Geotechnical resistance factors for ultimate limit state design of deep foundations in cohesive soils

Mehrangiz Naghibi and Gordon A. Fenton

Abstract: This paper investigates the ultimate limit state load and resistance factor design (LRFD) of deep foundations founded within purely cohesive soils. The geotechnical resistance factors required to produce deep foundation designs having a maximum acceptable failure probability are estimated as a function of site understanding and failure consequence. The probability theory developed in this paper, used to determine the resistance factors, is verified by a two-dimensional random field Monte Carlo simulation of a spatially variable cohesive soil. The agreement between theory and simulation is found to be very good, and the theory is then used to derive the required geotechnical resistance factors. The results presented in this paper can be used to complement current ultimate limit state design code calibration efforts for deep foundations in cohesive soils.

Key words: reliability-based design, load and resistance factor design, deep foundations, geotechnical resistance factor.

Résumé : Cet article étudie le facteur de conception de charge et résistance (FCCR) de l'état limite ultime de fondations profondes installées dans des sols purement cohésifs. Les facteurs de résistance géotechnique requis pour la conception de fondations profondes ayant une probabilité de rupture acceptable maximale sont estimés en fonction de la compréhension du site et des conséquences d'une rupture. La théorie des probabilités développée dans cet article, et utilisée pour déterminer les facteurs de résistance, est vérifiée par une simulation Monte Carlo avec un champ à deux dimensions pour un sol cohésif ayant une variation spatiale. La concordance entre la théorie et la simulation est très bonne; ainsi la théorie est utilisée pour obtenir les facteurs de résistance géotechnique requis. Les résultats présentés dans cet article peuvent être utilisés pour complémenter les efforts actuels de calibrage du code de conception à l'état limite ultime pour des fondations profondes dans des sols cohésifs.

Mots-clés : conception basée sur la fiabilité, facteur de conception de charge et résistance, fondations profondes, facteur de résistance géotechnique.

[Traduit par la Rédaction]

Introduction

Deep foundations, or piles, may fail through a punching shear failure, an ultimate limit state (ULS), where the load applied to the pile exceeds the shear strength of the surrounding soil (Fenton and Griffiths 2007). The soil supports the pile through a combination of end bearing and friction and (or) cohesion between the soil and the pile sides. In this paper, only cohesive resistance is considered, as would typically be found in a soil under total stress conditions (e.g., purely cohesive), and end bearing is ignored.

As the load on the pile is increased, the bond strength between the soil and the pile surface will eventually be exceeded, and the pile will slip through the surrounding soil. At this point, the ultimate resistance of the pile has been reached. The ultimate resistance of a pile due to cohesion, c, between the pile surface and its surrounding soil is given by

$$R_u = \int_0^H p\tau(z)\,dz \qquad [1]$$

where p is the effective perimeter length of the pile section, $\tau(z)$ is the ultimate shear stress acting on the surface of the pile at depth z (averaged around the perimeter), and H is the buried length of the pile. This paper looks specifically at the case where the soil is frictionless ($\phi = 0$) and the cohesion, $c = s_u$, is the undrained shear strength.

The ultimate shear stress acting between the soil under total stress conditions and the pile can be obtained by several methods. One commonly accepted procedure, the α method, is described briefly by Das (2000). According to the α method, the unit surface shear resistance in soils under total stress conditions can be represented by the equation,

$$\tau(z) = \alpha c(z) \qquad [2]$$

where $c(z)$ is the average soil cohesion around the pile perimeter at depth z, and α is an empirical adhesion factor, typically in the range of 0.3–1, as suggested by the Canadian Foundation Engineering Manual (CFEM) (Canadian Geotechnical Society 2006). For a normally consolidated clay with cohesion, c, less than about 33 kPa, the adhesion factor suggested by Das (2000) is 1.0. In general, the adhesion factor can be written as a function of the average cohesion, ta-

Received 2 April 2011. Accepted 18 August 2011. Published at www.nrcresearchpress.com/cgj on 2 November 2011.

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ken to be approximated by μ_c here, over the pile length (CFEM; Canadian Geotechnical Society 2006),

$$[3] \qquad \alpha = \begin{cases} 0.21 + 0.26P_a/\mu_c & \text{if } \mu_c \geq 33 \text{ kPa} \\ 1 & \text{if } \mu_c < 33 \text{ kPa} \end{cases}$$

where P_a is the standard atmospheric pressure (101.325 kPa).

Substituting eq. [2] into eq. [1], the ultimate cohesive resistance of a pile with length H and perimeter p becomes

$$[4] \qquad R_u = \int_0^H p\alpha c(z)\,dz$$

In the design of a pile, geotechnical engineers must find the effective perimeter, p, and buried length of the pile, H, required to avoid a cohesive resistance failure. In this paper, it is assumed that the pile type has been selected, so that p is known and the design involves determining H. As will be shown later, the value of p has no effect on the required resistance factors presented in this paper.

In any reliability-based design, uncertain quantities such as load and resistance are represented by random variables having some distribution. Distributions are usually specified by their mean, standard deviation, and some shape (e.g., normal or lognormal). The design itself, however, requires design parameters to be used. These design parameters are commonly referred to as *characteristic* or *nominal* values. In this paper, the word *characteristic* is preferred, since these values are generally obtained from an investigation aimed at characterizing the site.

In the design process, the pile length, H, is selected so as to ensure that the pile does not achieve any of the performance limit states. Only the ultimate limit state is considered here, and the design proceeds by ensuring that the factored load does not exceed the factored resistance,

$$[5] \qquad \varphi_{gu}\hat{R}_u \geq \sum_i I_i \alpha_i \hat{F}_i$$

where φ_{gu} is the ultimate geotechnical resistance factor, $\hat{R}_u$ is the characteristic ultimate geotechnical resistance based on characteristic (nominal) soil properties, I_i is an importance factor corresponding to the ith characteristic load effect, $\hat{F}_i$, and α_i is the ith load factor.

The importance factor, I_i, reflects the severity of the failure consequences and may be larger than 1.0 for important structures, such as hospitals, whose failure consequences are high and whose target probabilities of failure should be smaller than usual. Typical structures are generally designed using $I_i = 1$, which is the case assumed in this paper. Structures with low failure consequences (minimal risk of loss of life, injury, and (or) economic impact) may have $I_i < 1$.

In most modern codes, the importance factor is generally applied to highly variable site-specific loads, such as wind, snow, and seismic loads, and is designed to shift the factored load such that the overall probability of failure is lower for more important structures and higher for less important structures. Although the basic idea of adjusting the failure probability to match the failure consequence is definitely good, it is philosophically questionable as to whether this factor should be applied to the load side of eq. [5]. Loads are typically quite indifferent to the structure's importance, whereas the "designed" resistance is very much concerned with the structure's importance. In this paper, the effect of both resistance uncertainty and failure consequence are handled through the single resistance factor, and the importance factor is set to 1.0. The resistance factor in this paper can thus be thought of as the product of a geotechnical resistance factor (capturing site uncertainty) and a consequence factor (capturing failure consequence, the consequence factor is the inverse of the importance factor).

Only one load combination, dead plus live, will be considered in this paper,

$$[6] \qquad \alpha_T\hat{F} = \alpha_L\hat{F}_L + \alpha_D\hat{F}_D$$

where $\hat{F}_L$ is the characteristic live load, $\hat{F}_D$ is the characteristic dead load, α_L and α_D are the live and dead load factors, respectively, α_T is the equivalent total load factor, and $\hat{F} = \hat{F}_L + \hat{F}_D$ is the sum of characteristic live and dead loads. The load factors used in this paper will be as specified by the National Building Code of Canada (NBCC) (National Research Council Canada 2005) where $\alpha_L = 1.5$ and $\alpha_D = 1.25$. The theory presented here, however, is easily extended to other load combinations and factors, so long as their (possibly time-dependent) distributions are known.

