A method to assess risk reduction when utilizing geosynthetic clay liners (GCLs) with compacted soil liners

W.T. Menzies, G.A. Fenton, C.B. Lake, and D.V. Griffiths

Abstract: This paper presents an analytical solution developed to estimate probabilities of "failure" or advective flux "exceedance" for the case of a spatially variable geosynthetic clay liner (GCL) situated over a spatially variable compacted soil liner (CSL). The risk of utilizing such a liner system is assessed relative to a regulatory compacted clay-based soil liner. The analytical solution developed is validated over a range of parameters against random field simulation using the Local Average Subdivision method, and the analytical solution is shown to be in good agreement with simulation. The analytical solution is then used to examine the "probability of exceedance" for a spatially variable GCL and CSL combined liner system. It is shown that the use of a GCL can potentially result in a low probability of exceedance when used with a spatially varying, high hydraulic conductivity CSL. The risk of exceedance generally decreases as the hydraulic conductivity of the CSL decreases. An example problem is presented to demonstrate the capabilities of the analytical solution.

Key words: Monte Carlo simulation, spatial variability, compacted soil liner, geosynthetic clay liner, barrier.

Résumé : Cet article présente une solution analytique développée afin d'estimer les probabilités de « rupture » ou d'un « dépassement » des flux advectifs dans le cas d'un revêtement géosynthétique avec argile (GCL) ayant une variabilité spatiale et placé sur un revêtement de sol compacté (CSL) ayant aussi une variabilité spatiale. Le risque associé à l'utilisation d'un tel système de revêtement est évalué en comparaison avec un revêtement en sol fait d'argile compactée selon les normes actuelles. La solution analytique développée est validée par une gamme de paramètres, à partir de simulations aléatoires de terrain obtenues avec la méthode « moyenne locale de subdivision ». La solution analytique correspond bien aux simulations. La solution analytique est ensuite utilisée pour étudier la « probabilité de dépassement » d'un système de revêtement combiné GCL et CSL ayant une variabilité spatiale. Il est démontré que l'utilisation d'un GCL résulte potentiellement en une faible probabilité de dépassement lorsqu'utilisé avec un CSL présentant une variabilité spatiale et une conductivité hydraulique élevée. De façon générale, le risque de dépassement diminue lorsque la conductivité hydraulique de CSL diminue. Un exemple de problème est présenté afin de démontrer les capacités de la solution analytique.

Mots-clés : simulation de Monte Carlo, variabilité spatiale, revêtement de sol compacté, revêtement géosynthétique avec argile, barrière.

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Introduction

Numerous factors can influence the overall hydraulic conductivity, k, of intact compacted clay liners (CCLs). These factors may include natural variability in soil composition (e.g., grain size, Atterberg limits, and moisture content) and variation in compaction energy and techniques (Benson et al. 1994). Although much of the variability associated with hydraulic conductivity can be reduced by proper quality control and quality assurance programs, sometimes a borrow source of material will be "rejected" from consideration for use as a CCL even though it is actually acceptable. One example of this occurs when a portion of a borrow source is rejected because the variance of the material is considered too high and poses too much of a risk, even though the mean hydraulic conductivity of the borrow source material is less than a predefined regulatory value (e.g., 1×10^{-9} m/s). Another potential example leading to "rejection" of a portion of a borrow source occurs when the mean hydraulic conductivity of the material is above the regulatory value (e.g., 1×10^{-9} m/s). Regardless of the reason for rejection, the decision could require the transport of acceptable clay to the site from further distances, which will ultimately result in higher capital costs for a construction project. There is thus an economic advantage to being able to use these rejected soils, perhaps in combination with a geosynthetic clay liner (GCL) to reduce the hydraulic conductivity to acceptable levels. The economic advantage is especially pertinent if the regulatory agency requires a minimum thickness of the compacted soil liner (CSL) under the GCL.

Geosynthetic clay liners are commonly used to reduce the hydraulic flux through barrier systems. The GCLs are a manufactured product, with plant manufacturing quality control, which are hence subject to fewer construction irregularities than CSLs. The GCL variability (in terms of k) is thus

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usually less than that of a constructed CSL. Figure 1 illustrates how the hydraulic conductivity distribution of a CSL (Fig. 1*a*) compares to that of a combined GCL–CSL system (Fig. 1*b*). The combined GCL–CSL is composed of a GCL (mean k of GCL, $\mu_{k_G} = 1 \times 10^{-11}$; coefficient of variation of GCL, $\upsilon_{k_G} = 0.1$) overlying a CSL (mean k of CSL, $\mu_{k_C} = 9.0 \times 10^{-10}$; coefficient of variation of CSL, $\upsilon_{k_C} = 2.0$). The significantly reduced variability of the combined system seen in Fig. 1*b* is due to the small variability of the GCL.

Cross-sections through a CSL and a combined GCL–CSL system are shown in Figs. 2*a* and 2*b*, respectively. Note that the GCL is normally very thin compared to the CSL, and the overall thickness of both liners is approximately the same as the CSL alone. If one considers the probability distribution of k for the CSL in Fig. 2*a*, as shown in Fig. 1*a*, it can be seen that the average hydraulic conductivity of the CSL may be at or below the regulated value, assumed here to be 9.0×10^{-10} m/s, but there will be a probability that some portions of the CSL system are above the regulated value, as shown by the hatched area of Fig. 1*a*. For situations where there is a possibility that the regulatory hydraulic conductivity may be exceeded because of variability in the constructed CSL, placing a GCL over the CSL (see Fig. 2*b*) may significantly reduce the probability that flow will exceed the specified regulatory value. Although materials such as sand–bentonite liners can be utilized to meet hydraulic conductivity requirements, the use of GCLs in many situations is cost effective. To meet hydraulic conductivity requirements, GCLs over CSLs are now used in at least five of the seven operating second-generation landfills in the province of Nova Scotia. In these landfills, regulations require a minimum 1 m thick primary barrier system, and hence the GCL is placed over 1 m of native recompacted tills (CSL). The first "generation" cells of these landfills used sand–bentonite liners. The switch from sand–bentonite to GCLs has been mainly due to cost. For example, the use of GCLs resulted in a cost savings of approximately CAN\$15/m^2 (CAN\$790 000) for Cell 5 of the Otter Lake Landfill in Halifax, Nova Scotia (HRM 2008). Although the cost savings will be different in different areas of the world, the Nova Scotia experience highlights that GCLs can beneficial in terms of cost in some regulatory environments.

Simple hydraulic flux calculations can be performed with deterministic values of hydraulic conductivity to decide on the use of a GCL–CSL barrier depicted in Fig. 2*b*; however, there are currently no methods available to assess the level of risk associated with this type of system involving GCLs. Benson et al. (1994, 1999) have published work related to the influence of hydraulic conductivity variability on the field performance of noncombined clayey soil liners. However, there is a need to develop a risk-based analysis to provide some quantitative assessment of the level of risk that exists when using spatially variable CSLs and GCLs combinations in practice.

The main purpose of this paper is to develop a set of relatively simple analytical solutions that can be implemented into a spreadsheet to calculate the "probability of exceedance" of a GCL–CSL combined liner. In this paper, "exceedance" is defined as the condition that total steady-state flow through the liner, Q, is greater than the total steady-state flow through a regulatory liner, Q_R, e.g., through a 1 m thick liner with hydraulic conductivity of 1×10^{-9} m/s. The paper presents the theory leading up to the probabilistic analyses and verifies the analytical solutions with simulation using the Local Average Subdivision (LAS) (Fenton and Vanmarcke 1990) performed via the program mrflow3d (Griffiths and Fenton 1997). Insight is provided on how different practical factors, such as the mean and variability of hydraulic conductivity of the CSL and GCL, the thickness of the CSL, and the liner area, influence the probability of exceedance. Results are presented in the context of potential benefits of utilizing a GCL to reduce the probability of exceedance of the compacted soil liner.

Theory

Analytical solution

To assess the level of risk associated with using GCLs in combination with CSLs, one can develop an analytical solution that allows a comparison of the probability that flow through a GCL–CSL system will exceed that of some minimum regulatory CSL. Most CSLs for containment applications are constructed to a deterministic maximum hydraulic conductivity specification of 1×10^{-9} m/s as well as to some specified minimum thickness, H_C. The flux of water through a CSL is given by Darcy's Law, assuming flow in the z direction only:

$$[1] \qquad v_{az} = k_C i_z = k_C \frac{\Delta h_z}{H_C}$$

where v_{az} is the advective flux through the soil in the z direction; k_C is the hydraulic conductivity of the CSL (initially assumed deterministic); i_z is the gradient in the z direction (assumed deterministic); Δh_z is the difference in total head across the barrier (assumed deterministic); H_C is the thickness of the CSL in z direction.

Equation [1] essentially describes the flux of water through the CSL barrier in Fig. 2*a*.

For a combined liner system such as that shown in Fig. 2*b*, the effective hydraulic conductivity is the harmonic average of the individual layer hydraulic conductivities, assuming flow lines remain largely perpendicular to the layers. Hence, the saturated flow through the two-layer system shown in Fig. 2*b*, assuming flow is predominately in the z direction, can be calculated (e.g., Rowe et al. 2004) as

$$[2] \qquad v_{az} = \bar{k}\bar{i}_z = \bar{k}\frac{\Delta h_z}{H}$$

where $H = H_G + H_C$ is the total liner thickness in which H_G is the thickness of the GCL in the z direction, $\bar{i}_z$ is the total hydraulic gradient across the combined liner system, $\Delta h_z/H$, and $\bar{k}$ is the harmonic average hydraulic conductivity of the GCL and CSL. The harmonic average is calculated as (Terzaghi 1944):

$$[3] \qquad \bar{k} = \frac{H_G + H_C}{(H_G/k_G) + (H_C/k_C)}$$

where k_G is the hydraulic conductivity of the GCL.

Equation [2] can be used to calculate the flux of water through the GCL–CSL combined system shown in Fig. 2*b*.

Fig. 1. Probability density function of (*a*) CSL k and (*b*) combined GCL–CSL $\bar{k}$.

