older established lines reassessed, it is desirable to minimize costs while maintaining an acceptably low probability of failure. Electrical power providers are beginning to recognize the value of reliability-based design. For example, Hydro Quebec has been using probability based design techniques for new tower designs and upgrade of older transmission lines for many years now [3]. This task involves minimizing the sum of initial capital and transmission line failure costs while maintaining an acceptable reliability. Ghannoum [4], [5] suggests that the optimum annual transmission line failure probability should range from about 0.01 to 0.001, depending on the consequences of failure. Acceptable failure probabilities in this range correspond to climatic loads having return periods of roughly 50 to 500 years.

This paper will develop a reliability-based design methodology aimed at minimizing the total expected cost of a transmission tower system over a design life of 50 years. The design decision variable considered will be the conductor span between transmission towers, assuming level terrain, and the decision will be made on the basis of climatic loads as they affect the possibility of tower structural failure. The case study is hypothetical, and only considers one of myriad design decisions and a single decision criterion, but serves to demonstrate a simulation approach to reliability-based design.

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In contrast to traditional design, a reliability-based approach needs to identify the random quantities on both the load and resistance side, and to gather enough data to allow the distributions of these random quantities to be estimated. In addition, because loads and strengths are both time varying and because maintenance/failure costs accrue with time, the system lifetime needs to be carefully considered. In other words, while traditional designs typically need only mean (or characteristic) load and resistance parameters, along with empirically based safety factors, a reliability-based approach also needs to know about; construction, maintenance, and failure costs; the complete distribution of all random load and resistance parameters; and how all of these parameters vary with time. This represents quite a bit of additionally required information, much of which is not currently known. It is anticipated that, as reliability-based design tools such as proposed in this paper become readily available, the data-base required to estimate distributions will become available in the years to come.

The main point of this paper is to present a simulation-based methodology for transmission line design. The simulation algorithm used to optimize a single design variable (span length) for a single limit state (tower failure) is presented just before the conclusions. The rest of the paper develops the background information needed for the simulation and is organized as follows: In Section II, the mathematical concepts behind reliability-based design are explained. These concepts form the theoretical basis of the simulation algorithm. In Section III, the mean tower resistance is determined by finite element analysis and a random resistance model is proposed. Random climatic ice and wind loads are then characterized in Section IV and used in determining the total load applied to a tower during an ice storm. If the total applied load exceeds the ultimate tower resistance, the tower fails, and failure consequences are discussed in Section V. Section VI describes the simulation methodology and presents simulation results from which reliability-based design conclusions are drawn and recommendations made in Section VII.

The methodology developed here forms just one tool in a transmission line designer's repertoire. The current approach to transmission line design involves the following steps: 1) mapping out the terrain and determining environmental loads; 2) locating the tower positions according to the terrain, loads, maximum sag, and other conductor demands; and 3) selection of suspension and dead-end towers to safely support the conductor loads. The last step would typically be done using finite element analysis (as in Section III). The results of this paper can be used to aid in step 3), i.e., in the optimal reliability-based selection of towers. If the methodology proposed here is applied to a variety of tower designs to obtain an optimal span for each, then a table of tower types versus optimal spans could be developed. Towers having optimal spans closest to the spans required by step 2) could then be selected.

II. Risk-Based Design Methodology

In the following, the word "risk" will be assumed to be the product of consequence, in monetary terms, and probability of failure (the complement of reliability). In other words, risk is the expected cost of failure. Thus, to determine the risk associated with a design, both the probability of failure and the cost of failure must be known. To estimate the probability of failure of a transmission line, the individual components of the line must be examined. These components include transmission towers, conductors, and various pieces of hardware. Failure of any one of the components can result in failure of the entire transmission line system.

In its simplest form, the probability of failure of a system can be estimated by considering just two random variables: resistance, R, and load, L. If the load exceeds the resistance then the system fails, with probability $p_f = \mathrm{P}[R < L]$. If both R and L are independent and lognormally distributed (the latter is commonly assumed in structural engineering since resistance and load are both non-negative and the lognormal distribution has a long tail in the direction of extremes, which is conservative), then the probability of failure can be expressed as

$$\begin{aligned} p_f &= \mathrm{P}[R < L] = \mathrm{P}\left[\frac{R}{L} < 1\right] = \mathrm{P}[\ln R - \ln L < 0] \\ &= \Phi\left(\frac{0 - (\mu_{\ln R} - \mu_{\ln L})}{\sqrt{\sigma^2_{\ln R} + \sigma^2_{\ln L}}}\right) = \Phi(-\beta) \end{aligned} \tag{1}$$

where $\mu_{\ln R}$ and $\mu_{\ln L}$ are the means of ln R and ln L, respectively, $\sigma^2_{\ln R}$ and $\sigma^2_{\ln L}$ are their variances, and Φ is the standard normal cumulative distribution function. The reliability index, β, is defined as the number of standard deviations between the mean of $(\ln R - \ln L)$ and the failure point, which in this case is 0

$$\beta = \frac{\mu_{\ln R} - \mu_{\ln L}}{\sqrt{\sigma^2_{\ln R} + \sigma^2_{\ln L}}} \tag{2}$$

assuming R and L are independent. Once the probability of failure has been established, the next step is to assess the consequence of failure expressed in monetary terms. If C_f is defined as the cost of failure, then the total system cost, C, over the system lifetime is given by

$$C = C_o + C_m + C_f X_f \tag{3}$$

where C_o is the initial cost of the system, C_m is the lifetime maintenance cost, and X_f is a Bernoulli random variable having value 1 if failure occurs and value 0 if not. The probability that failure occurs is $p_f = \mathrm{P}[X_f = 1] = \mathrm{E}[X_f]$ which can be computed by means of (1). The total expected lifetime system cost is obtained by taking the expectation of (3)

$$\mathrm{E}[C] = C_o + C_m + C_f p_f \tag{4}$$

where it is assumed that the cost components are non-random. The task in a risk-based design is developing a design which minimizes (4) while maintaining a reasonably low failure probability. The authors note that basing a design purely on the minimization of (4) can sometimes lead to unacceptable failure probabilities—often solutions which *almost* minimize (4) have much lower failure probabilities and so are preferred.