In some cases, the characteristic load values used in a design are defined to be the means, but they can be more generally defined in terms of the means as

$$[7a] \qquad \hat{F}_L = k_L\mu_L$$

$$[7b] \qquad \hat{F}_D = k_D\mu_D$$

where μ_L and μ_D are the means of the live and dead loads, and k_L and k_D are live and dead load bias factors, respectively (Fenton and Griffiths 2008). For typical multistorey office buildings, Allen (1975) estimates $k_L = 1.41$, based on a 30 year lifetime. Becker (1996*b*) estimates k_D to be 1.18. The characteristic loads, $\hat{F}_L$ and $\hat{F}_D$, are thus obtained as $\hat{F}_L = 1.41\mu_L$ and $\hat{F}_D = 1.18\mu_D$.

The characteristic ultimate geotechnical resistance, $\hat{R}_u$, is determined using characteristic soil properties, in this case characteristic values of the soil's cohesion, c. To obtain the characteristic soil properties, the soil is assumed to be sampled over a single column somewhere in the vicinity of the pile, for example, by a single cone penetration test (CPT) sounding or field vane test taken near the pile, which yields a sequence of m observed cohesion values, $\hat{c}_1, \hat{c}_2, \ldots, \hat{c}_m$. The characteristic value of the cohesion, $\hat{c}$, is defined in this paper to be an arithmetic average of the sampled observations, $\hat{c}_i$,

$$[8] \qquad \hat{c} = \frac{1}{m}\sum_{i=1}^{m}\hat{c}_i$$

The characteristic ultimate geotechnical resistance, $\hat{R}_u$, can now be obtained from eq. [4] by letting $c(z) = \hat{c}$,

$$[9] \qquad \hat{R}_u = pH\alpha\hat{c}$$

To determine the geotechnical resistance factor, φ_{gu}, required to achieve a certain acceptable reliability, the failure probability of the pile must be estimated. This probability will depend on the load distribution, the load factors selected, the resistance factor, and the resistance distribution. The resistance distribution is discussed in the section "Random soil model", and the load distribution is discussed in the section "Random load model". The section "Theoretical approach to estimating probability of failure" presents a theoretical failure probability model, and the section "Validation of theory via Monte Carlo simulation" assesses the quality of the theoretically predicted failure probability using a Monte Carlo simulation.

The load and resistance factor design (LRFD) approach involves selecting a maximum acceptable failure probability level, p_m. The choice of p_m derives from a consideration of acceptable risk and directly influences the value of φ_{gu}. Different levels of p_m may be considered to reflect the "importance" of the supported structure; p_m may be much smaller for a hospital than for an uninhabited storage warehouse.

The choice of a maximum acceptable failure probability, p_m, should consider the margin of safety implicit in current foundation designs and the levels of reliability for geotechnical design as reported in the literature. The values of p_m for foundation designs should be nearly the same or somewhat less than that of the supported structure because of the difficulties and high expense of foundation repairs. A literature review of the suggested maximum acceptable failure probability for foundations is listed in Table 1.

Meyerhof (1995) suggests that a typical lifetime failure probability for a foundation is around 10^{-4}, and so the numbers in Table 1 range on the high side of that suggested by Meyerhof. However, foundations are normally supported by more than a single pile, and multiple piles provide at least some degree of system redundancy, which serves to reduce the system failure probability. That is, if a single pile in a pile group happens to be placed in an exceptionally low-strength region and fails, its load will be transferred to surrounding piles having greater resistances, and the overall foundation is less likely to fail. If it is assumed that Meyerhof's 1995 estimate is for the entire foundation system, then the required failure probability for a single pile would be greater than his suggested system failure probability of 10^{-4}. The Federal Highway Administration (FHWA) (Allen 2005) suggests that a reasonable value of maximum acceptable failure probability for single driven piles within a redundant group may be in the range of 10^{-2}–10^{-3}. Although more research is no doubt required to determine the failure levels appropriate for redundant pile systems, the National Cooperative Highway Research Program (NCHRP) reports (Barker et al. 1991; Paikowsky 2004) are based on a lifetime failure probability of about 10^{-3} for an individual pile, which suggests that the NCHRP is also considering pile redundancy.

In this paper, four maximum acceptable failure probabilities for an individual pile, 10^{-2}, 10^{-3}, 10^{-4}, and 10^{-5}, will be considered. The failure probabilities, 10^{-3}, 10^{-4}, and 10^{-5}, might be appropriate for designs involving low (e.g., storage facilities), medium (typical structures), and high (e.g., hospitals and schools) failure consequence structures, respectively. The geotechnical resistance factors required to achieve these maximum acceptable failure probabilities will be recommended in the section "Geotechnical resistance factors".

Table 1. Literature review of lifetime probabilities of failure of foundations.

Source	p_m
Meyerhof (1970, 1993, 1995)	10^{-2}–10^{-4}
Simpson et al. (1981)	10^{-3}
Barker et al. (1991)	10^{-2}–10^{-4}
Becker (1996*a*)	10^{-3}–10^{-4}

Random soil model

The soil cohesion, c, is assumed to be lognormally distributed with mean, μ_c, standard deviation, σ_c, and some spatial correlation structure. The lognormal distribution is selected because it is commonly used to represent nonnegative soil properties and has a simple relationship with the normal; a lognormally distributed random field can be obtained from a normally distributed random field, $G_{\ln c}(z)$, having zero mean, unit variance, and spatial correlation length, θ, through the transformation

$$[10]\qquad c(z) = \exp[\mu_{\ln c} + \sigma_{\ln c} G_{\ln c}(z)]$$

where z is the depth at which c is desired. The mean and variance of ln$c$ are obtained from the specified mean and variance of cohesion using the transformations

$$[11]\qquad \sigma^2_{\ln c} = \ln(1 + v_c^2), \quad \mu_{\ln c} = \ln(\mu_c) - \frac{1}{2}\sigma^2_{\ln c}$$

where $v_c = \sigma_c/\mu_c$ is the coefficient of variation of the cohesion.

The correlation coefficient between the log cohesion at some point z_1 and a second point z_2, is specified by a correlation function, ρ. In this study, a simple exponentially decaying (Markovian) correlation function will be assumed, having the form

$$[12]\qquad \rho(t) = \exp\left\{-\frac{2|t|}{\theta}\right\}$$

where $t = z_1 – z_2$ is the distance between the two points.

It should be noted that the correlation function selected above acts between values of ln$c$ because ln$c$ is normally distributed, and a normally distributed random field is simply defined by its mean and covariance structure. In practice, the correlation length, θ, can be estimated by evaluating spatial statistics of the log cohesion data directly (see, e.g., Fenton 1999).

The spatial correlation length, θ, appearing in eq. [12], is loosely defined as the separation distance within which two values of ln$c$ are significantly correlated. Mathematically, θ is defined as the area under the correlation function, $\rho(t)$ (Vanmarcke 1984). The spatial correlation function, $\rho(t)$, has a corresponding variance reduction function, $\gamma(H)$, which specifies how the variance is reduced upon local averaging of ln$c$ over some length H and is defined by

$$[13]\qquad \gamma(H) = \frac{1}{H^2}\int_0^H\int_0^H \rho(z_1 - z_2)\,dz_1\,dz_2$$

In the illustrative example presented later in this paper, the average cohesion, μ_c, is assumed to be 50 kPa, and so its corresponding adhesion factor, α, is given by eq. [3] to be $\alpha = 0.74$. As will be shown later, both μ_c and α cancel out of the failure probability prediction equations, and so the choice in their values is entirely arbitrary and will make no difference to the required geotechnical resistance factor used in the design process.