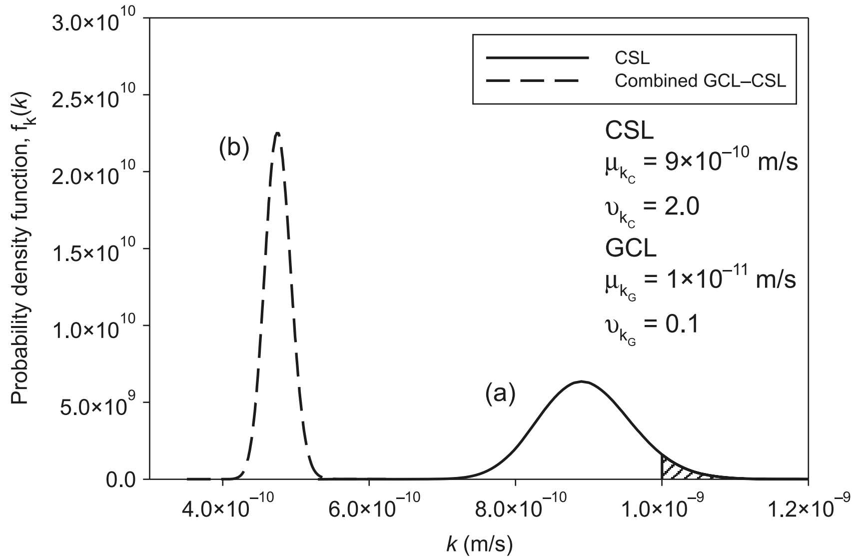
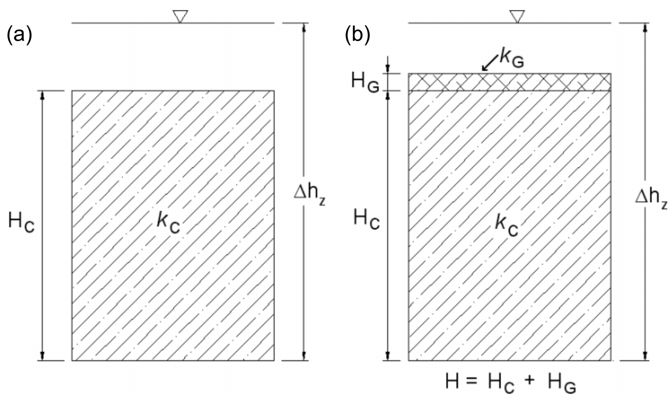


Fig. 2. Diagram of (*a*) CSL and (*b*) combined GCL–CSL.



As presented, both eqs. [1] and [2] assume deterministic values of the hydraulic conductivity for each of the barrier systems, and thus these equations do not allow us to directly assess risk associated with the barrier system presented in Fig. 2*a*, nor the potential reduction in risk by using the barrier system in Fig. 2*b*.

In this paper, the exercise will be to examine if the total advective flow, Q, through the barriers shown in Figs. 2*a* or 2*b* exceeds a predetermined regulatory value, Q_R. In most practical cases, the thickness of the regulatory barrier system, H_R, is written into the regulation, and it is the hydraulic conductivity of the constructed barrier that is the main uncertainty. The probability of exceedance, $P(E)$ of the barrier, relates to the probability that flow will be greater than Q_R. Probability of exceedance is defined as

$$[4] \qquad P(E) = P(Q > Q_R) = P(v_{az}A > v_R A)$$

where A is the plan area of the liner and $v_R = k_R i_R$ is the regulatory advective flux through the soil in the z direction. The term v_R is the product of the regulatory hydraulic conductivity, k_R, and the hydraulic gradient, $i_R = \Delta h_z/H_R$, where H_R is the regulatory liner thickness.

Substituting eq. [2] into eq. [4] gives

$$[5] \qquad P(E) = P(\bar{k}\bar{i}_z > k_R i_R)$$

Further substituting eq. [3] into eq. [5], the probability of exceedance becomes

$$[6] \qquad P(E) = P\left\{\left[\frac{H_G + H_C}{(H_G/k_G) + (H_C/k_C)}\right]\left(\frac{\Delta h_z}{H_C + H_G}\right) > k_R \frac{\Delta h_z}{H_R}\right\} = P\left(\frac{H_G}{k_G} + \frac{H_C}{k_C} < \frac{H_R}{k_R}\right)$$

For simplicity, the ratio of H/k is expressed using the variable w' (the prime is used to distinguish between this ratio at a specific location in the liner and the spatial geometric average of this ratio; the latter is soon to be defined and used for the remainder of the paper) such that

$$[7] \qquad w' = w'_G + w'_C = \frac{H_G}{k_G} + \frac{H_C}{k_C}$$

which gives

$$[8] \qquad P(E) = P\left(w'_G + w'_C < w_R\right)$$

where $w_R = H_R/k_R$ is deterministic.

In this paper, the distribution of k_C is assumed to be lognormal (Bogardi et al. 1989, 1990; Benson et al. 1994), and a geometric average (Fenton and Griffiths 1993) is used to best describe the equivalent total (or block) hydraulic conductivity of each barrier layer. It should be noted that based on a statistical analysis of hydraulic conductivity for 57 different landfills, Benson (1993) found that the lognormal distribution is approximately correct for CSLs. In addition, the geometric average of k is simply e raised to the power of the arithmetic average of $\ln k$, which tends to be a lognormal distribution by the Central Limit Theorem regardless of the underlying distribution of k. Thus, the assumption of lognormally distributed hydraulic conductivity is reasonable. Any distribution is an approximation, but the lognormal assumption is generally conservative because it has a long positive tail, allowing occasionally high conductivities. The following relationships exist between the mean, μ_{k_C}, and the coefficient of variation, v_{k_C}, of hydraulic conductivity and the parameters of the lognormal distribution:

$$[9a] \qquad \mu_{\ln k_C} = \ln\mu_{k_C} - \frac{1}{2}\sigma^2_{\ln k_C}$$

$$[9b] \qquad \sigma^2_{\ln k_C} = \ln(1 + \upsilon^2_{k_C})$$

where μ_{k_C} is the mean k of CSL; $\upsilon_{k_C} = \sigma_{k_C}/\mu_{k_C}$, the coefficient of variation of k of CSL; $\sigma_{k_C} = \mu_{k_C}v_{k_C}$, the standard deviation of k of CSL; $\mu_{\ln k_C}$ is the mean of ln$k$ of CSL; $\sigma^2_{\ln k_C}$ is the variance of ln$k$ of CSL.

Similar relationships hold for the GCL (with subscript "G" replacing subscript "C"), where it is assumed that k_G is also lognormally distributed.

If k_C is lognormally distributed, then w'_C is also, where

$$[10a] \qquad w'_C = \frac{H_C}{k_C}$$

so that

$$[10b] \qquad \ln w'_C = \ln H_C - \ln k_C$$

is normally distributed. Equation [10b] can be used to determine the parameters of lognormally distributed w'_C as follows: $\mu_{\ln w'_C} = \ln H_C - \mu_{\ln k_C}$ and $\sigma_{\ln w'_C} = \sigma_{\ln k_C}$.

Equations [9] and [10] specify the distribution of hydraulic conductivity and the thickness to hydraulic conductivity ratio, respectively, at any point in the barrier. Work by Fenton and Griffiths (1993) suggests that the equivalent, or block hydraulic conductivity for flow in two or three dimensions is best represented by using a geometric average of the hydraulic conductivity. Over a domain, D, the geometric average of the "point" ratio w'_C, is defined by

$$[11] \qquad w_C = \exp\left(\frac{1}{D}\int \ln w'_C \,\mathrm{d}x\right) = \exp\left[\frac{1}{D}\int_D (\ln H_C - \ln k_C)\,\mathrm{d}x\right]$$

and the geometric average will be used to represent the entire liner in the remainder of this paper. In the case of the CSL, the domain D is equal to $A \times H_C$, which is the CSL volume. If k_C is lognormally distributed, as assumed, then w_C is also lognormally distributed with parameters,

$$[12a] \qquad \mu_{\ln w_C} = \ln H_C - \mu_{\ln k_C}$$

$$[12b] \qquad \sigma^2_{\ln w_C} = \sigma^2_{\ln k_C}\gamma_C(T_1, T_2, T_3)$$

and similarly for w_G, where $\gamma_C(T_1, T_2, T_3)$ is a three-dimensional variance function (Vanmarcke 1983) for the CSL, which gives the fraction of variance reduction due to local averaging; T_1, T_2, and T_3 are the length, width, and depth of the averaging domain (i.e., of the CSL), respectively, and the domain D used in eq. [11] is equal to the product, $D = T_1 \times T_2 \times T_3$. A similar variance function, $\gamma_G(T_1, T_2, T_3)$, exists for the GCL where T_1, T_2, and T_3 are the corresponding dimensions of the GCL.

The thickness to hydraulic conductivity ratios appearing in eqs. [6]–[8] are now replaced by their geometric averages so that the probability of exceedance is now defined by

$$[13] \qquad P(E) = P(w_G + w_C < w_R)$$

In the simulation results reported in this paper, the correlation function of the random hydraulic conductivity field is assumed to be Markovian, with correlation coefficient, ρ, exponentially decaying with separation distance:

$$[14] \qquad \rho(\tau_1, \tau_2, \tau_3) = \exp\left[-2\sqrt{\left(\frac{\tau_1}{\theta_1}\right)^2 + \left(\frac{\tau_2}{\theta_2}\right)^2 + \left(\frac{\tau_3}{\theta_3}\right)^2}\right]$$

where θ_i is the scale of fluctuation in the ith direction; τ_i is the distance between points in the ith direction for which the correlation coefficient is desired.

The corresponding variance function

$$[15] \qquad \gamma(T_1, T_2, T_3) = \frac{8}{T_1^2T_2^2T_3^2}\int_0^{T_1}\int_0^{T_2}\int_0^{T_3}(T_1-\tau_1)(T_2-\tau_2)(T_3-\tau_3)\rho(\tau_1, \tau_2, \tau_3)\,\mathrm{d}\tau_3\,\mathrm{d}\tau_2\,\mathrm{d}\tau_1$$

is calculated in this paper using 20-point Gauss quadrature (see, for example, Fenton and Griffiths 2008 for details). A method to approximate the variance function for simple hand calculations is provided in Appendix A.