While application of the basic concept, given above, to optimize a design is relatively straightforward, its mathematical formulation in the case of a transmission line is more complex. First of all, the resistance of a tower to loads is at least bivariate—the

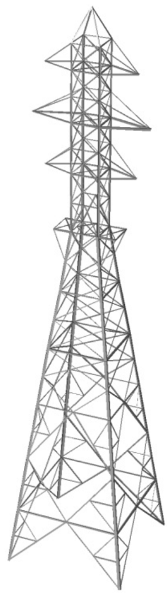

Fig. 1. Double circuit standard suspension tower.

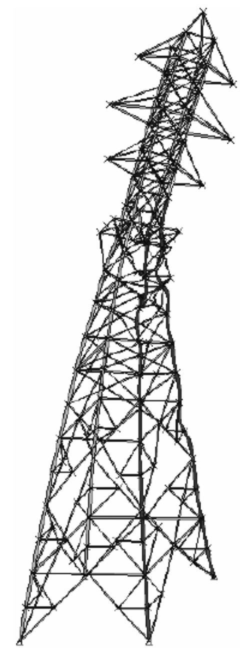

Fig. 2. Tower collapse mechanism.

ultimate horizontal load, R_H, that the tower can withstand depends on the magnitude of the total applied vertical load, V. In other words, the ultimate horizontal resistance, R_H, and the applied vertical load, V, are both random and dependent—as V increases, R_H decreases. Secondly, the load components, which in this study are ice and wind loads, are correlated—as the ice thickness on the tower and conductors (which contribute to the vertical load) increases, the horizontal wind load component also increases due to the larger cross-sectional area of the accreted ice. Thus, the probability of tower failure is a function of at least four cross-correlated random variables. While this probability can be approximated by defining a multi-variate failure function and then applying first-order reliability methods [6], a much simpler approach (and much less restrictive in terms of approximations) is to estimate the failure probability by simulation, an approach also used by Ghannoum [4], [5]. The simulation approach will be adopted in this paper. The various random components of the tower resistance and the environmental loads acting on the tower are discussed in the next two sections.

III. Tower Resistance

It is assumed that the transmission towers are double circuit standard suspension lattice structures, each constructed following traditional deterministic design criteria, supporting six 27.762 mm Condor conductors and one ground wire. The conductors are supported at six locations, at the ends of each of the three cross-arms, and the ground wire supported at the top of the tower. All applied loads were assumed to be transmitted to the tower through these seven attachment points for simplicity. Fig. 1 shows the configuration of the transmission tower.

The double circuit standard suspension tower was modeled using the finite element program SACS 5.2 [7]. The model consisted of 184 joints and 496 steel angle members. Tower loads were assumed to be transmitted largely axially along the elements. However, all tower joints were fixed, with the exception of the tower base and lower cross-arm members, since joint fixity closely represented the actual bolted system used in the tower and led to increased numerical stability in the nonlinear tower response.

Most often tower foundations consist of earth grillages or anchor bolts/rock anchors. Pinning the base was thought to suitably simulate these foundation types. The lower cross-arm member ends were not fixed because as the tower was loaded vertically from the insulator attachment points (end of the crossarms), buckling of the horizontal members caused instability in the analysis. To avoid premature and unrealistic buckling of the lower cross-arm members, which led to numerical failure of the finite element analysis, the ends of these members were released with respect to rotations about directions perpendicular to the member axis. Plastic deformation in a tower element occurred if stress in the element at any point exceeded its yield stress, in compliance with Von Mise's Stress Criterion. The formation of a hinge in a failed member allowed for plasticity to develop throughout the tower until overall loss of lateral and vertical stiffness resulted in tower collapse. The tower stiffness matrix was continually updated as the loads were incrementally applied, and stiffness iterations continued until strict convergence tolerances were reached. Fig. 2 illustrates a tower collapse.