Random load model

The load acting on a foundation is typically composed of dead loads, which are largely static, and live loads, which are largely dynamic. Dead loads are relatively well defined and can be computed by multiplying volumes by characteristic unit weights. The mean and variance of dead loads are thus reasonably well known. On the other hand, live loads are more difficult to characterize probabilistically. A typical definition of a live load is the maximum dynamic load (e.g., wind, vehicle, bookshelf loads) that a structure will experience during its design life. Note that the distribution of live load depends on the design lifetime. Dead and live loads will be denoted as F_D and F_L, respectively. Assuming that the total load, F, is equal to the sum of the maximum lifetime live load, F_L, and the static dead load, F_D, i.e.,

[14] $$F = F_L + F_D$$

then the mean and variance of F, assuming dead and live loads are independent, are given by

[15*a*] $$\mu_F = \mu_L + \mu_D$$

[15*b*] $$\sigma_F^2 = \sigma_L^2 + \sigma_D^2$$

The dead and live loads are assumed to be lognormally distributed. The total load, $F = F_L + F_D$, is also assumed to be lognormally distributed, which was found to be reasonable by Fenton et al. (2008).

The total load distribution has parameters,

[16*a*] $$\mu_{\ln F} = \ln(\mu_F) - \frac{1}{2}\sigma_{\ln F}^2$$

[16*b*] $$\sigma_{\ln F}^2 = \ln\left(1 + \frac{\sigma_F^2}{\mu_F^2}\right)$$

The example problem presented in this study involves a pile supporting loads having means and standard deviations shown in Table 2. As with the cohesion mean, it will be shown later that the resistance factors are also independent of the load means, μ_L and μ_D. Thus, the values of μ_L and μ_D are arbitrary, having no effect on the paper's results, and were selected here mainly to ensure that the designed pile length, H, doesn't exceed the depth of the soil model used in the simulation.

Assuming bias factors, $k_D = 1.18$ (Becker 1996*b*) and $k_L = 1.41$ (Allen 1975), and importance factor, $I_i = 1.0$, gives the characteristic live load, $\hat{F}_L = 1.41\mu_L = 28.2$ kN, dead load, $\hat{F}_D = 1.18\mu_D = 70.8$ kN, and characteristic total factored design load, $\alpha_L\hat{F}_L + \alpha_D\hat{F}_D = 1.5\hat{F}_L + 1.25\hat{F}_D = (1.5)(28.2) + (1.25)(70.8) = 130.8$ kN.

Theoretical approach to estimating probability of failure

To estimate the probability of failure of a pile, the soil is first modeled as a spatially varying random field. In general, cohesion will vary in all three dimensions, but there is little advantage in considering the third dimension, since piles are essentially one-dimensional, and only the second dimension is needed to consider distance between a sample and the pile location. Hence, this study considers a two-dimensional random field in which the pile is placed vertically at a certain position, and soil samples, as in CPT or standard penetration test (SPT) sounding, are taken vertically at some, possibly different, position, as illustrated in Fig. 1. The theoretical approximation to the probability of pile failure in soils under total stress conditions is explained as follows.

When the soil properties are spatially variable, as they are in reality, then it is proposed that eq. [4] can be replaced by

[17] $$R_u = pH\alpha\bar{c}$$

where $\bar{c}$ is the equivalent cohesion, defined as the uniform cohesion value that leads to the same ultimate strength as observed in the spatially varying soil over a pile of length, H. It is hypothesized here that $\bar{c}$ is the arithmetic average of the spatially variable cohesion over the pile length H,

[18] $$\bar{c} = \frac{1}{H}\int_0^H c(z)\,\mathrm{d}z \simeq \frac{1}{n}\sum_{i=1}^{n}\bar{c}_i$$

where $c(z)$ is interpreted as an average cohesion around the pile perimeter at depth z. If the pile is broken up into a series of n elements (as will be done in the simulation), the average is determined using the sum at the right of eq. [18], where $\bar{c}_i$ is the local average of $c(z)$ over the ith element, for $i = 1, \ldots, n$.

The required minimum design pile length, H, can be obtained by substituting eq. [9] into eq. [5] (taking $I_i = 1.0$),

[19] $$\varphi_{gu}pH\alpha\hat{c} = \alpha_L\hat{F}_L + \alpha_D\hat{F}_D \rightarrow H = \frac{\alpha_L\hat{F}_L + \alpha_D\hat{F}_D}{\varphi_{gu}p\alpha\hat{c}}$$

By further substituting eq. [19] into eq. [17], the ultimate resistance, R_u, can be estimated to be

[20] $$R_u = \left(\frac{\alpha_L\hat{F}_L + \alpha_D\hat{F}_D}{\varphi_{gu}}\right)\left(\frac{\bar{c}}{\hat{c}}\right)$$

The reliability-based design goal is to find the required length H such that the probability of the actual load, F, exceeding the actual resistance, R_u, is less than some maximum acceptable failure probability, p_m. The actual failure probability, p_f, is

[21] $$p_f = \mathrm{P}[F > R_u]$$

and a successful design methodology will have $p_f \leq p_m$. Substituting eq. [20] into eq. [21] leads to

[22] $$p_f = \mathrm{P}\left[F > \left(\frac{\alpha_L\hat{F}_L + \alpha_D\hat{F}_D}{\varphi_{gu}}\right)\left(\frac{\bar{c}}{\hat{c}}\right)\right] = \mathrm{P}\left[\frac{F\hat{c}}{\bar{c}} > \frac{\alpha_L\hat{F}_L + \alpha_D\hat{F}_D}{\varphi_{gu}}\right]$$

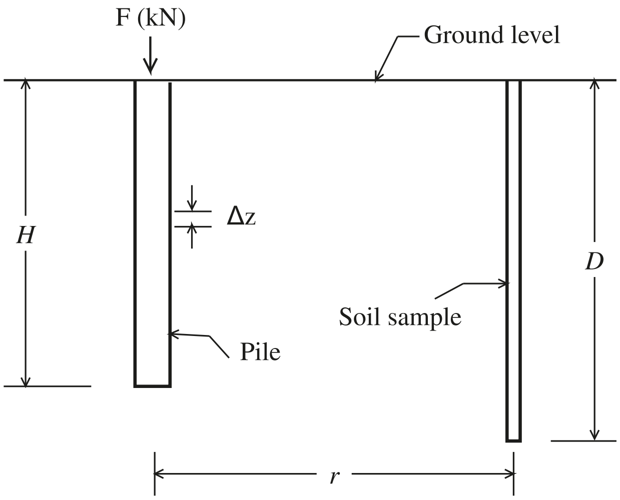
where in the last step, all random quantities were moved to the left-hand side (LHS) of the inequality.

Note that the design parameters, α (adhesion factor) and p (perimeter length), have both cancelled out of the above fail-

Table 2. Load distribution parameters.