For all cases investigated in this paper, the scale of fluctuation, θ, is assumed to be 1 m in all directions. Benson et al. (1994) suggested that the scale of fluctuation for compacted clays is likely between 1 and 3 m based on the work of Benson (1991), so that the assumption made here is deemed to be reasonable. The more traditional approach to probabilistic flow analysis assumes that the scale of fluctuation is infinite in all directions, corresponding to k_C and k_G being treated as single random variables, rather than random fields. Because geometric averaging of a random field is low-value dominated, the layer-wise geometric averages considered in this study will have lower conductivities, on average, than would be found using an infinite scale of fluctuation. In other words, the exceedance probabilities found later in this paper are lower than would be found using an infinite scale of fluctuation, the latter being deemed overly conservative by the authors.

To estimate the probability of exceedance, the distribution of $w = w_G + w_C$, which is the sum of two lognormally distributed random variables, needs to be determined. Though the sum of lognormally distributed random variables is not lognormal, nor does it follow any other common distribution, it will be shown via simulation in the next section that the lognormal distribution provides a reasonable approximation to the distribution of w. Thus, w will be assumed to be lognormally distributed, and the task is to determine its parameters, $\mu_{\ln w}$ and $\sigma_{\ln w}$. To do this, the means and variances of

$\ln w_C$ and $\ln w_G$ must first be transformed to the means and variances of w_C and w_G as follows:

$$[16a] \qquad \mu_{w_C} = \exp\left(\mu_{\ln w_C} + \frac{\sigma^2_{\ln w_C}}{2}\right)$$

$$[16b] \qquad \sigma^2_{w_C} = \left[\exp\left(\sigma^2_{\ln w_C}\right) - 1\right]\exp\left(2\mu_{\ln w_C} + \sigma^2_{\ln w_C}\right)$$

A similar transformation can be performed for the GCL to obtain the mean and variance of w_G.

Assuming independence between w_G and w_C, the mean and variance of the sum $w = w_G + w_C$, is given by

$$[17a] \qquad \mu_w = \mu_{w_G} + \mu_{w_C}$$

$$[17b] \qquad \sigma^2_w = \sigma^2_{w_G} + \sigma^2_{w_C}$$

Finally, assuming w is at least approximately lognormally distributed, the mean and variance of w can be transformed back to the parameters of the lognormal distribution,

$$[18a] \qquad \mu_{\ln w} = \ln\mu_w - \frac{1}{2}\sigma^2_{\ln w}$$

$$[18b] \qquad \sigma^2_{\ln w} = \ln\left(1 + \frac{\sigma^2_w}{\mu^2_w}\right)$$

so that the probability of exceedance is obtained from eq. [13] as

$$[19] \qquad P(E) = \Phi\left(\frac{\ln w_R - \mu_{\ln w}}{\sigma_{\ln w}}\right)$$

where $w_R = H_R/k_R$, and Φ is the cumulative distribution function of the standard normal variate.

Simulation to verify analytical solution

Although the theory presented in the previous section can be used to estimate the probability of exceedance through a GCL–CSL combined liner via eq. [19], the theory involves a number of assumptions that must be validated. The assumptions needing validation are as follows:

(1) The equivalent hydraulic conductivity of each liner layer (CSL and GCL) is a geometric average of the random hydraulic conductivity fields.
(2) The equivalent hydraulic conductivity of the combined layered barrier is the harmonic average of the two geometric layer averages.
(3) The distribution of w is approximately lognormal.

Assumptions 1 and 2 have been verified, at least approximately, elsewhere (Terzaghi 1944; Fenton and Griffiths 1993). Assumption 3 can be investigated via simulation. Since the simulation does not involve any of the assumptions made in the theory, agreement between the simulation results and the theoretical results would then imply that the assumptions made in the theory are reasonable.

The soil barrier is simulated by generating spatially-varying lognormally distributed hydraulic conductivity fields in three dimensions for each soil layer, which are characterized by hydraulic conductivity mean, variance, and correlation function. Hydraulic conductivity realizations are created using the Local Average Subdivision (LAS) method (Fenton and Vanmarcke 1990). For the present study, LAS is used to generate two independent hydraulic conductivity fields: one for the CSL and one for the GCL. For each realization performed, LAS generates a correlated field of "local averages" of hydraulic conductivity values. The geometric average of these hydraulic conductivity values is then calculated and recorded. This type of averaging retains lognormal distribution of k_C and k_G. In the post-processing stage, each realization of the geometric average hydraulic conductivity of the CSL is harmonically averaged with a corresponding realization of the geometric average hydraulic conductivity of the GCL. After this is completed for 5000 simulations, the distribution of the combined liner hydraulic conductivity can be estimated. Figure 3*a* shows a typical histogram developed from the simulation (solid line) compared to the analytical solution (dashed line). It can be seen that the assumption of a lognormal distribution fitted to the mean and standard deviation of the simulated data matches the distribution assumed in the analytical solution. Note that $w = w_G + w_C$ decreases as k_G and (or) k_C increase (see, for example, eq. [7], and so the probability of exceedance of the liner system is shown on the histogram as the area under the curve to the left of the regulatory value (hatched area). For example, in Fig. 3*a*, the area under the entire curve is 1.0, and the probability of exceedance of the combined GCL–CSL liner is estimated from the histogram as 0.20. Figure 3*b* is an example of a histogram for which the analytical solution and simulation show poorer agreement. The $P(E)$ obtained from the two histograms of Fig. 3 are shown as points a and b on Fig. 4. This figure represents $P(E)$ for a variety of simulations compared to $P(E)$ obtained by the analytical solution for a range of assumed means and variances. Perfect agreement between simulation and the analytical solution would plot exactly on the diagonal line. Figure 4 presents 117 points; all possible combinations of a 1 m CSL with coefficient of variation of 0.5, 2.0, and 5.0, and mean hydraulic conductivities (m/s) of 1×10^{-10}, 5×10^{-10}, 1×10^{-9}, 5×10^{-9}, 1×10^{-8}, 5×10^{-8}, 1×10^{-7}, 5×10^{-7}, 1×10^{-6}, 5×10^{-6}, 1×10^{-5}, 1×10^{-5}, 1×10^{-4} and a 10 mm GCL with coefficient of variation of 0.05, 0.2, and 0.1 and a mean hydraulic conductivity of 1×10^{-11} m/s. All points show approximate agreement between theory and simulation. The points that vary away from the line do so at high CSL mean k and GCL coefficients of variation of 0.05, implying that the model generated may begin to break down at these parameters. However, even for the cases that don't show perfect agreement, it appears as if the analytical solutions match reasonably well with the simulation analyses, and the assumption of a lognormal distribution for w is reasonable. These observations provide some confidence in the derivation of the analytical solutions provided earlier.

Utilizing analytical solutions to examine probability of exceedance for a range of parameters

Given the suitability of the analytical solutions for calculating the probability of exceedance through a GCL–CSL combined liner, it becomes a fairly easy exercise to examine

Fig. 3. Typical probability density function of combined liner *w*. (*a*) Good fit; (*b*) poorest fit.

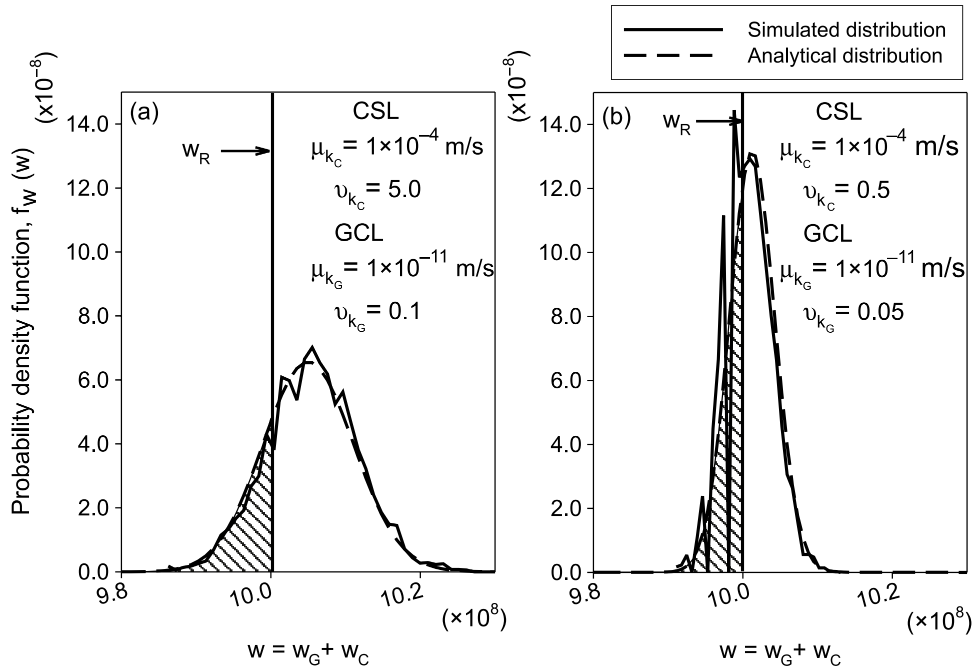
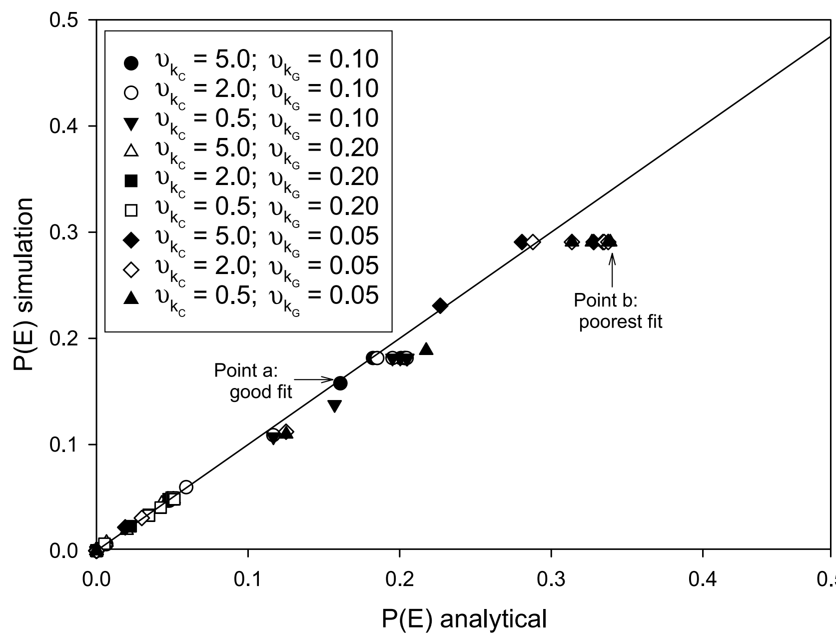


Fig. 4. Comparison of analytical solution and simulated results.



some practical factors that can influence the risk of exceedance for using a GCL–CSL combined system. As stated earlier in the paper, a deterministic hydraulic flux calculation can be performed for a GCL–CSL combined system using eq. [2]. However, this calculation does not allow material variability and risk to enter into the decision-making exercise. The following sections examine the probability of exceedance for a GCL–CSL combined system (see Fig. 2*b*) for a range of mean hydraulic conductivities of the CSL and GCL. Also presented are the influences of CSL and GCL variance, CSL thickness, and barrier area on the probability of exceedance.