In Fig. 2, a vertical load, V, and a horizontal load were applied evenly at the six conductor and ground wire points and increased incrementally until the tower failed. In the following, the word "conductor" will be assumed to include the ground wire. Thus, as far as structural loads are concerned, the tower will be assumed to be loaded by seven "conductors" in total. Loads acting on the tower consisted of self-weight of the tower, wind, and ice loads on the conductors and on the tower itself. The application of the load at the conductor points is, to some extent, an approximation. Wind, ice, and self-weight loads within

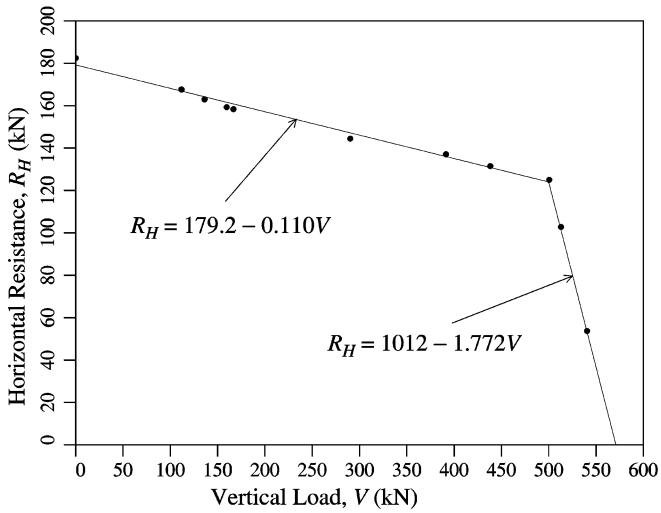

Fig. 3. $R_H - V$ resistance curve of the tower.

the tower itself will act on every member individually—the assumption that the resultants act at the conductor points simplifies the analysis. Since the tower typically failed at a point well below the conductor points, this approximation was deemed to have a negligible effect on the estimated tower resistance. Only the tower loads (self-weight, wind, and ice) above the average failure region were considered since only these tower weights contribute to the failure of the members in the failure region. In other words, self-weight, wind, and ice loads acting on the tower below the point where the tower fails are merely transmitted to the foundation and do not contribute to failure of the tower. However, every tower failure will be different and so the actual proportion of the tower loads which contribute to failure will be random. In this paper it will be assumed that there exists a fixed point above which tower loads contribute to failure and below which they do not. It was observed that the tower typically failed at about half-height, as suggested in Fig. 2. Therefore the fixed failure point will be considered to be at the tower half-height.

Fig. 3 illustrates the relationship between the ultimate horizontal resistance of the tower, R_H, and vertical applied load, V, as determined by the finite element analysis described above. The black dots are the (V, R_H) combinations at which tower failure just occurs. The numerically determined points in Fig. 3 demonstrate a bilinear relationship between horizontal and vertical capacities. Once the vertically applied load exceeds about 500 kN, the horizontal resistance capacity of the tower falls off rapidly. This suggests that the following bilinear relationship can be used to predict the deterministic ultimate capacity for this particular tower

$$R_H = \begin{cases} 179.2 - 0.1107V, & \text{for } 0 < V \leq 500 \text{ kN} \\ 1012 - 1.772V, & \text{for } 500 < V \leq 570\text{kN} \\ 0, & \text{otherwise} \end{cases}. \quad (5)$$

The above bilinear curve was fit to the finite element results, as shown by the solid line in Fig. 3, and it clearly shows an excellent agreement with the finite element results. Failure of the tower is now assumed to occur if the combination of vertical and horizontal loads lies above the ultimate capacity curve depicted in Fig. 3.

If a tower were to be constructed in the field and tested to destruction, there is little question that due to material and construction variability, its strength would be somewhat different than that suggested by the finite element results of Fig. 3 and (5). In other words, the actual strength of towers constructed in the field will be random and should be modeled as such. As in a typical regression analysis (despite the lack of actual experimental data in this paper) the R_H component of the tower resistance will be taken as random, with mean given by (5). Because full-scale experimental data on tower strength is lacking, the slope variability, along with the variability in the cut-off point (500 kN in Fig. 3), are unknown. It will be assumed for simplicity here that the response of R_H in the range $V > 500$ kN is non-random and that for $V \leq 500$ only the intercept (having mean 179.2 kN) is random. That is, the bilinear relationship between the horizontal and vertical tower resistance will be assumed to be

$$R_H = \begin{cases} Y - 0.1107V, & \text{for } 0 < V \leq 500 \text{ kN} \\ 1012 - 1.772V, & \text{for } 500 < V \leq 570\text{kN} \\ 0, & \text{otherwise} \end{cases} \quad (6)$$

where Y is assumed to be lognormally distributed with mean 179.2 kN and coefficient of variation selected based on engineering judgment to be 0.2 [8].

Over time, the tower strength will degrade due to corrosion, fatigue, and lack of proper maintenance. For simplicity, and due to the lack of information about how steel lattice towers degrade with time, it will be further assumed that loss of structural capacity results in a reduction in the parameter Y in (6). The value of Y will be replaced by Y_i for each year $i = 0, 1, 2, \ldots, n_y$, where n_y is the design lifetime, according to

$$Y_i = Y f^i. \quad (7)$$

The value $Y_0 = Y$ is the initial horizontal tower strength intercept and

$$f = \exp\left\{\frac{1}{n_y}\ln(1 - s_{\text{loss}})\right\} = (1 - s_{\text{loss}})^{1/n_y} \quad (8)$$

is the fractional annual loss in tower strength. The parameter s_{loss} is the total fractional strength loss over the design life, n_y. Weathering steel was found to lose somewhat less than 20% thickness under worst case condition over 60 years by [9]. On the basis of their study, the parameter s_{loss} will be taken to be 0.2 in this paper, which is probably conservative if weathering steel is used in the towers over a 50 year design life, and may not be if other types of steel are used (e.g., galvanized, depending on the galvanizing thickness and quality of application, especially at connections).

IV. Tower Loads

A. Wind

The total horizontal load applied to the transmission tower arises from wind loads on the conductors and on the tower itself.