μ_L (kN)	μ_D (kN)	σ_L (kN)	σ_D (kN)	μ_F (kN)	σ_F (kN)	$\mu_{\ln F}$	$\sigma_{\ln F}$
20	60	6	9	80	10.82	4.4	0.14

Fig. 1. Relative locations of pile and soil samples.



ure probability estimate. This means that the values of these parameters will not affect the required resistance factors obtained in this study. It is also instructive to investigate how varying the means of the load and cohesion might influence the failure probability, and hence the required resistance factor. If eq. [22] is rearranged so that the load and resistance terms are grouped together, one gets

$$[23] \qquad p_f = P\left[\frac{F}{\alpha_L \hat{F}_L + \alpha_D \hat{F}_D} > \frac{1}{\varphi_{gu}}\left(\frac{\bar{c}}{\hat{c}}\right)\right]$$

which involves comparing the distribution of the load term on the LHS of the inequality to the distribution of the cohesive term on the right-hand side (RHS). Interest is in how these two quantities are affected by changes in their means. The mean of the load term is (see eq. [15*a*] and discussion at the end of section "Random load model")

$$[24] \qquad E\left[\frac{F}{\alpha_L \hat{F}_L + \alpha_D \hat{F}_D}\right] = \frac{\mu_L + \mu_D}{\alpha_L k_L \mu_L + \alpha_D k_D \mu_D} = \frac{1 + R_{D/L}}{\alpha_L k_L + \alpha_D k_D R_{D/L}}$$

where E is the expectation operator. From eq. [24] it can be seen that the mean of the load term depends only on the ratio of the dead to live load means, $R_{D/L} = \mu_D/\mu_L$, and not on the actual load means. Thus, the load means can be scaled by any common amount without affecting the overall pile failure probability predicted here.

The RHS of the inequality in eq. [23] involves the random quantity $\bar{c}/\hat{c}$, which has mean

$$[25] \qquad E\left[\frac{\bar{c}}{\hat{c}}\right] = \frac{\exp\left[(1/2)\left(\sigma_{\hat{c}}^2 - \sigma_{\bar{c}}^2\right)\right]\sqrt{\left(1 + \nu_{\hat{c}}^2\right)\left(1 + \nu_{\bar{c}}^2\right)}}{\exp\left[\rho\sqrt{\ln\left(1 + \nu_{\hat{c}}^2\right)\ln\left(1 + \nu_{\bar{c}}^2\right)}\right]}$$

where $\nu = \sigma/\mu$ is the coefficient of variation of the subscripted variable, and ρ is the correlation coefficient between $\ln\hat{c}$ and $\ln\bar{c}$. Other than the correlation coefficient, eq. [25] depends only on the variance and coefficient of variation of the cohesion. If the sample length is (at least approximately) the same as the pile length, then $\sigma_{\hat{c}}^2 = \sigma_{\bar{c}}^2$, and the mean of $\bar{c}/\hat{c}$ depends only on ρ and the coefficient of variation. Thus, so long as the coefficient of variation is held constant, the failure probability (and, hence, resistance factor) presented here is basically independent of the choice in the mean of the cohesion.

Returning now to the computation of the failure probability in eq. [22], the following two quantities are defined as

$$[26a] \qquad W = \frac{F\hat{c}}{\bar{c}}$$

$$[26b] \qquad \hat{Q} = \alpha_L \hat{F}_L + \alpha_D \hat{F}_D$$

so that eq. [22] can be written as

$$[27] \qquad p_f = P\left[W > \frac{\hat{Q}}{\varphi_{gu}}\right]$$

The solution to eq. [27] involves the determination of the distribution of *W*. If the random load, *F*, and cohesion values, $\hat{c}$ and $\bar{c}$, are all assumed to be lognormally distributed, which is a reasonable assumption (see, e.g., Fenton et al. 2008), then *W* will also be lognormally distributed and its parameters can be determined by considering the individual distributions of *F*, $\ln\hat{c}$, and $\ln\bar{c}$.

If *W* is lognormally distributed, then

$$[28] \qquad \ln W = \ln F + \ln\hat{c} - \ln\bar{c}$$

is normally distributed and p_f can be found from

$$[29] \qquad p_f = P[W > \hat{Q}/\varphi_{gu}] = P[\ln W > \ln(\hat{Q}/\varphi_{gu})] = 1 - \Phi\left[\frac{\ln(\hat{Q}/\varphi_{gu}) - \mu_{\ln W}}{\sigma_{\ln W}}\right]$$

where Φ is the standard normal cumulative distribution function.

The failure probability p_f in eq. [29] can be estimated once the mean and variance of $\ln W$ are determined. The mean and variance of $\ln W$ are

$$[30a] \qquad \mu_{\ln W} = \mu_{\ln F} + \mu_{\ln\hat{c}} - \mu_{\ln\bar{c}}$$

$$[30b] \qquad \sigma_{\ln W}^2 = \sigma_{\ln F}^2 + \sigma_{\ln\hat{c}}^2 + \sigma_{\ln\bar{c}}^2 - 2\mathrm{Cov}(\ln\hat{c},\ \ln\bar{c})$$

where the total load, *F*, and cohesion, *c*, are assumed to be independent. The components of eq. [30] can now be computed as follows:

1. As discussed in the section "Random load model", the total load, *F*, is equal to the sum of the live load, F_L, and

the dead load, F_D, i.e., $F = F_L + F_D$, and the mean and variance of ln$F$ can be evaluated using eqs. [15] and [16].

2. With reference to eq. [8],

$$[31a] \qquad \mu_{\ln\hat{c}} = \mathrm{E}[\ln\hat{c}] = \mathrm{E}\left[\ln\left(\frac{1}{m}\sum_{i=1}^{m}\hat{c}_i\right)\right] \simeq \ln(\mu_c)$$

$$[31b] \qquad \sigma^2_{\ln\hat{c}} \simeq \frac{\sigma^2_{\ln c}}{m^2}\sum_{i=1}^{m}\sum_{j=1}^{m}\rho(z_i^o - z_j^o)$$

where z_i^o is the spatial location of the center of the ith soil sample (i = 1, 2, …, m), and ρ is the correlation function defined by eq. [12]. Both equations make use of first-order Taylor series approximations (see Naghibi (2010) for more details). A further approximation occurs in the variance (eq. [31b] because of the fact that correlation coefficients between the local averages associated with observations are approximated by correlation coefficients between the local average centers. Assuming that ln$\hat{c}$ actually represents a local average of ln$c$ over the sample domain of length, D (see Fig. 1), then $\sigma^2_{\ln\hat{c}}$ is probably more accurately computed as

$$[32] \qquad \sigma^2_{\ln\hat{c}} = \sigma^2_{\ln c}\gamma(D)$$

where $\gamma(D)$ is the variance reduction function, given by eq. [13], that measures the reduction in variance due to local averaging over the sample domain D. In this research, the sample domain, D, is assumed to be $D = \Delta z \times m$, where m is the number of observations over sample domain D, and Δz is the vertical dimension of a single soil "sample" (e.g., the spacing between measurements).

3. With reference to eq. [18] and using many of the same arguments as in the previous item (see Naghibi (2010) for details),

$$[33a] \qquad \mu_{\ln\bar{c}} = \mathrm{E}\left[\ln\left(\frac{1}{H}\int_0^H c(z)\,\mathrm{d}z\right)\right] \simeq \ln(\mu_c)$$

$$[33b] \qquad \sigma^2_{\ln\bar{c}} \simeq \sigma^2_{\ln c}\gamma(H)$$

where $\gamma(H)$ is defined by eq. [13].