Influence of CSL and GCL mean hydraulic conductivity on probability of exceedance

A variety of soils exist in nature that can potentially be used for barrier systems, especially when used as some form of combined barrier system with GCLs. In this study, mean CSL hydraulic conductivities were varied from 1×10^{-10} to 1×10^{-4} m/s to represent soil types ranging from low permeability clays to coarse sands (Das 2002). Three GCL conductivities (1×10^{-12} m/s, 1×10^{-11} m/s, and 5×10^{-11} m/s) were chosen to use in combination with the CSL to provide harmonic average hydraulic conductivities in the vicinity of 1×10^{-9} m/s at high CSL *k*. Figure 5 plots

CSL mean k versus probability of exceedance for these GCL values. Each point on Fig. 5 represents one calculation for a given set of parameters, μ_{k_C}, $\sigma^2_{k_C}$, μ_{k_G}, $\sigma^2_{k_G}$. Lines drawn on the figure simply represent the trend of the data between points and do not present lines of statistical fit. For example, consider a GCL–CSL combined liner system in which

- the mean GCL hydraulic conductivity is 1×10^{-11} m/s,
- GCL coefficient of variation is 0.1,
- CSL mean hydraulic conductivity is 1×10^{-4} m/s,
- CSL coefficient of variation is 0.5, and
- GCL and CSL scales of fluctuation are 1 m in all three directions.

One can see from Fig. 5*b* that the probability of exceedance for this system is calculated as 0.2. In other words, given the variability statistics of the CSL and the GCL combined, there is a 20% probability that the flow will be higher than that of a regulatory liner system 1 m thick with a deterministic hydraulic conductivity of 1×10^{-9} m/s. In a similar manner, it can be seen for the same GCL used in combination with a CSL of mean hydraulic conductivity of 1×10^{-8} m/s (CSL coefficient of variation of 0.5), the probability of exceedance is less than 1×10^{-10}. A similar exercise can be performed for any of the three GCL mean hydraulic conductivities and the CSL mean hydraulic conductivities chosen. If other GCL mean hydraulic conductivity values were required to be used in the analysis, one could generate similar plots using the analytical solution presented earlier. An example of such an exercise is presented in a following section. It should be noted that the results shown in Fig. 5 are for a plan area of 20 m $\times$ 20 m. Because some probabilities of exceedance are very low (less than 10^{-10}), the same results are presented on both arithmetic (I, upper plots) and logarithmic (II, lower plots) y-axis scales.

Based on the results shown in Figure 5, the following points can be noted:

- For the GCL hydraulic conductivities examined, as the mean hydraulic conductivity of the CSL increases above the regulatory value of 1×10^{-9} m/s, the probability of exceedance also increases. The reason is that as k increases for the CSL, the harmonic average of the CSL and GCL combined liner system starts to approach the regulatory hydraulic conductivity.
- For the range of mean k examined for the CSL, as the mean hydraulic conductivity of the GCL decreases to 10^{-12} m/s, the risk of exceedance approaches zero. This implies that for very low values of hydraulic conductivity of the GCL, the probability of exceedance posed by the risk of high k values of the CSL become minimal.
- For "marginal" k soils (near but slightly above the regulatory value of 1×10^{-9} m/s), a GCL with k values of 1×10^{-11} m/s and below can significantly reduce the probability of exceedance for the GCL–CSL combined liner properties examined.

Although some of these observations could be obtained from a simple deterministic analysis of the problem, the methods employed in this paper allow a quantitative aspect of risk to be introduced into the decision-making exercise.

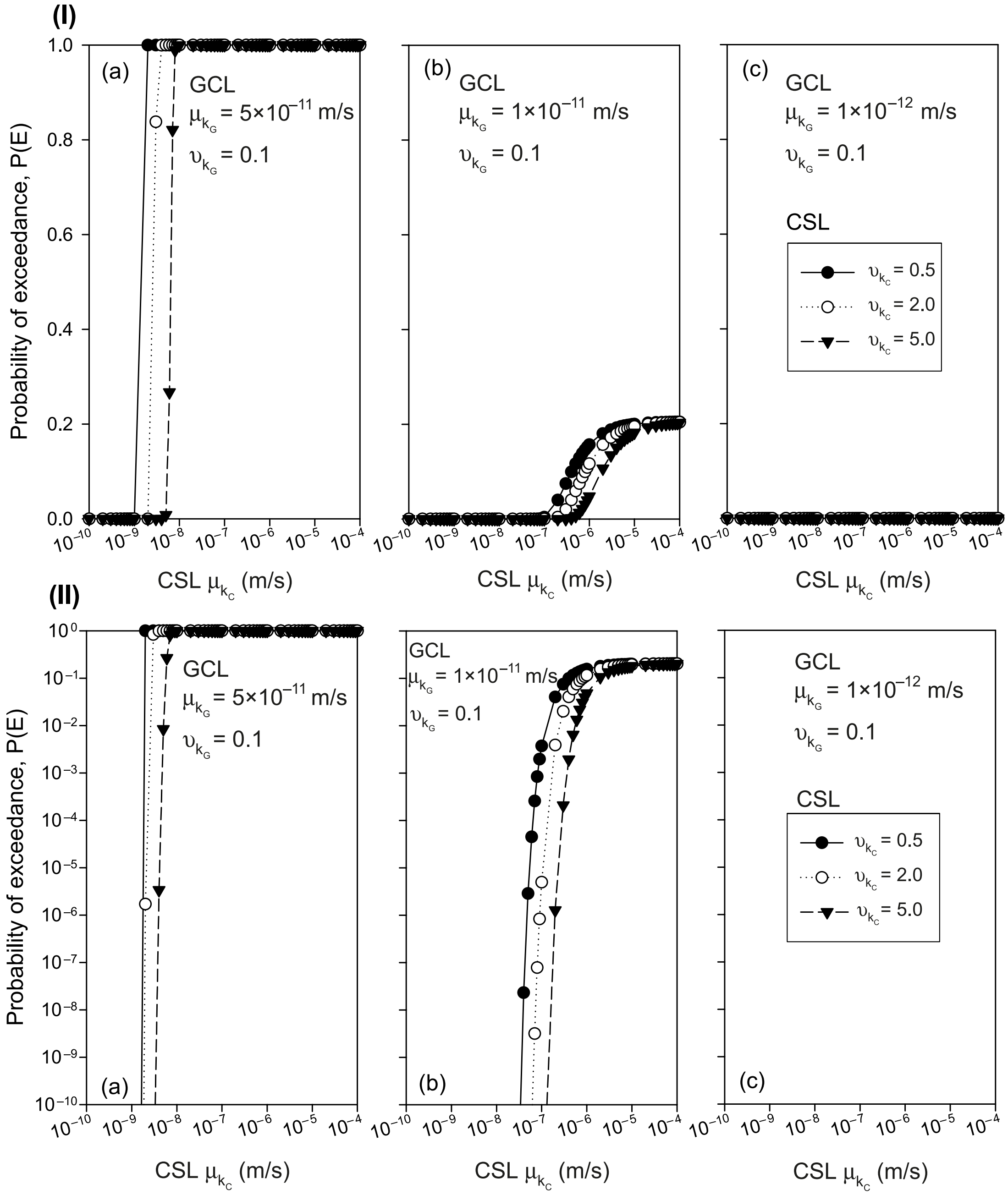
Influence of CSL and GCL hydraulic conductivity variance on probability of exceedance

The influences of GCL and CSL variance on probability of exceedance for a GCL–CSL combined liner system are counterintuitive to what one might initially assume. As shown in Fig. 6, for a given mean hydraulic conductivity of the GCL and CSL, increasing the GCL or CSL variance reduces the probability of exceedance. This does not mean that one should strive for more variability in the GCL or CSL; it simply means that by "fixing" the mean hydraulic conductivity of the CSL and (or) GCL, there is a greater chance of obtaining lower hydraulic conductivity values as the variance increases. This concept was previously discussed by Benson and Daniel (1994). They investigated the decrease in probability of exceedance with increasing variance in a study that examined the minimum thickness of CSLs. Although their study calculated the probability of exceedance based on first passage times and flux distributions, they found the same influence of coefficient of variation on CSL performance. Benson and Daniel (1994) explained the decrease in probability of exceedance with increasing coefficient of variation through the use of Fig. 7. They explain that increasing the coefficient of variation produces greater positive skew of the lognormal k distribution for a given mean hydraulic conductivity of the individual material. As a result, an increase in coefficient of variation causes only a small change in the probability of high k, but a large increase in the probability of low k. In other words, it is more likely to get lower values of k in the flow path through the liner. This reasoning applies to both GCL and CSL k; hence, for any given pair of GCL and CSL k, decreasing the variance of either material will result in an increase in the probability of exceedance for a given mean hydraulic conductivity. Similar to the plots in Fig. 7 that are based on the work presented by Benson and Daniel (1994), Fig. 8 presents probability distributions of the GCL–CSL combined liner with changing GCL coefficient of variation. The increased variance of the GCL forces a wider distribution, as expected. However, because of the shift in the mean hydraulic conductivity, the increased variance results in a lower probability of exceedance (i.e., the area to the left of w_R decreases).