It will be assumed conservatively that all winds act horizontally in a direction perpendicular to the conductor span.

Hourly windspeeds (km/hr), without consideration of direction, were obtained from Environment Canada [10] at the Halifax, Nova Scotia, airport from January 1961 to April 2008. The hourly windspeeds are based on 2-min average windspeeds recorded once per hour. As far as the authors are able to determine, the mean gust duration which results in a tower failure is unknown. However, considering typical conductor spans, it is reasonable to assume that a 2-min duration extreme wind is sufficient to transmit all extreme loads to the tower and result in failure if the 2-min average wind load exceeds the tower capacity.

The hourly 2-min windspeed data were then analyzed to extract the daily maximum 2-min average windspeeds over the 1961–2008 period. Although the hourly 2-min windspeeds could be modeled and simulated directly, there is significant temporal correlation (i.e., if the windspeed is high in one hour, it is very likely to be still high in the next hour), which would complicate the simulation. To simplify the following analysis it will be assumed that the daily maximum 2-min windspeeds are independent. The estimated correlation coefficient between daily maximum 2-min average windspeeds drops rapidly with time at Halifax, and is only about 30% after one day, and negligible thereafter. The assumption of independence between daily maximum windspeeds is reasonable and conservative (i.e., independence leads to slightly higher probabilities of failure).

The daily maximum (2-min average) windspeeds can be seen in Fig. 4 to be approximately lognormally distributed. A chi-square goodness-of-fit test rejected the hypothesis that the windspeed distribution is truly lognormal at the 5% significance level, but this is not surprising for such a large sample size—the period from 1961 to 2008 contains more than 17 000 days. With such a large sample, very small discrepancies between the empirical and fitted distributions become significant. The results depicted in Fig. 4 nevertheless suggest that a lognormal distribution is reasonable and will be adopted in this study.

The mean maximum (2-min average) daily windspeed was estimated to be 28.94 km/hr with a standard deviation of 9.53 km/hr at the Halifax airport, with corresponding lognormal distribution parameters $\mu_{\ln W}$ and $\sigma_{\ln W}$ estimated to be 3.31 and 0.321, respectively. These statistics will be assumed to apply over the entire tower transmission line being considered due to the persistence of large scale weather patterns (or, at least, larger scale than the transmission line length).

The horizontal wind load, H, acting on the tower and its supported conductors is calculated by multiplying the cross sectional area of the tower members and conductors (including ice) by the wind pressure, q_o. The wind pressure, N/m^2, is related to windspeed through the relationship [11]

$$q_o = 0.05\rho W^2 \tag{9}$$

where ρ is the air mass density (1.293 kg/m^3 at 0° C) and W is the windspeed (km/hr).

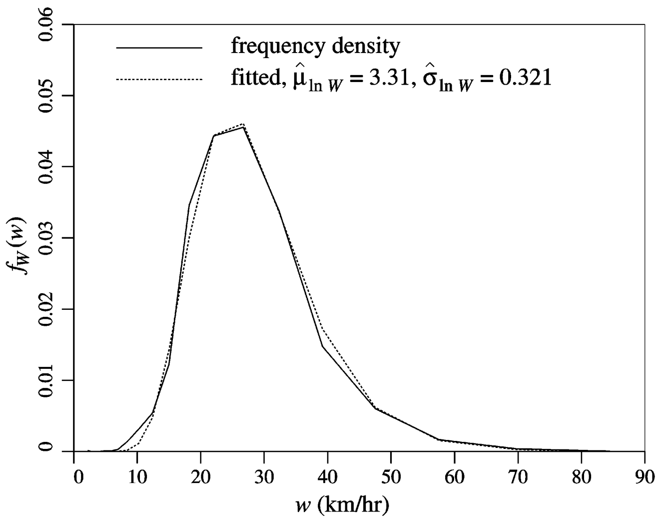

Fig. 4. Frequency-density plot of daily maximum 2-min average windspeed at Halifax airport with superimposed lognormal fit.

B. Ice

Ice accumulation on a tower and its conductors is difficult to measure and quantify. Ice accretion amounts are usually determined using one of various ice accretion models. Reference ice loads in CSA 22.3 No. 1-01 for Overhead Systems [11] are estimated using the Chainé and Castonguay [12] ice accretion model which employs precipitation amounts in designated geographical areas to determine an equivalent radial ice thickness defined as the probable ice thickness accretion around a conductor.

The Chainé and Castonguay model was used to calculate ice accretion thickness at the Halifax International Airport for ten historical ice storms over a ten-year period [12]. From this limited set of estimated ice thicknesses, the mean ice thickness on conductors and tower elements during an ice storm was estimated to be 13.3 mm. Such a small sample size does not allow for a reasonable estimation of the complete ice thickness distribution, but since ice thickness cannot be negative and there is no arbitrary upper bound, it will be assumed here conservatively that the ice thickness during each storm is lognormally distributed. Estimates used to determine the 50-year return period ice distribution [13] suggest a large coefficient of variation of ice thickness of 50% and so an ice thickness standard deviation of 6.6 mm will be used in this study.

The ice layer is assumed to accrete evenly on all exposed surfaces and any irregularities due to the wind, gravity, and exposure are ignored. Fig. 5 shows a cross-section of a typical tower angle member (left) and a conductor (right) with an accreted ice layer of thickness T.