4. The covariance in eq. [30] between the arithmetic average of the observed cohesion values over the sample domain, $D = \Delta z \times m$, and the cohesion along the pile length, H, is obtained as follows (Naghibi 2010):

$$[34] \qquad \mathrm{Cov}(\ln\hat{c},\ \ln\bar{c}) \simeq \frac{\sigma^2_{\ln c}}{mH}\sum_{i=1}^{m}\int_0^H \rho\left[\sqrt{r^2 + (z - z_i^o)^2}\right]\mathrm{d}z \simeq \sigma^2_{\ln c}\gamma_{HD}$$

where γ_{HD} is the average correlation coefficient between the cohesion samples over domain D and the cohesion along the pile of length H, and ρ is the correlation function between ln$c(z_i^o)$ and ln$c(z)$. In detail, γ_{HD} is defined by

$$[35] \qquad \gamma_{HD} \simeq \frac{1}{mH}\sum_{i=1}^{m}\int_0^H \rho\left[\sqrt{r^2 + (z - z_i^o)^2}\right]\mathrm{d}z$$

where r is the horizontal distance between the pile centerline and the centerline of the soil sample column, as shown in Fig. 1. The approximation in the covariance arises both because a first-order Taylor series approximation is used and because correlation coefficients between local averages associated with observations are approximated by correlation coefficients between the local average centers.

Substituting eqs. [16], [31]–[34] into eq. [30] leads to

$$[36a] \qquad \mu_{\ln W} = \mu_{\ln F}$$

$$[36b] \qquad \sigma^2_{\ln W} \simeq \sigma^2_{\ln F} + \sigma^2_{\ln c}[\gamma(D) + \gamma(H) - 2\gamma_{HD}]$$

which allows the probability of failure to be obtained using eq. [29]. The argument to Φ in eq. [29] is the reliability index,

$$[37] \qquad \beta = \frac{\ln(q/\varphi_{gu}) - \mu_{\ln W}}{\sigma_{\ln W}}$$

If the reliability index is specified through knowledge of p_m, then the geotechnical resistance factor is determined by

$$[38] \qquad \varphi_{gu} = \exp(\ln q - \mu_{\ln W} - \beta\sigma_{\ln W})$$

Validation of theory via Monte Carlo simulation

In this section, probabilistic analyses of pile capacities using Monte Carlo simulation are performed. The objective is to investigate the failure probability of a pile in soils under total stress conditions with spatially varying cohesion field, c, via simulation to validate the theory developed in the previous section. The simulation essentially proceeds by carrying out a series of hypothetical designs on simulated soil fields and checking to see what fraction of the designs fail. In practice, the accuracy of the Monte Carlo method depends on how well the assumed probability distribution fits the real stochastic process. If the fit is reasonable, the accuracy increases with the number of simulation runs, i.e., improved results will be obtained as the number of simulation realizations increases. In detail, the steps involved in the Monte Carlo simulation are as follows;

1. The cohesion, c, of a soil mass is simulated as a spatially variable random field using the local average subdivision (LAS) method (Fenton and Vanmarcke 1990). The number of soil cells in the X and Y directions are assumed to be 128 × 128, and each cell size is taken to be 0.1 × 0.1. The cohesion is assumed to be lognormally distributed, with mean 50 kPa and coefficient of variation, v_c, ranging from 0.1 to 0.5 The correlation length is varied from 0 to 50 m.
2. The simulated soil is sampled along a vertical line through the soil at some distance, r, from the pile. These virtually sampled soil properties are used to estimate the characteristic cohesion, $\hat{c}$, according to eq. [8]. Three sampling distances are considered: the first is at r = 0 m, which means that the samples are taken at the pile location. In this case, uncertainty about the pile resistance only arises if the pile extends below the sampling

depth. Typically, probabilities of failure when $r = 0$ m are very small. The other two sample distances considered are $r = 4.5$ m and $r = 9.0$ m, corresponding to reducing understanding of the soil conditions at the pile location. These rather arbitrary distances were based on preliminary random field simulations, which happened to involve fields 9 m in width. However, it is really the ratio, r/θ, that governs the failure probability, and a wide range in the correlation length, θ, has been considered.

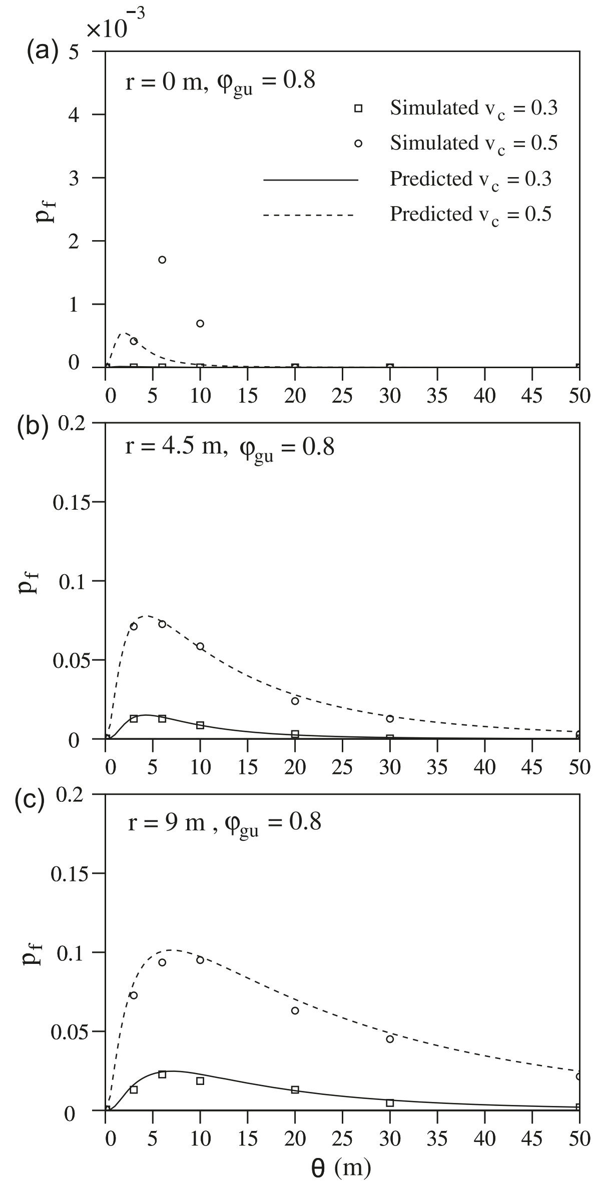
3. Once the characteristic cohesion has been established, the required design pile length, H, is calculated using eq. [19].
4. Dead and live loads, F_D and F_L, are simulated as independent lognormally distributed random variables and then added to produce the actual total load on the pile, $F = F_L + F_D$ The means and standard deviations of the dead and live loads assumed in this simulation are given in Table 2.
5. The “true” ultimate pile resistance, R_u, is computed according to eq. [4] using the averaged cohesion values simulated along either side of the pile.
6. The ultimate resistance, R_u, and total load F are compared. If $F > R_u$, then the pile, as designed in step 3, is assumed to have failed.
7. The entire process from steps 1 to 6 is repeated n_{sim} times (where $n_{sim} = 10\,000$ in the present study). If n_f of these repetitions result in a pile failure, then an estimate of the probability of failure is $p_f = n_f / n_{sim}$.
8. Repeating steps 1–7 using various values of φ_{gu} in the design step allows plots of failure probability versus geotechnical resistance factor to be produced for the various sampling distances, coefficient of variation of the cohesion, and correlation length.