Based on the results shown in Figs. 6 and 8, the following points can be noted:

- The influence of the CSL and GCL variances are counterintuitive to what one would naturally assume. Increasing CSL or GCL variance reduces the probability of exceedance for any fixed pair of CSL and GCL mean hydraulic conductivities because of a decreasing geometric average (the geometric average is low-value dominated; Fenton and Griffiths 2008). This does not necessarily imply that less quality control on the construction of a liner is better. In the field, increased variance would most likely be accompanied with an increase in CSL mean k, effectively increasing the probability of exceedance.
- For a given mean and variance of the GCL, as the mean k of the CSL increases above 1×10^{-6} m/s, the variance of the CSL has little effect on the probability of exceedance.
- For values of CSL k less than 1×10^{-8} m/s, the probability of exceedance approaches zero when GCL k is less than 1×10^{-11} m/s. The CSL variance appears to play a

Fig. 5. Probability of exceedance versus CSL mean hydraulic conductivity for a liner of area 20 m × 20 m with GCL mean hydraulic conductivities of (*a*) 5×10^{-11} m/s, (*b*) 1×10^{-11} m/s, and (*c*) 1×10^{-12} m/s (in which all probabilities are less than 1×10^{-20}). Plots in I use a linear vertical scale, while plots in II use a logarithmic vertical scale.



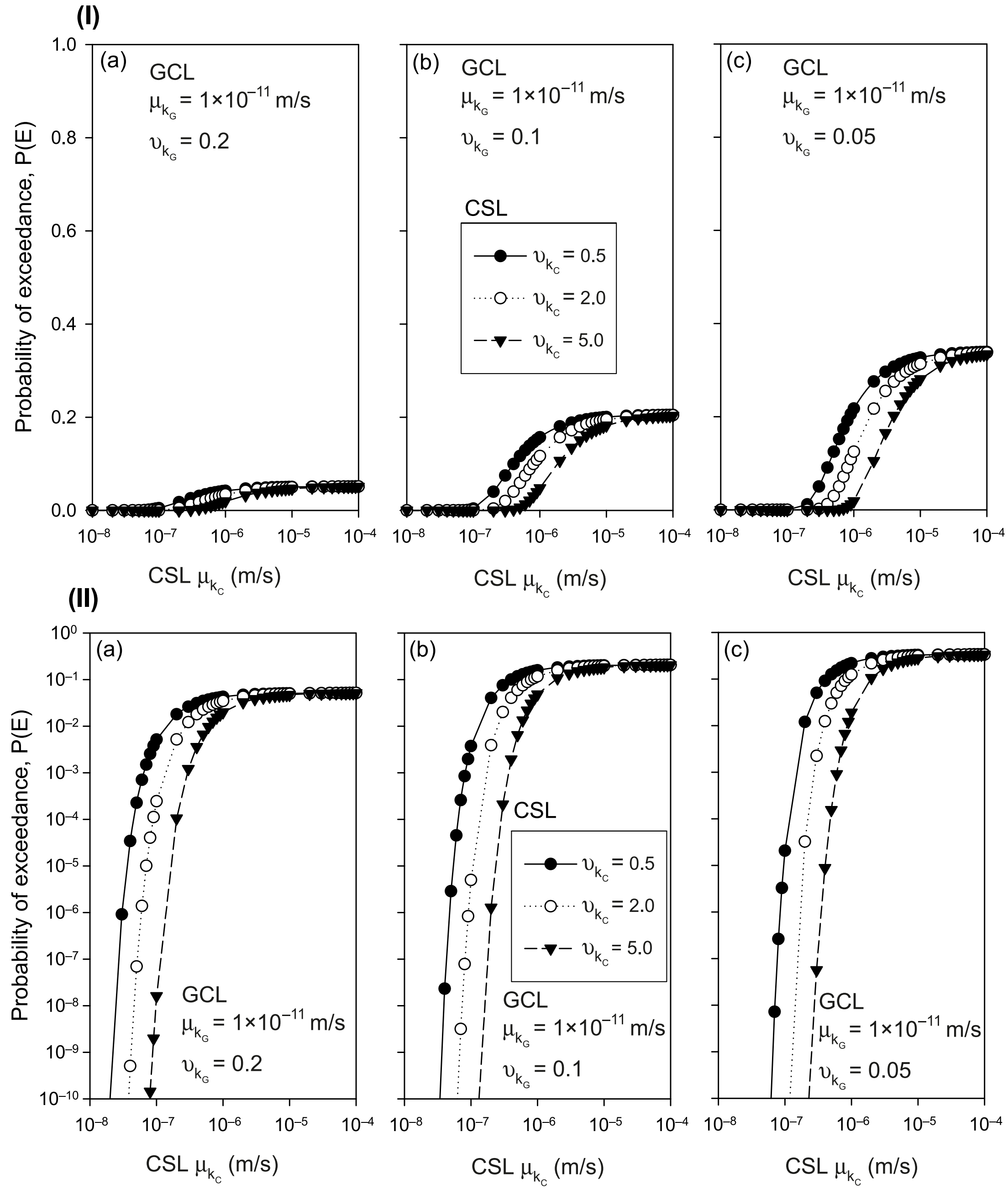
more significant role when the CSL mean k is more than 1×10^{-6} m/s.

- For moderate values of CSL k, between 1×10^{-8} and 1×10^{-5} m/s, the probability of exceedance is influenced by both CSL and GCL variance.

Influence of CSL thickness on probability of exceedance

Most regulations require that CSL thicknesses are in the range of 0.6–1.0 m. For the purposes of this study, H_R was assumed to be 1 m and the regulatory hydraulic conductivity was assumed to be 1×10^{-9} m/s. The influence of CSL thickness when used in a combined liner system with a GCL was then considered in terms of how reducing H_C could influence probability of exceedance. The CSL thickness was varied from 0.5 to 1.0 m. The case of no CSL (i.e., only GCL) was also considered. The CSL was assumed to have a coefficient of variation of 2.0, and was used combined with a 10 mm GCL with mean hydraulic conductivity 1×10^{-11} and coefficient of variation of 0.1. The results shown in Fig. 9 are for a 20 m × 20 m liner. Figure 9 shows that as the mean CSL hydraulic conductivity decreases from 10^{-4} m/s, the CSL thickness has an influence over the probability of exceedance, and the probability of exceedance is limited to approximately 0.2 for the conditions shown;

Fig. 6. Probability of exceedance versus CSL mean hydraulic conductivity for liner of area 20 m × 20 m with GCL mean hydraulic conductivity 1×10^{-11} m/s and coefficients of variation (*a*) $v_{k_G} = 0.2$, (*b*) $v_{k_G} = 0.1$, and (*c*) $v_{k_G} = 0.05$. Plots in I use a linear vertical scale, while plots in II use a logarithmic vertical scale.



which is also the probability of exceedance when no CSL is present (i.e., GCL only). The results of a 100 m × 100 m liner with the same specifications as the previous figure are shown in Fig. 10. In this case, the probability of exceedance is only significantly greater than zero for hydraulic conductivities greater than 1×10^{-6} m/s. A probability of exceedance limit of 4.6×10^{-5} is approached when no CSL is present.

Figures 9 and 10 exhibit two trends in the probability of exceedance for the range of CSL thicknesses considered.

- Increased liner thickness reduces the probability of exceedance of the combined liner system.
- At high CSL hydraulic conductivities, a GCL–CSL combined liner approaches a limiting value of probability of exceedance set by the condition of having no CSL.

The difference between these two figures is due to the influence that liner area has on the probability of exceedance, which will be discussed in the following section. It should be noted that these conclusions are based on advective flux considerations only and do not consider the influence of thickness on contaminant migration mechanisms such as diffusion, sorption, etc. It should also be noted that this analysis of liner thickness does not take into consideration the properties of individual lift thicknesses, similar to the analysis provided by Benson and Daniel (1994).

Fig. 7. Probability density functions of hydraulic conductivity.

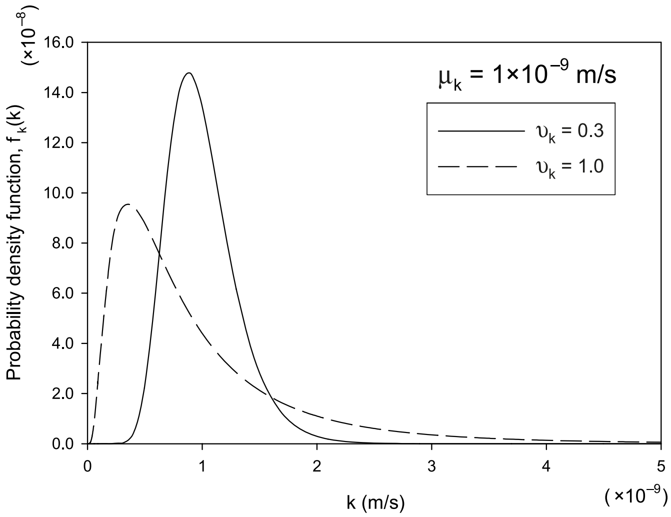
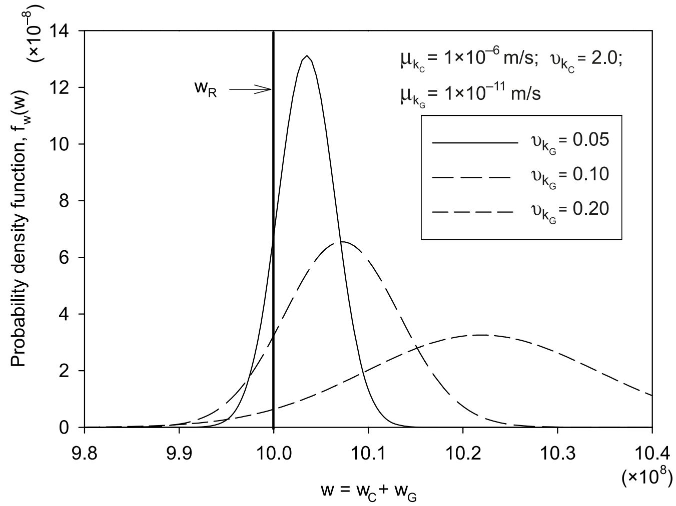


Fig. 8. Typical probability density functions for combined soil liner, *w*, with different GCL variances.