The ice is assumed to have force density $r_i = 8996$ N/m^2. The average cross-sectional area of ice surrounding a tower member, A_i, is given by

$$A_i = T\left[4b_{\text{ave}} + T\left(\frac{5}{4}\pi - 1\right)\right] \tag{10}$$

where T is the random ice thickness and b_{ave} is the average flange length of all tower members above the tower failure point.

Fig. 5. Ice accretion—typical tower member on left and conductor on right.

In (10), all tower members are assumed to be angle members having cross-section as shown on the left of Fig. 5. The total vertical ice load on the tower, V_{si}, above the failure point, is thus

$$V_{si} = r_i L_s A_i \tag{11}$$

where L_s is the total length of structural members above the failure point.

The vertical weight of ice on a single conductor is

$$V_{ci} = \pi T(d+T)Lr_i \tag{12}$$

where d is the conductor diameter and L is the conductor span (distance between towers).

C. Total Vertical Load

Given the ice loads specified above, the total weight of the iced structure above the failure point, is obtained by combining (11) with the tower self-weight

$$\begin{aligned} V_s &= V_{\text{steel}} + V_{si} \\ &= V_{\text{steel}} + r_i L_s T\left[4b_{\text{ave}} + T\left(\frac{5}{4}\pi - 1\right)\right] \end{aligned} \tag{13}$$

where V_{steel} is the total weight of the structural members making up the tower above the failure point. The total vertical load contributed to the tower by a single conductor V_c can be computed by combining (12) with the self-weight of the conductors

$$V_c = L[V_{\text{cond}} + r_i \pi T(d+T)] \tag{14}$$

in which V_{cond} is the unit length conductor weight and L is the span between towers.

Finally, the total vertical load applied to the tower above the failure point is

$$V = n_c V_c + V_s \tag{15}$$

where n_c is the number of conductors (including the ground wire).

D. Total Horizontal Load

The total horizontal load is the product of the extreme wind pressure and the projected cross-sectional area of the iced conductor and the structural portions of the tower above the failure point. It will be assumed that two faces of the tower are exposed to the wind and that the leading face does not significantly shelter the downwind face. If the extreme windspeed during the ice storm, W, is used in (9) to compute q_o, the total horizontal load on the tower is computed as

$$H = q_o[n_c L(d+2T) + 0.5L_s(b_{\text{ave}} + 2T)] \tag{16}$$

where n_c is the number of conductors (including the ground wire), d is the conductor diameter (assumed to be the same for the ground wire, which is generally conservative), T is the ice thickness, L is the span between towers, and L_s is the total length of all tower members above the failure point, each having average flange width b_{ave}.

V. Failure Consequences

For applied vertical load, V, computed using (15), the ultimate resistance of the tower to horizontal load, R_H, can be computed using (6). If the horizontal load, H, computed using (16) exceeds R_H the tower is assumed to fail. Tower failure results in a replacement and an outage cost.

The initial cost of constructing a tower, C_{tow}, was estimated to be approximately \$250 000.00. The cost of replacing a failed tower, C_{rep}, which includes tower removal and clean up is estimated to be approximately \$400 000. Maintenance costs, C_{mnt}, were estimated at approximately \$1625 per year per tower [14]. Since the risk assessment will be performed using simulation, it is relatively easy to include the time value of money and compute all costs in present day dollars. Note, however, that ignoring the time value of money would be conservative, since the present day value of future costs is always less than or equal to the future cost, i.e., ignoring the time value of money leads to higher present day costs.

Outage costs were estimated from a study completed by the Power System Research Group at the University of Saskatchewan [15]. The study was based on customer surveys implemented to determine short term impacts of power outages. Residential, commercial and industrial customers were included in the survey. Power interruption costs for each customer category were inflated to 2008 dollars and multiplied by an estimate of the number of residential, commercial and industrial customers located in a downtown urban area such as Halifax, Nova Scotia. In this study the total power interruption (outage) cost, C_{out}, to consumers (all categories combined) was assumed to be \$840 000 per hour that the power is interrupted.

Outage duration was assumed to be random with lognormal distribution. If a tower fails during an ice storm, it was assumed that the mean outage time is 8 hours with standard deviation 2 hours [16]. Since outage time typically increases as the magnitude of the damage to the entire transmission line increases, it is reasonable to assume that outage time will increase with the number of towers failed by the storm. The following relationship between number of failed towers, N_{f_i}, in the ith year and total outage time in the same year, T_{out_i}, was assumed in this study

$$T_{\text{out}_i} = \sum_{j=1}^{N_{f_i}} \frac{T_j}{j} \tag{17}$$

where $T_1, T_2, \ldots$ are random independent lognormally distributed outage times, each having mean 8 hr and standard deviation 2 hr. If one tower fails along the line ($N_{f_i} = 1$) then the outage time is T_1 with mean 8 hours. However, if two towers fail ($N_{f_i} = 2$) then the *additional* outage time due to failure of the second tower is reduced to $T_2/2$ (with mean 4 hr) because repair crews will already be mobilized and temporary structures will be being put into place. Similarly, for a higher number of failed towers during the same storm, more repair crews will be mobilized and the additional outage time per failed tower will be reduced.

The authors note that little evidence, beyond discussions with Nova Scotia Power [14], was available in the literature regarding the distribution of outage times and its relationship with the severity of damage to the transmission line. The outage time distribution suggested above is based on reasonable engineering judgment only.