The failure probabilities estimated by theory, via eq. [29], can be superimposed on the simulation-based failure probability plots, allowing a direct comparison of the methods. Figure 2 illustrates the agreement between theory and simulation. Given all the approximations made in the theory, the agreement is considered to be excellent, allowing the geotechnical resistance factors to be computed theoretically with reasonable confidence, even at probability levels that the simulation cannot estimate — the simulation involved only 10 000 realizations and so cannot properly resolve probabilities less than about 10^{-4}.

It is immediately clear from Fig. 2 that the probability of failure, p_f, increases with soil variability, v_c, which is to be expected. Also, as expected, the probabilities of failure are smaller when the soil is sampled directly at the pile than when sampled some distance away from the pile centerline. This means that considerable construction savings may be achieved by improving the sampling scheme, especially when significant soil variability exists.

The largest discrepancies between theory and simulation occur for the small probabilities when the sampling point is at the pile location ($r = 0$ m). The discrepancies at very small probabilities may be largely due to estimator error in the simulations. For example, if a simulation has 17 failures out of 10 000, as in the highest point in Fig. 2*a*, the estimated probability of failure is $p_f = 0.0017$, which has standard error, $\sigma_{\hat{p}_f} = \sqrt{(0.0017)(0.9983)/10\,000} \simeq 0.0004$, and the 95% confidence interval on p_f is $0.0017 \pm 1.96(0.0004) =$

Fig. 2. Comparison of failure probabilities estimated by simulation (10 000 realizations) and theory for geotechnical resistance factor ($\varphi_{gu} = 0.8$) and three sampling locations: (*a*) $r = 0$ m; (*b*) $r = 4.5$ m; (*c*) $r = 9$ m.



[0.0009, 0.0025], which is quite wide. In fact, if only five failures are observed, then the 95% confidence interval on p_f is [0.0001, 0.0009]. In other words, the simulation results cannot be trusted for p_f values less than about 0.001.

The generally good agreement between simulation and theory suggests that the theory can be used to reliably estimate the pile failure probabilities. The theory will be used in the following section to provide recommendations regarding required geotechnical resistance factors for certain target probabilities of failure.

Geotechnical resistance factors

In this section, the geotechnical resistance factors, φ_{gu}, required to achieve four maximum acceptable failure probability levels (10^{-2}, 10^{-3}, 10^{-4}, and 10^{-5}) are proposed. The corresponding reliability indices, β, of these four target probabilities are approximately 2.3, 3.1, 3.7, and 4.3, respectively.

Figures 3–5 show the geotechnical resistance factors required for the cases where the soil is sampled at the pile location, at a distance of 4.5 m and at a distance of 9 m from the pile centerline, respectively.

Figure 3 corresponds to sampling at the pile location where the design conditions are so well understood that the geotechnical resistance factor exceeds 1.0 when $p_m \geq 10^{-3}$. For this reason, the $p_m = 10^{-2}$ and 10^{-3} cases are not shown in Fig. 3.

The worse case (lowest) geotechnical resistance factors occur when the correlation length, θ, is between about 1 and 10 m. This worst case is important, since the correlation length is very hard to estimate and will be unknown for most sites. In other words, in the absence of knowledge about the correlation length, the lowest geotechnical resistance factor in these plots, at the worst case correlation length, should be used.

To explain why a worst case exists, the nature of the correlation length must be considered. The correlation length, θ, measures the distance within which soil properties are significantly correlated. Low values of θ lead to soil properties that vary rapidly in space, while high values mean that the soil properties vary only slowly with position. A large correlation length, of say $\theta = 50$ m, means that soil samples taken well within 50 m from the pile location will be quite representative of the soil properties at the pile location. In other words, lower failure probabilities are expected when the soil is sampled well within the distance θ from the pile.

Alternatively, when θ is very small (say, 0.01 m), then any soil sample will consist of a large number of independent "observations" whose average tends to be equal to the true mean. Since the pile also averages the soil properties, the pile then "sees" the same true mean value predicted by the soil sample, assuming that the soil's random properties are "stationary". In other words, when θ is small and stationarity holds, the sample will accurately reflect the average conditions along the pile, and in this case, the failure probability is again low.

At intermediate correlation lengths, soil samples become less accurate estimators of conditions along the pile, and so the probability of failure increases, or conversely, the required geotechnical resistance factor decreases. Thus, the minimum required geotechnical resistance factor will occur at some correlation length between zero and infinity. In general, the authors have found that the worst-case correlation length occurs when θ is approximately equal to the distance from the pile to the sampling location. Notice in Figs. 3–5 that the worst-case correlation length does show an increase as the distance to the sample location, r, increases.

The smallest geotechnical resistance factors correspond to the worst-case correlation length at the smallest acceptable failure probability shown, $p_m = 10^{-5}$, when the soil is sampled 9 m away from the pile centerline (Fig. 5). When the cohesion coefficient of variation is relatively large, $v_c = 0.5$, the worst-case values of φ_{gu} dip down to 0.15 to achieve $p_m = 10^{-5}$. In other words, there will be a significant construction cost penalty if a high-reliability pile is to be designed using a site investigation, which is insufficient to reduce the residual variability to less than $v_c = 0.5$.

The worst-case geotechnical resistance factors required to achieve the indicated maximum acceptable failure probabilities, as seen in Figs. 3–5, are summarized in Table 3. Some of the geotechnical resistance factors recommended in this study for $p_m = 10^{-2}$ are greater than 1.0, which may be because the load factors provide too much safety for the larger acceptable failure probabilities when the site is well understood.

Table 4 compares the geotechnical resistance factors recommended in this paper with those recommended by the CFEM (Canadian Geotechnical Society 1992, 2006), NBCC (National Research Council Canada 2005), Canadian Highway Bridge Design Code (CHBDC) (Canadian Standards Association 2006), Australian Standard Bridge Design Part 3 (AS 5100.3 (Standards Australia 2004)), two American Association of State Highway and Transportation Officials codes (AASHTO 2002, 2007), and a NCHRP report (Paikowsky 2004). The 2006 CFEM (Canadian Geotechnical Society 2006) references both the 2005 NBCC and the 2006 CHBDC, rather than provide its own list of resistance factors, and so the 2006 CFEM does not explicitly appear in Table 4. The geotechnical resistance factors recommended in this paper (first six rows of Table 4) correspond to the cases where $v_c = 0.5$, $r = 4.5$ m and 9.0 m, and maximum acceptable failure probabilities of $p_m = 10^{-3}$, 10^{-4}, and 10^{-5}.

A review of eq. [5] shows that the load factors and resistance factors used in a design cannot be selected independently. For a fixed level of uncertainty in the loads and resistances, the system reliability remains constant when the load factor is changed only if the resistance factor is correspondingly changed so that their ratio remains constant. In other words, to properly compare the recommended geotechnical resistance factors, φ_{gu}, with values presented by other codes and the literature, it is the ratio of the total load factor to the geotechnical resistance factor, α_T/φ_{gu}, which must be compared. This ratio is essentially an estimate of the "safety factor" commonly used in traditional geotechnical design.