Influence of liner area on probability of exceedance

In practice, liner systems are used in projects varying in size from hundreds of square metres (small lagoons) to thousands or even millions of square metres (large landfills). The size of the problem will inherently affect the probability of exceedance owing to averaging of the spatial variability (both the mean and variance decrease with increasing averaging area). For this reason, it is prudent to investigate the effect, if any, of the liner area.

When comparing Fig. 5 (20 m $\times$ 20 m) to Fig. 11 (100 m$\times$100 m), larger area liners have a lower probability of exceedance than smaller liners of the same composition. This emphasises the importance of material quality and construction techniques for small, lagoon size, projects. For large liners, it is more likely that there are more regions of soil with high *k*; however, their impact on total flow through the liner system is significantly less than that of smaller liners. A 1 m^2 area with high *k* would let a higher proportion of total flow through on a small 20 m $\times$ 20 m liner than on a larger 100 m $\times$ 100 m liner. Mathematically, increasing the area reduces the three-dimensional variance function used in both simulation and the analytical solutions described previously. The decrease in variance produces a narrowing of the lognormal distribution, which results in a lower probability of the histogram spanning across the regulated value, as shown in Fig. 12. Hence, the probability of exceedance becomes either close to zero (less than 1×10^{-20}) or one at a large area, depending on the harmonic average hydraulic conductivity, as shown in Fig. 11. What these figures also show is that for areas larger than the 20 m $\times$ 20 m examined in Fig. 5, the GCL can eliminate risk of exceedance for a wide variety of *k* of the CSL. It appears from Fig. 12 that the mean of *w* is not changing with increasing liner area but is decreasing very gradually as the area increases (not noticeable from scale).

Using the analytical solution: an example

To ensure clarity of the methodology proposed in this paper, it is useful to demonstrate the use of the analytical solution via an example. Consider the case where there is a regulatory requirement to build a 1 m thick clay liner with hydraulic conductivity not exceeding 1×10^{-9} m/s. The liner is proposed to occupy a plan area of 20 m $\times$ 20 m. Testing of the soil to be used in the CSL revealed a mean hydraulic conductivity of a readily available clayey soil to be 2×10^{-9} m/s with a coefficient of variation of 2.0. To take advantage of this site material, a GCL 10 mm thick with a mean *k* of 1×10^{-11} m/s and a coefficient of variation of 0.1 is being considered to be placed on the 1 m CSL material described above. Assume further that the scale of fluctuation is 1 m in all directions for both materials. If the liner fails to achieve the required advective flux calculated based on these regulatory requirements, it will need to be replaced, and remedial activities will cost an additional CAN\$10/m^2. The purpose of this example is to determine, from a risk standpoint, whether there is a rational basis to add the GCL to the liner system. For this problem, we will assume the CSL and GCL cost CAN\$25/m^2 and CAN\$8/m^2 to install, respectively.

Given the mean and variance of the CSL hydraulic conductivity, the mean and variance of ln*k* of the CSL using eqs. [9*a*] and [9*b*] are as follows:

$$\sigma^2_{\ln k_C} = \ln\left(1 + \upsilon^2_{k_C}\right) = \ln\left(1 + 2.0^2\right) = 1.609$$

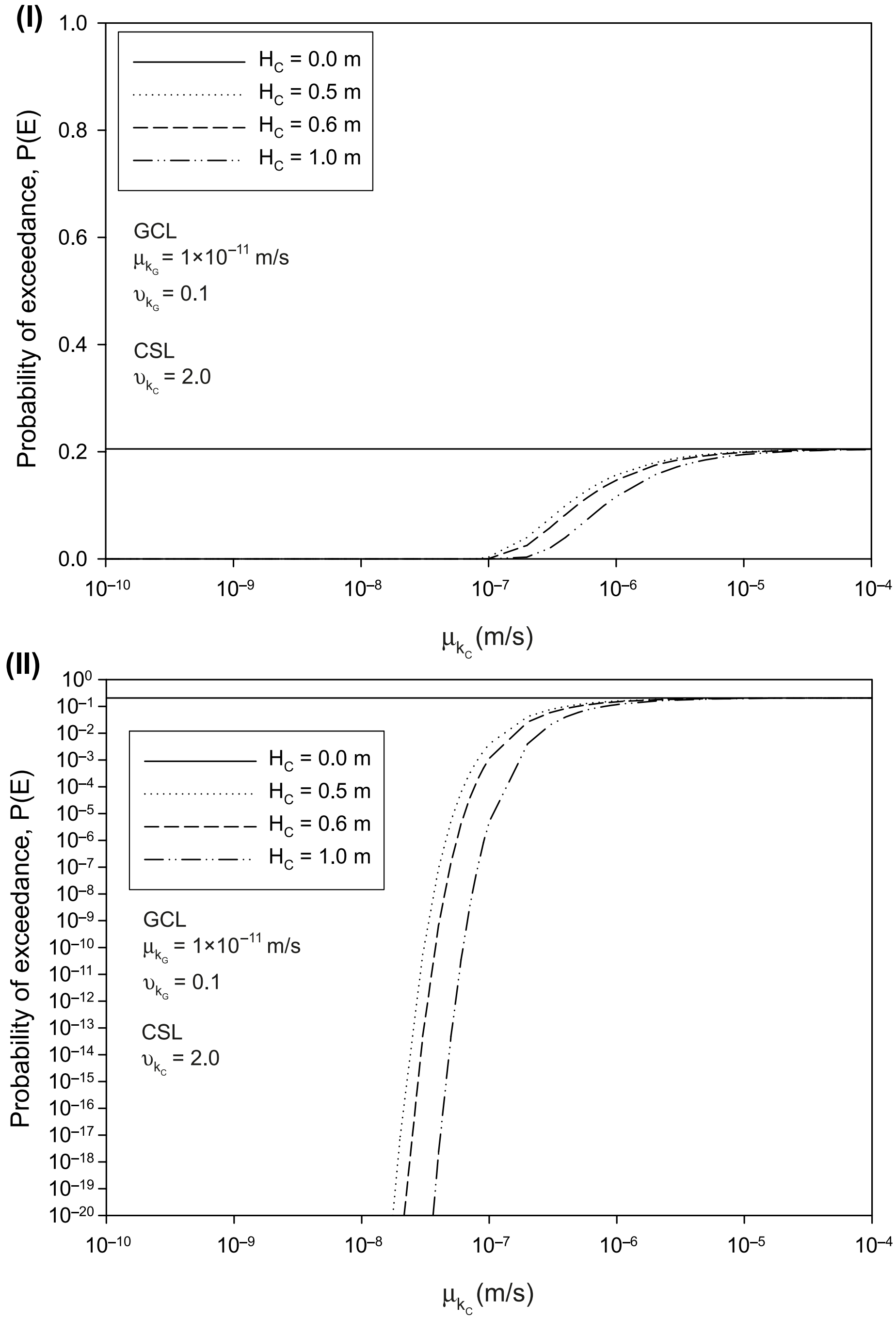
$$\mu_{\ln k_C} = \ln\mu_{k_C} - \frac{1}{2}\sigma^2_{\ln k_C} = \ln\left(2 \times 10^{-9}\right) - \frac{1}{2}(1.609) = -20.835$$

Given the scale of fluctuation and dimensions of the CSL, the variance function is evaluated as $3.075\,55 \times 10^{-3}$ ($3.682\,86 \times 10^{-3}$ for the GCL) from eq. [15]; the mean and variance of lnw_C, using eqs. [12*a*] and [12*b*], are then

$$\mu_{\ln w_C} = \ln H_C - \mu_{\ln k_C} = \ln(1) + 20.835 = 20.835$$

$$\sigma^2_{\ln w_C} = \sigma^2_{\ln k_C}\gamma_C(T_1,\ T_2,\ T_3) = 1.609 \times 3.075\,55 \times 10^{-3} = 4.95 \times 10^{-3}$$

Fig. 9. Probability of exceedance with varying CSL thickness; liner area of 20 m × 20 m.



Similarly, $\mu_{\ln w_G}$ and $\sigma^2_{\ln w_G}$ can be calculated to be 20.728 and 3.66×10^{-5}, respectively.