The total cost of transmission line failure in the i'th year of service is given by

$$C_i = N_{f_i} C_{\text{rep}} + T_{\text{out}_i} C_{\text{out}}. \tag{18}$$

Note that the number of failed towers in the i'th year of service, N_{f_i}, is a random variable. The number of failed towers depends on whether the ultimate horizontal capacity (6) is exceeded at each tower along the line for the given vertical and horizontal loads during the assumed annual ice storm.

The total cost of the transmission line over its lifetime is given by

$$C = n_t(C_{\text{tow}} + n_y C_{\text{mnt}}) + \sum_{i=1}^{n_y} C_i \tag{19}$$

where n_t is the number of towers supporting the transmission line connecting the generating station with customers and n_y is the design life of the transmission system in years. If the current value of money were to be taken into account, at constant interest rate p, (19) would become

$$C = n_t C_{\text{tow}} + \sum_{i=1}^{n_y} (n_t C_{\text{mnt}} + C_i)(1+p)^{-i}. \tag{20}$$

Minimizing the expected total cost is a goal of a reliability-based design, along with ensuring an acceptable probability of failure. The expected value of (20) is

$$\text{E}[C] = n_t C_{\text{tow}} + \sum_{i=1}^{n_y} \{n_t C_{\text{mnt}} + \text{E}[C_i]\}(1+p)^{-i}. \tag{21}$$

The calculation of the expected value of C_i is complicated by the fact that the number of towers that fail in the i'th year of service, N_{f_i}, and the corresponding outage time, T_{out_i}, are both random variables which are dependent (see (17)). Because the joint distribution between N_{f_i} and T_{out_i} is unknown, and because the ultimate tower resistance, R_H, is random and dependent on a random vertical applied load, V, the expected total cost (21), as well as the lifetime probability of transmission line failure, will be obtained using simulation, as mentioned earlier and described in the next section.

VI. Simulation

To illustrate the reliability-based design approach proposed in this paper, a simple hypothetical example of a 20-km-long transmission line supplying power to a small, urban area (approximately the size of downtown Halifax, Nova Scotia) will be considered. Many of the parameters of this example have already been suggested above. The target design life of the transmission system will be $n_y = 50$ years. The design decision variable will be the span length between towers and the optimum span length will be determined by considering the total expected cost and the probability of failure of each possible span length based on the structural resistance of the tower. If the transmission line is constructed on level terrain with no obstructions, the span length will be approximately the same for all towers. Otherwise the optimum span length suggested by this analysis may be considered conservatively to be the maximum distance between towers. Optimizing a design based on unequal spans along the line is certainly possible via simulation, involving the simulation of various proposed tower placement schemes and selecting the design with the lowest total expected cost or which achieves an acceptable lifetime reliability. Only horizontal loads acting perpendicular to the conductor spans are considered, and so the effect of longitudinal tension loads, dead-ends, and direction changes are ignored. All towers are assumed to have the same distribution of structural resistance to the applied horizontal loads. If dead-end towers (designed to support differing longitudinal tension loads) are stronger laterally, then this is a conservative assumption.

It is assumed that one ice storm will occur each year during the assumed 50 year tower life span. Storm duration (length of time that the tower structure and supported conductors are covered with ice) is assumed to be two days and nonrandom. The ice storm frequency and duration assumptions are, admittedly, a weak point of the analysis, since ice storms may or may not occur annually, may occur more than once in a year, and may last longer than two days. If it is assumed that, on average, the region under investigation will have 50 ice storms in 50 years (which may be conservative given global warming) then the assumption of one storm per year is a reasonable approximation. In addition, ice storms have random duration (e.g., the ice storm in northeastern North America that struck in the winter of 1998 lasted for six days). Ice storm duration could be randomized, but no information on the distribution of ice storm duration was found in the literature. A fixed duration of two days was decided upon for the purposes of this paper, noting that the ice storm duration only affects the number of independent maximum daily windspeeds seen by the towers. The ice thickness is simulated independently of the storm duration. For the assumed storm duration, the tower and conductors will experience two daily maximum wind loadings, based on 2-min averages, as discussed in Section IV.A. It will be conservatively assumed that the maximum ice thickness is present throughout the duration of the storm.

The simulation will estimate the expected total lifetime cost of the transmission line [see (21)] and the probability of failure, p_f. The probability of failure is defined as the probability of one or more towers failing sometime in the lifetime of the system. Once a single tower has failed, the system is assumed to have failed its lifetime design objective, and further failures in the same lifetime do not further contribute to the failure probability estimate.

Altogether the simulation will consider four random variables. These are:

- the lognormally distributed maximum daily windspeed—it is assumed that the scale of the storm is considerably larger than the 20 km length of the transmission line, so that the same winds are assumed to applied to the entire line. In other words, it is assumed that each tower along the line will see the same daily maximum 2-min average windspeed—this is a conservative assumption.
- the lognormally distributed ice thickness—the same ice thickness is assumed to apply to all towers and conductors along the line for the same reason given for the wind.
- the tower's ultimate horizontal resistance intercept, Y, [see (6)]—each tower in the line is assumed to have independent random ultimate horizontal resistance.
- the lognormally distributed outage duration, given one or more tower failures, during each ice storm [see (17)].

The conductor span is varied in a series of steps from a minimum span of 100 m to a maximum span of 1400 m for demonstration purposes. For each span length, the simulation proceeds as follows:

A) The number of towers is computed using the length of the transmission line divided by the conductor span: $n_t = 1 + \mathrm{int}(D_{\mathrm{src}}/L)$ where L is the conductor span, 100 m, 200 m, etc. and D_{src} is the distance from source to destination (in this case 20 000 m).