The dead load factor, $\alpha_D = 1.25$, and live load factor, $\alpha_L = 1.5$, used in this paper, are as specified by the NBCC (National Research Council Canada 2005). Bias factors of $k_D = 1.18$ (Becker 1996*b*), $k_L = 1.41$ (Allen 1975), and the ratio of dead to live load means $R_{D/L} = 3.0$, are assumed here. The characteristic dead to live load ratio, $\hat{R}_{D/L}$, and the total load factor, α_T, in this paper are then

$$[39a] \qquad \hat{R}_{D/L} = \frac{\hat{F}_D}{\hat{F}_D} = \frac{k_D \mu_D}{k_L \mu_L} = \frac{1.18(3\mu_L)}{1.41\mu_L} = \frac{1.18(3)}{1.41} = 2.5$$

$$[39b] \qquad \alpha_T = \frac{\alpha_L \hat{F}_L + \alpha_D \hat{F}_D}{\hat{F}_L + \hat{F}_D} = \frac{\alpha_L + \alpha_D \hat{R}_{D/L}}{1 + \hat{R}_{D/L}} = \frac{1.5 + 1.25(2.5)}{1 + 2.5} = 1.32$$

As can be seen in Table 4, the total load factors used in the CFEM (Canadian Geotechnical Society 1992), CHBDC (Canadian Standards Association 2006), and NBCC (National Research Council Canada 2005), which is now referred to by the CFEM (Canadian Geotechnical Society 1992, 2006) are all very close to the total load factor used in the current study and differ only because of a slight difference in the character-

Fig. 3. Geotechnical resistance factors when the soil has been sampled at the pile location (r = 0 m) (note the reduced vertical scale): (a) $p_m = 10^{-4}$; (b) $p_m = 10^{-5}$.

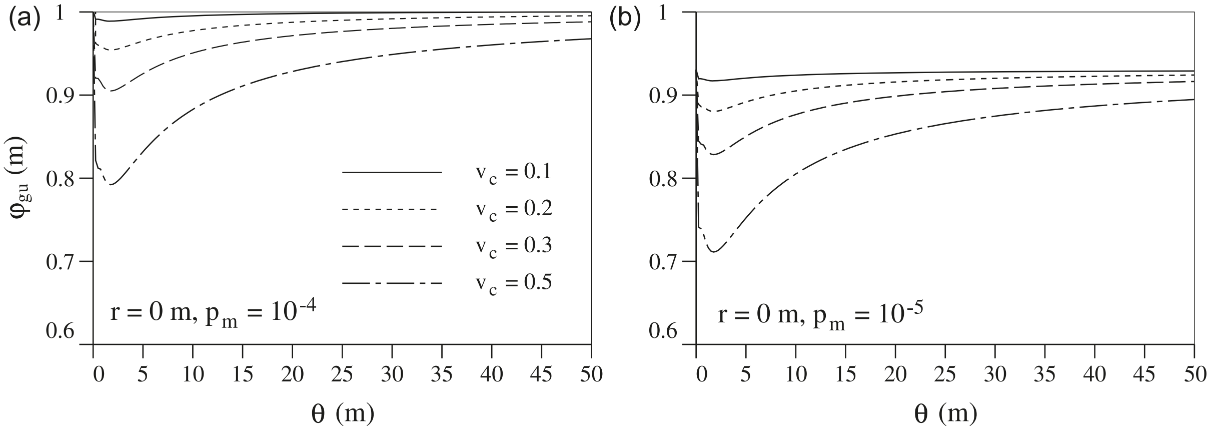
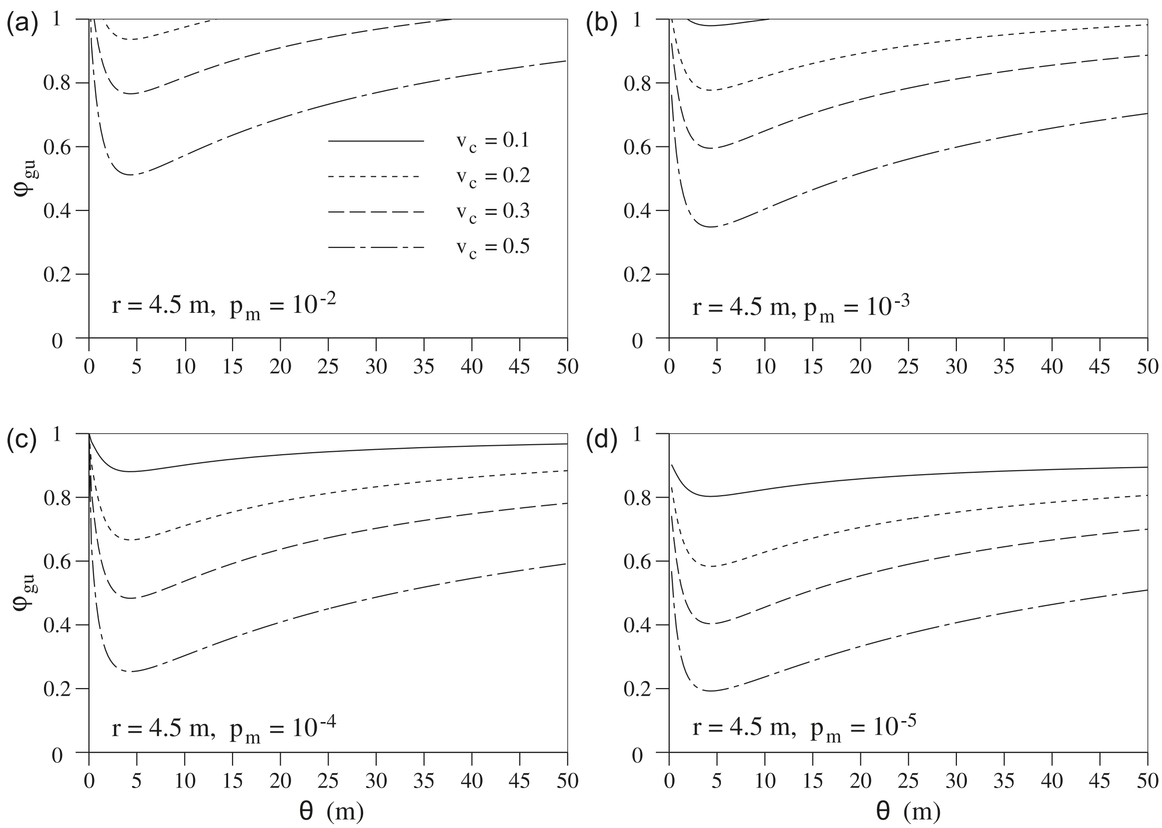


Fig. 4. Geotechnical resistance factors when the soil has been sampled (r = 4.5 m) from the pile centerline: (a) $p_m = 10^{-2}$; (b) $p_m = 10^{-3}$; (c) $p_m = 10^{-4}$; (d) $p_m = 10^{-5}$.



istic dead to live ratio, $R_{D/L}$, and differing load factors in the CHBDC. The ratio of the total load factor to the resistance factor, α_T/φ_{gu}, in the CFEM (Canadian Geotechnical Society 1992) is close to that recommended here for r = 4.5 and $p_m = 10^{-4}$, while the same ratios in the NBCC (National Research Council Canada 2005) and CHBDC (Canadian Standards Association 2006) are very close to those recommended here for either r = 4.5 and $p_m = 10^{-5}$ or r = 9.0 and $p_m = 10^{-4}$.

Similarly, the total load factor used in the Australian standard, AS5100.3 (Standards Australia 2004), is very close to the total load factor used in the current study. The range of ratios of the total load factor to the resistance factor, α_T/φ_{gu}, specified by the Australian standard, AS5100.3 (Standards Australia 2004), covers pretty much the same range as recommended here for all but the lowest failure probability, $p_m = 10^{-5}$.