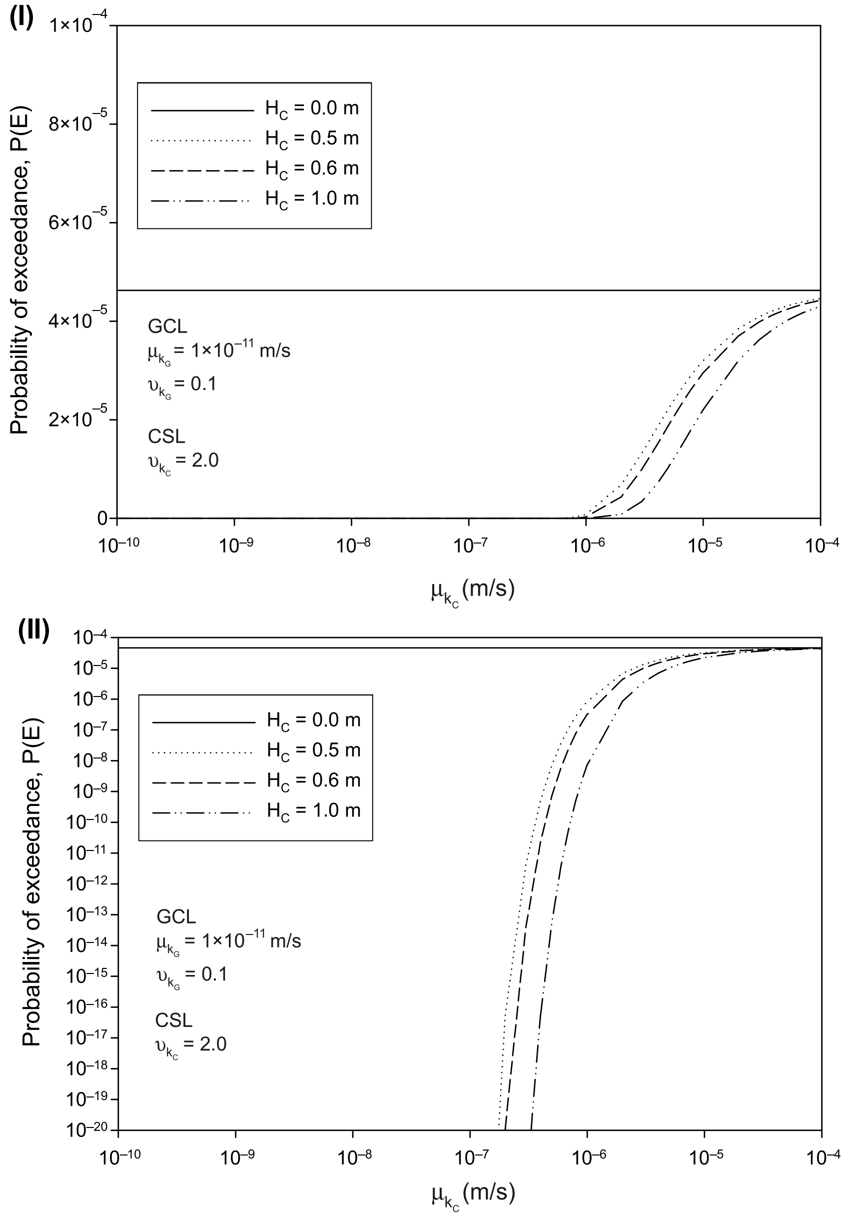
At this point, the probability of exceedance for the CSL alone can be computed using the lognormal distribution:

$$P(E) = \Phi\left[\frac{\ln(H_R/k_R) - \mu_{\ln w_C}}{\sigma_{\ln w_C}}\right] = \Phi\left\{\frac{\ln\left[1/(1 \times 10^{-9})\right] - 20.835}{\sqrt{4.95 \times 10^{-3}}}\right\} = \Phi(-1.586) = 0.056\,39$$

For the combined liner, the mean and variance of w_C and w_G can be calculated using eqs. [16*a*] and [16*b*].

$$\mu_{w_C} = \exp\left(\mu_{\ln w_C} + \frac{\sigma^2_{\ln w_C}}{2}\right) = \exp\left(20.835 + \frac{4.95 \times 10^{-3}}{2}\right) = 1.12 \times 10^9$$

Fig. 10. Probability of exceedance with varying CSL thickness; liner area of 100 m × 100 m.



$$\begin{aligned}\sigma^2_{w_C} &= \left[\exp(\sigma^2_{\ln w_C}) - 1\right]\exp\left(2\mu_{\ln w_C} + \sigma^2_{\ln w_C}\right)\\ &= \left[\exp(4.95\times10^{-3}) - 1\right]\exp\left[2(20.835) + 4.95\times10^{-3}\right]\\ &= 6.23\times10^{15}\end{aligned}$$

Similarly, μ_{w_G} and $\sigma^2_{w_G}$ can be calculated as 1.01×10^9 and 3.70×10^{13}, respectively.

Given independence between the CSL and GCL, the mean and variance of w is computed using eqs. [17*a*] and [17*b*] as follows:

$$\begin{aligned}\mu_w &= \mu_{w_G} + \mu_{w_C}\\ &= 1.12\times10^9 + 1.01\times10^9 = 2.13\times10^9\end{aligned}$$

$$\begin{aligned}\sigma^2_w &= \sigma^2_{w_C} + \sigma^2_{w_G}\\ &= 6.23\times10^{15} + 3.70\times10^{13} = 6.27\times10^{15}\end{aligned}$$

Assuming the distribution of w is approximately lognormal, the mean and variance of $\ln w$ is calculated with eqs. [18*a*] and [18*b*], respectively.

$$\begin{aligned}\mu_{\ln w} &= \ln\mu_w - \frac{1}{2}\sigma^2_{\ln w}\\ &= \ln\left(2.13\times10^9\right) - \frac{1}{2}\left(1.39\times10^{-3}\right) = 21.477\end{aligned}$$

$$\begin{aligned}\sigma^2_{\ln w} &= \ln\left(1 + \frac{\sigma^2_w}{\mu^2_w}\right)\\ &= \ln\left[1 + \frac{6.27\times10^{15}}{(2.13\times10^9)^2}\right] = 1.3866\times10^{-3}\end{aligned}$$

Finally, the probability of exceedance for the combined liner can be calculated (shown in Fig. 5*b*(I)).

Fig. 11. Probability of exceedance versus CSL mean hydraulic conductivity for liner of area 100 m × 100 m with GCL mean hydraulic conductivities of (*a*) 5×10^{-11}, (*b*) 1×10^{-11}, and (*c*) 1×10^{-12} m/s. All probabilities are either 1.0 or less than 1×10^{-20}.

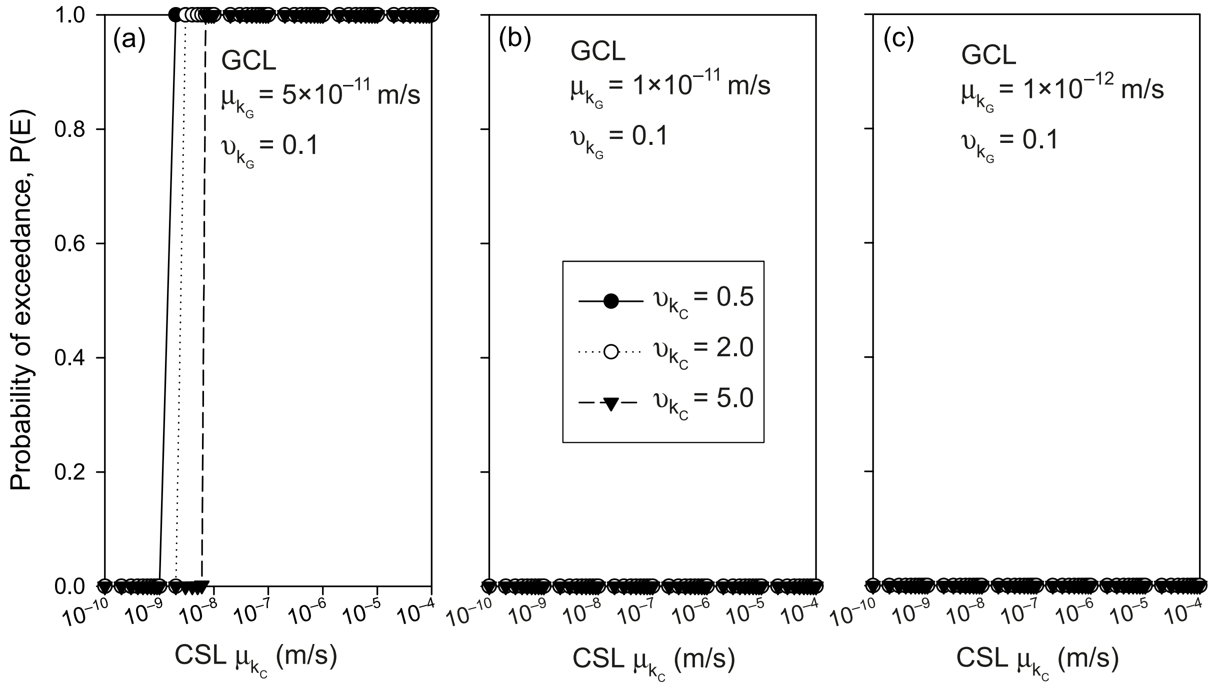
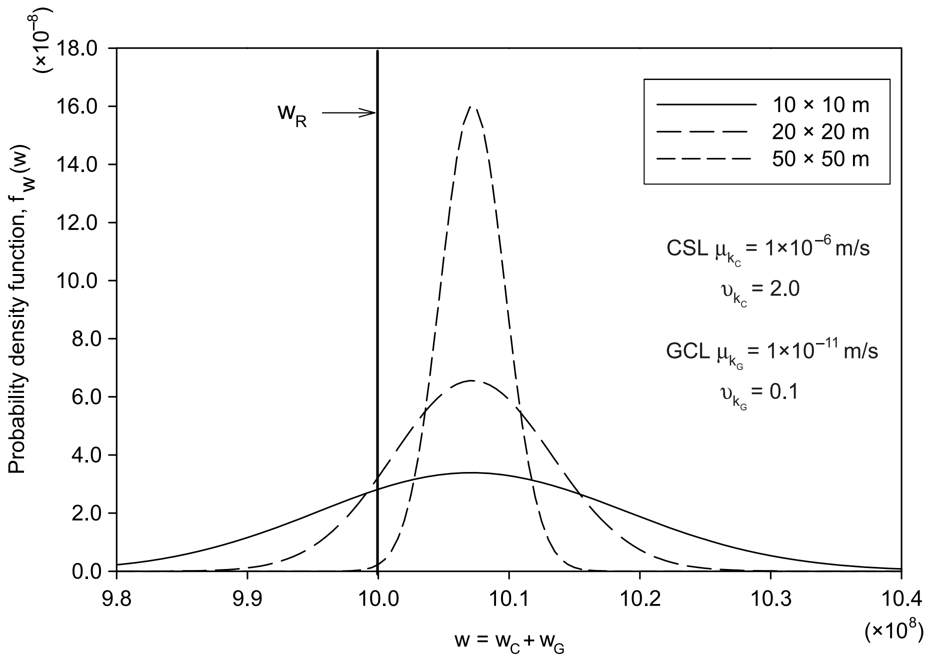


Fig. 12. Probability density function of combined liner with changing area.