B) Initialize the total lifetime cost, C, of the line to $n_t C_{\mathrm{tow}}$ and initialize the count of the number of lifetime failures, j_f, to zero.

C) Assuming tower strengths to be independent, simulate a lognormally distributed ultimate horizontal resistance intercept, Y, for each tower [see (6)].

D) For each year $i = 0, 1, \ldots, n_y$, over the lifetime of the system, perform the following steps.
   1) Increment the total lifetime cost, C, by $n_t C_{\mathrm{mnt}}(1+p)^{-i}$
   2) Simulate one storm, assumed to act on the entire transmission line equally. This step involves the simulation of a lognormally distributed ice thickness and two independent lognormally distributed maximum daily windspeeds for each day of the storm. The maximum of these two windspeeds is applied to the transmission line.
   3) Compute the total horizontal and vertical loads acting on the towers based on the simulated wind and ice thickness in the previous step [see (15) and (16)]. The same loads are applied to all towers along the line.
   4) Decrement the tower resistance according to (7).
   5) For each tower along the line, apply the simulated vertical load, V, and check if the applied horizontal load, H, exceeds the ultimate tower resistance, R_H, computed by (6) for the tower under consideration. If $H > R_H$, the tower is assumed to fail. Count the number of failed towers, n_f, along the line. Increment the total lifetime cost, C, by $n_f C_{\mathrm{rep}}(1+p)^{-i}$
   6) If $n_f > 0$, simulate an outage time according to (17) and increment the total lifetime cost, C, by $T_{\mathrm{out}_i} C_{\mathrm{out}}(1+p)^{-i}$.
   7) If $n_f > 0$ *for the first time* in this lifetime analysis, increment the probability counter, j_f, by 1.

E) Repeat from step B n_{sim} times, where n_{sim} is the number of simulations performed for each span length considered (20 000 in this paper). Averaging the total lifetime costs determined over the n_{sim} realizations produces an estimate of the expected total lifetime costs. The failure probability is estimated as j_f/n_{sim}.

F) Repeat from step A for the next conductor span under consideration.

Following the above simulation, plots of span length versus probability of failure and expected total lifetime cost can be produced. The span length corresponding to the lowest point in the expected total lifetime cost curve is desirable, so long as it is not accompanied by an unacceptable failure probability nor any other unacceptable design requirement (e.g., other limit states).

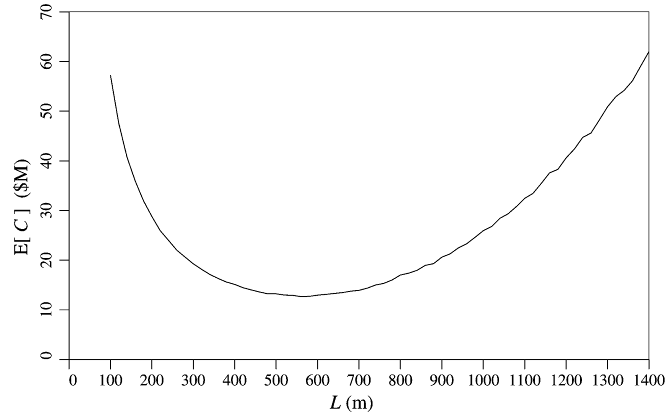

Fig. 6. Expected total lifetime cost versus span length.

VII. Results and Conclusions

Fig. 6 shows how the total expected cost varies with span length for this particular case study. All parameters used in the simulation can be found in Appendix A. Initially, the total expected cost falls rapidly as the span length increases. In this region, the rapid reduction in number of towers required as span length increases outweighs the loss in system reliability. However, the costs do begin to climb again as the system failure probability starts to become significant and outage costs climb. Fig. 7 shows how the lifetime failure probability, defined as the probability that one or more towers fail sometime during the 20 km line's lifetime (50 years), increases as the span length increases.

If the design were to proceed based purely on minimizing the expected total cost, then the optimum span length is suggested by Fig. 6 to be 560 m, having a expected total cost of 12.7 $ M. However, the lifetime failure probability corresponding to $L = 560$ m is $p_f = 0.26$ which may be considered to be too high. To put this lifetime failure probability in perspective, the *annual* target failure probabilities of 0.01 to 0.001, as suggested

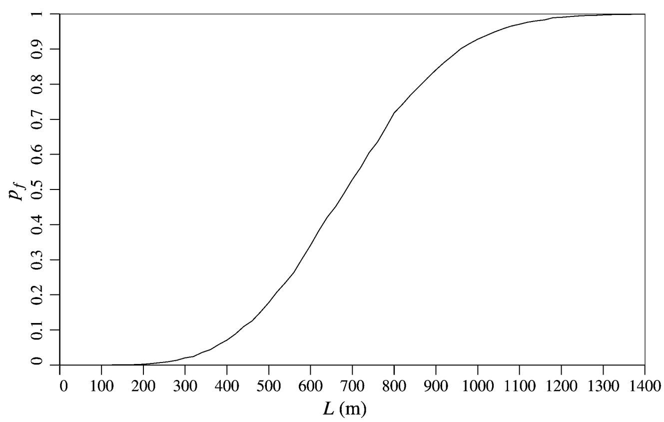

Fig. 7. Lifetime system failure probability versus span length.

by Ghannoum [4], [5], correspond to 50-year lifetime failure probabilities of 0.395 to 0.049 (assuming each year constitutes a Bernoulli trial). Thus, a lifetime failure probability of 0.26 may actually be quite reasonable.

On the basis of Fig. 7, if the target annual failure probability of 0.01 were desired (lifetime $p_f = 0.395$), then the span could be increased to 630 m, with corresponding increase in total expected cost to 13.2 \$ M. Alternatively, if the target annual failure probability of 0.001 were preferred (lifetime $p_f = 0.049$), then the span length would have to be reduced to 370 m, with expected total cost of 16.0 \$ M.

It is, of course, well known that design decisions, such as span length, are not only dependent on a single limit state, such as the horizontal structural resistance of the transmission towers. In general other limit states influence the design decision. For example, in the case of span length, the structural resistance of the supporting towers might be considered an ultimate limit state. Tensile failure of the conductors might also be considered an ultimate limit state. Excessive sag of the conductor between towers might be considered a serviceability limit state (in that this would not necessarily entail service failure, but might be just as debilitating in the long run). In the case considered here, the maximum span suggested by sag calculations, is estimated to be about 315 m (this is yet another risk-based design decision that needs further investigation).

Although the above "ultimate limit state" risk-based analysis is less useful in the event that another "limit state" takes precedence, the basic idea applies to all limit states under consideration. If the span length is governed by another limit state, plots such as Figs. 6 and 7 can be used to match towers to the governing span length if similar plots are produced over a variety of tower designs.

The significant advantage to simulation-based plots of expected total cost and failure probability, against the design parameter under investigation (regardless of the limit state), such as shown in Figs. 6 and 7, is that these plots allow the explicit consideration of the tradeoff between cost and reliability. For example, even if sag does govern the maximum span, the above plots can still be used to estimate tower structural reliability and expected system cost. The owner, designer, and public can use plots such as these to make informed decisions about how to achieve transmission line reliability and the cost of achieving such target reliabilities.

It is recognized that the design parameter considered in this paper, the span length, is just one of thousands that go into the design of a transmission line. The simulation itself is also somewhat idealized (e.g., constant span length between towers, wind and ice loads the same on all towers, a fixed failure point, all towers assumed to have the same resistance distribution, etc.). The results given in Figs. 6 and 7 should not be assumed to hold true for the general case and should primarily be viewed as part of a methodology. This simulation-based methodology is relatively easily altered, at least conceptually, to consider:

- other design parameters, such as the tower resistance, penalties due to conductor sag, etc.;
- more complicated load types and their spatial/temporal distributions (e.g., the effect of terrain on local wind gust magnitude, spatial variability of climatic loads, the effect of climate change on ice loads, etc.);
- more realistic tower spacing, including the effects of dead-ends;
- the effect of longitudinal tension forces and their possible unequal loss.

The algorithms given in this paper can be considered another step towards a fully reliability-based design of power transmission systems. The simulation software used in this paper is written in Fortran on a Linux platform and is freely available at www.engmath.dal.ca/tower.

APPENDIX I

See Tables I–VI below.

TABLE I
TOWER AND TRANSMISSION LINE PARAMETERS

Parameter	Value	Description
L_s	305.6	total length of all members above failure point (m)
d	0.027762	conductor diameter (m)
b_{ave}	0.0698	average member flange length (m)
V_{steel}	20500	weight of tower steel above failure point (N)
V_{cond}	14.9	unit weight of 27.762 mm conductor (N/m)
n_c	7	number of conductors (including ground wire)
s_{loss}	0.2	lifetime tower strength fractional loss
D_{src}	20000	length of transmission line (m)

TABLE II
TOWER STRENGTH PARAMETERS

Parameter	Value	Description
μ_Y	179200	mean horizontal resistance intercept (N)
σ_Y	35840	standard deviation of horizontal resistance intercept (N)
s_1	-0.1107	initial tower $R_H - V$ resistance slope
V_1	500000	vertical load breakpoint in tower $R_H - V$ resistance (N)
b_2	1012000	intercept of second $R_H - V$ tower resistance line (N)
s_2	-1.772	slope of second $R_H - V$ tower resistance line (N)
$V_{\max}$	570000	maximum vertical load at $R_H = 0$ (N)

TABLE III
OUTAGE DURATION STATISTICS

Parameter	Value	Description
μ_{out}	8	mean outage duration for a single tower failure (hours)
σ_{out}	2	standard deviation of outage duration (single tower) (hours)

TABLE IV
WIND STATISTICS

Parameter	Value	Description
ρ	1.293	air mass density (kg/m^3)
μ_W	28.94	mean daily max two minute average windspeed (km/hr)
σ_W	9.53	standard deviation of daily max two minute average windspeed (km/hr)

TABLE V
ICE STATISTICS

Parameter	Value	Description
n	2	number of effective days each ice storm lasts
r_i	8995.8	weight of ice (N/m^3)
μ_i	0.01326	mean ice thickness during storm (m)
σ_i	0.00663	standard deviation of ice thickness (m)

TABLE VI
COSTS

Parameter	Value	Description
C_{tow}	0.25	cost of constructing a new tower ($M)
C_{mnt}	0.001625	maintenance cost per tower per year ($M)
C_{rep}	0.4	cost of replacing a tower ($M)
C_{out}	0.84	outage cost per hour ($M/hr)
p	0.04	annual interest rate

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