$$
\begin{aligned}
P(E) &= \Phi\left[\frac{\ln(H_R/k_R) - \mu_{\ln w}}{\sigma_{\ln w}}\right] \\
&= \Phi\left\{\frac{\ln\left[1/(1 \times 10^{-9})\right] - 21.477}{\sqrt{1.39 \times 10^{-3}}}\right\} \\
&= \Phi(-20.234) = 2.45 \times 10^{-91}
\end{aligned}
$$

The total expected cost of each liner system can be calculated as

$$
\text{Total expected cost} = (\text{cost of liner}) + P(E) \times (\text{cost of repair})
$$

For no GCL, the total expected cost would be

$$E(\text{cost}) = (\text{CAN\$25/m}^2)(20\text{ m})^2 + 0.056(\text{CAN\$25/m}^2 + \text{CAN\$10/m}^2)(20\text{ m})^2 = \text{CAN\$10 784}$$

and for the combined GCL–CSL, the total expected cost would be

$$E(\text{cost}) = (\text{CAN\$25/m}^2 + \text{CAN\$8/m}^2)(20\text{ m})^2 + (2.5\times 10^{-91})(\text{CAN\$25/m}^2 + \text{CAN\$8/m}^2 + \text{CAN\$10/m}^2)(20\text{ m})^2 = \text{CAN\$13 200}$$

For the given problem, choosing the CSL alone represents the lowest cost; however, there is a relatively high probability of exceedance of 5.6%. In many situations, such a high probability of exceedance would likely be unacceptable, and using a combined liner system, one with a GCL, would provide an adequate reduction in risk (i.e., for a minimal increase in cost).

Conclusions

A set of relatively simple analytical solutions were developed, and verified with simulation, to predict the level of risk reduction for adding a GCL to a CSL in terms of advective flux exceeding a regulated condition. It was shown, for the assumed properties of CSL and GCL, that decreases in the mean k of a CSL resulted in lower probabilities of exceedance, as expected. Typically, large changes in CSL k are required for noticeable changes in harmonic average k of the combined liner system. However, in some instances with large areas (i.e., greater than 100 m × 100 m), slight increases in CSL k result in the probability of exceedance changing from close to zero (less than 1×10^{-6}) to one. The latter corresponds to cases where the harmonic average of the means is greater than the regulatory value.

In all situations considered throughout this study, GCL k has a dramatic effect on the probability of exceedance of the combined liner. As expected, decreasing GCL k produces lower probabilities of advective flux exceeding a regulated value. To produce intermediate probabilities of exceedance (i.e., values much greater than zero but less than one), GCL k must be chosen to produce a harmonic average k sufficiently close to the regulated value.

Coefficient of variation has a counterintuitive influence on the probability of exceedance for both CSLs and GCLs. An increase of either CSL or GCL variance produces a decrease in probability of advective flux exceeding a regulated value. Typically, GCL variance has a more significant influence than CSL variance on probability of exceedance at high CSL k, greater than 1×10^{-5} m/s; and CSL variance has a more significant influence on probability of exceedance than GCL variance at low CSL k, less than 1×10^{-8} m/s.

The influence of CSL thickness is as expected; increasing CSL thickness results in a decrease in probability of exceedance. For all CSL thicknesses considered in this paper, the probability of exceedance approaches the condition of having only a GCL at high CSL k, greater than 1×10^{-5} m/s.

Liner area has an effect on the probability of advective flux exceeding a regulated value. Increased liner areas reduce the probability of exceedance to either values exceptionally close to zero (less than 1×10^{-6}) or one. This is caused by a narrowing of the probability density function resulting from a decrease in variance due to the application of the variance function, which continually decays with constant scale of fluctuation and increasing area.

The analytical solution presented by the authors will allow professionals to quantify the level of risk for GCL–CSL combined systems, with respect to flow. An example problem was provided to assist in this regard. It is hoped that the technique presented herein will allow one to make rational decisions when evaluating such liner systems.

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List of Symbols

A plan area of the liner
D averaging domain of the continuous random field
H thickness of combined liner
H_C thickness of CSL in z direction
H_G thickness of GCL in z direction
H_R thickness of the regulatory soil barrier system
Δh_z difference in total head across liner system
i_R regulatory hydraulic gradient in the z direction
i_z hydraulic gradient in the z direction
$\bar{i}_z$ hydraulic gradient across the combined soil liner system in the z direction
k hydraulic conductivity
$\bar{k}$ harmonic average hydraulic conductivity of the combined soil liner system
k_C hydraulic conductivity of the CSL
k_G hydraulic conductivity of the GCL
k_R regulatory hydraulic conductivity
P correlation function
$P(E)$ probability of exceedance
Q total flow through the liner
Q_R total flow through a regulatory liner
T_i dimension of the averaging domain in the ith direction
v_a advective flux through the soil in the z direction
v_R regulatory advective flux through the soil liner in the z direction
w geometric average ratio of thickness to hydraulic conductivity of the combined soil liner
w_C geometric average ratio of thickness to hydraulic conductivity of the CSL
w_G geometric average ratio of thickness to hydraulic conductivity of the GCL
w_R ratio of regulatory thickness to hydraulic conductivity of the soil liner
w' ratio of thickness to hydraulic conductivity of the combined soil liner at a point
w'_C ratio of thickness to hydraulic conductivity of the CSL at a point
w'_G ratio of thickness to hydraulic conductivity of the GCL at a point
γ variance reduction function of CSL (γ_C) or GCL (γ_G)
θ_i correlation length in the ith direction
μ_{k_C} mean hydraulic conductivity of the CSL
μ_{k_G} mean hydraulic conductivity of the GCL
$\mu_{\ln k_C}$ mean log hydraulic conductivity of the CSL
$\mu_{\ln k_G}$ mean log hydraulic conductivity of the GCL
$\mu_{\ln w}$ mean of the log geometric average of thickness to hydraulic conductivity ratio of the composite soil
$\mu_{\ln w_C}$ mean of the log geometric average of thickness to hydraulic conductivity ratio of the CSL
$\mu_{\ln w'_C}$ mean of the log thickness to hydraulic conductivity ratio at a point in the CSL
$\mu_{\ln w_G}$ mean of the log geometric average of thickness to hydraulic conductivity ratio of the GCL
μ_w mean of the geometric average of thickness to hydraulic conductivity ratio of the composite soil liner
μ_{w_C} mean of the geometric average of thickness to hydraulic conductivity ratio of the CSL
μ_{w_G} mean of the geometric average of thickness to hydraulic conductivity ratio of the GCL
ρ correlation coefficient between log hydraulic conductivity at two points in the liner
σ_{k_C} standard deviation of the hydraulic conductivity of the CSL ($\sigma_{k_C} = v_{k_C}\mu_{k_C}$)
σ_{k_G} standard deviation of the hydraulic conductivity of the GCL ($\sigma_{k_G} = v_{k_G}\mu_{k_G}$)
$\sigma_{\ln k_C}$ standard deviation of the log hydraulic conductivity of the CSL
$\sigma_{\ln k_G}$ standard deviation of the log hydraulic conductivity of the GCL
$\sigma_{\ln w}$ standard deviation of the log geometric average of thickness to hydraulic conductivity ratio of the composite soil liner
$\sigma_{\ln w_C}$ standard deviation of the log geometric average of thickness to hydraulic conductivity ratio of the CSL
$\sigma_{\ln w'_C}$ standard deviation of the log thickness to hydraulic conductivity ratio at a point in the CSL
$\sigma_{\ln w_G}$ standard deviation of the log geometric average of thickness to hydraulic conductivity ratio of the GCL
σ_w standard deviation of the geometric average of thickness to hydraulic conductivity ratio of the composite soil liner
σ_{w_C} standard deviation of the thickness to hydraulic conductivity ratio of the CSL
σ_{w_G} standard deviation of the thickness to hydraulic conductivity ratio of the GCL
τ_i distance between points in the ith direction for which the correlation coefficient is desired
υ_{k_C} coefficient of variation of CSL ($\upsilon_{k_C} = \sigma_{k_C}/\mu_{k_C}$)
υ_{k_G} coefficient of variation of GSL ($\upsilon_{k_G} = \sigma_{k_G}/\mu_{k_G}$)
Φ standard normal cumulative density function

Appendix A

An approximation to the three-dimensional Markovian variance function can be made by assuming the correlation structure to be separable. Although not a perfect match, the following simplification will produce a variance function that follows the same trend and generates values in the same order of magnitude as those used throughout this paper via eq. [15]:

$$[A1] \qquad \gamma(T_1, T_2, T_3) \approx \gamma(T_1)\gamma(T_2)\gamma(T_3)$$

which has the separable correlation function

$$[A2] \qquad \rho(\tau_1, \tau_2, \tau_3) = \exp\left[-2\left(\frac{|\tau_1|}{\theta_1} + \frac{|\tau_2|}{\theta_2} + \frac{|\tau_3|}{\theta_3}\right)\right] = \exp\left(-2\frac{|\tau_1|}{\theta_1}\right)\exp\left(-2\frac{|\tau_2|}{\theta_2}\right) \times \exp\left(-2\frac{|\tau_3|}{\theta_3}\right)$$

from which the corresponding variance functions, $\gamma(T_i)$, can be calculated explicitly to be

$$[A3] \qquad \gamma(T_i) = \frac{2}{T_i^2}\int_0^{T_i}(T_i - \tau_i)\rho(\tau_i)\,\mathrm{d}\tau_i = \frac{\theta_i^2}{2T_i^2}\left[\frac{2|T_i|}{\theta_i} + \exp\left(-\frac{2|T_i|}{\theta_i}\right) - 1\right]$$

Substituting eq. [$A3$] into eq. [$A1$], for each coordinate direction, provides a reasonable approximation to the numerical integration involved in eq. [15].

List of Symbols

T_i dimension of the averaging domain in the ith direction

γ variance reduction function of CSL (γ_C) or GCL (γ_G)

θ_i correlation length in the ith direction

ρ correlation coefficient between log hydraulic conductivity at two points in the liner

τ_i distance between points in the ith direction for which the correlation coefficient is desired