The ratio, α_T/φ_{gu}, used in AASHTO (2002) corresponds approximately to the ratio found here for $p_m = 10^{-3}$, while AASHTO (2007) seems to have decided on a higher level of safety, corresponding approximately to the $p_m = 10^{-5}$ results found here. The ratio, α_T/φ_{gu}, used in NCHRP 507 (Paikowsky 2004) are based on a target reliability index of 3.0 (p_m = 0.0013). The total load factor, α_T, considered in NCHRP 507

Fig. 5. Geotechnical resistance factors when the soil has been sampled (r = 9 m) from the pile centerline: (*a*) $p_m = 10^{-2}$; (*b*) $p_m = 10^{-3}$; (*c*) $p_m = 10^{-4}$; (*d*) $p_m = 10^{-5}$.

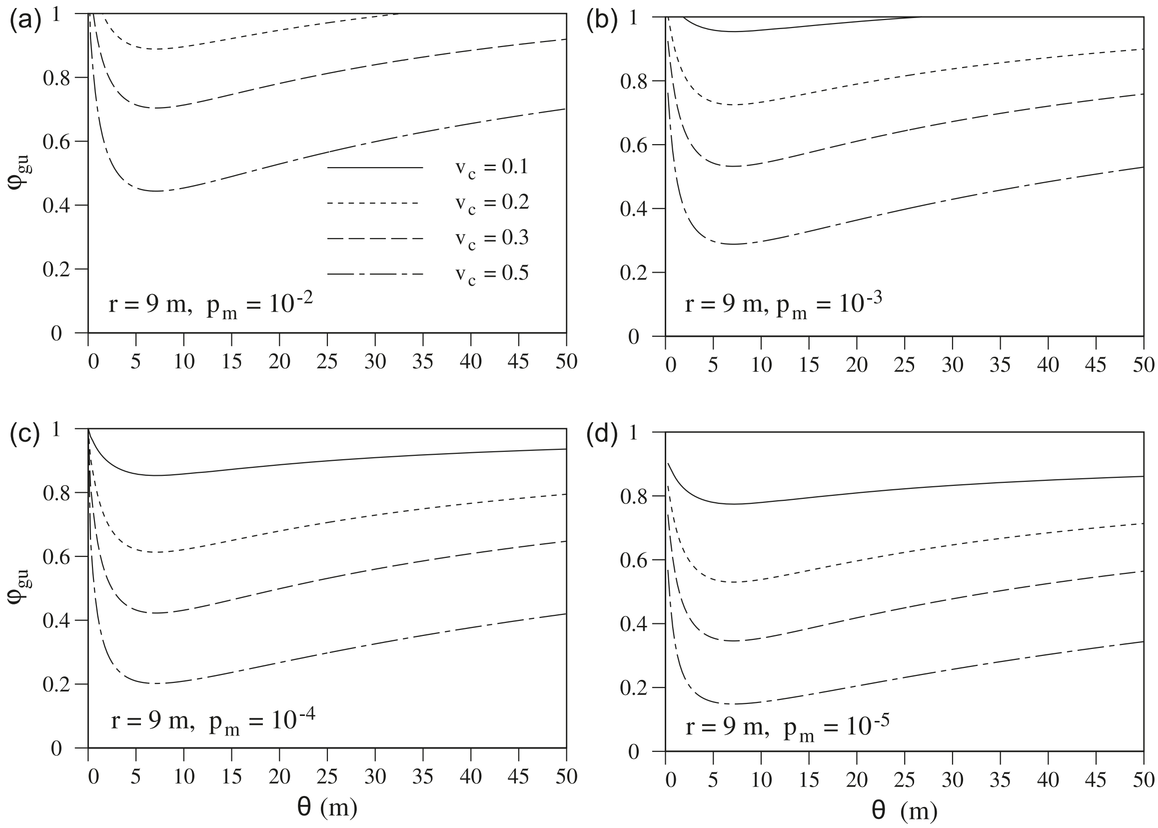


Table 3. Worst-case geotechnical resistance factors for various coefficients of variation, v_c, distance to sampling location, r, and acceptable failure probabilities, p_m.

		Geotechnical resistance factor			
r (m)	v_c	$p_m = 10^{-2}$	$p_m = 10^{-3}$	$p_m = 10^{-4}$	$p_m = 10^{-5}$
0.0	0.1	1.20	1.08	0.99	0.92
0.0	0.2	1.17	1.05	0.95	0.88
0.0	0.3	1.13	1.00	0.91	0.83
0.0	0.5	1.04	0.90	0.79	0.71
4.5	0.1	1.15	0.98	0.88	0.80
4.5	0.2	0.94	0.78	0.66	0.58
4.5	0.3	0.78	0.60	0.49	0.41
4.5	0.5	0.51	0.35	0.25	0.20
9.0	0.1	1.09	0.95	0.85	0.77
9.0	0.2	0.89	0.73	0.61	0.53
9.0	0.3	0.70	0.53	0.42	0.36
9.0	0.5	0.43	0.29	0.20	0.15

is within 10% of the value used in this research. However, the range in ratios α_T/φ_{gu} used in NCHRP 507 are much higher than those discovered here, due to very low resistance factors. The reason for this apparent conservatism is unknown to the authors.

In general, with the exception of NCHRP 507 (Paikowsky 2004), the published code ratios, α_T/φ_{gu}, correspond very well with those recommended in this research over the range of sampling distances and maximum acceptable failure probabilities considered in Table 4. The more modern Canadian codes (NBCC (National Research Council Canada 2005) and CHBDC (Canadian Standards Association 2006)) correspond to a level of site understanding about equivalent to the r = 9 m case considered here and to a maximum acceptable failure probability a bit lower than $p_m = 10^{-4}$.

Conclusions

This paper proposes reliability-based provisions for the ultimate limit state LRFD of deep foundations in soils under total stress conditions, with $\phi = 0$ and $c = s_u$ being the undrained shear strength. The load factors are as used in the

Table 4. Comparison of geotechnical resistance factors recommended in this study (first six rows) with those recommended in other sources.

Source	$R_{D/L}$	α_L	α_D	α_T	φ_{gu}	α_T/φ_{gu}
r = 4.5 m, $p_m = 10^{-3}$	2.5	1.50	1.25	1.32	0.60	2.20
r = 4.5 m, $p_m = 10^{-4}$	2.5	1.50	1.25	1.32	0.49	2.69
r = 4.5 m, $p_m = 10^{-5}$	2.5	1.50	1.25	1.32	0.41	3.22
r = 9.0 m, $p_m = 10^{-3}$	2.5	1.50	1.25	1.32	0.53	2.49
r = 9.0 m, $p_m = 10^{-4}$	2.5	1.50	1.25	1.32	0.42	3.14
r = 9.0 m, $p_m = 10^{-5}$	2.5	1.50	1.25	1.32	0.36	3.67
CFEM (1992)[a]	3.0	1.50	1.25	1.31	0.50	2.62
NBCC (2005)[b]	3.0	1.50	1.25	1.31	0.40	3.28
CHBDC (2006)[c]	3.0	1.70	1.20	1.33	0.40	3.33
AS 5100.3 (2004)[d]	3.0	1.80	1.20	1.35	0.45–0.55	2.45–3.00
AASHTO (2002)	3.7	2.86	1.30	1.63	0.70	2.33
AASHTO (2007)	3.7	1.75	1.25	1.36	0.35	3.89
NCHRP 507 (2004)[e]	2.0	1.75	1.25	1.42	0.15–0.30	4.73–9.47

[a]Canadian Geotechnical Society (1992).
[b]National Research Council Canada (2005).
[c]Canadian Standards Association (2006).
[d]Standards Australia (2004).
[e]Paikowsky (2004).

NBCC (National Research Council Canada 2005). A mathematical theory was developed to theoretically estimate the probability of pile failure. The theory assumes a statistically stationary random soil with lognormally distributed cohesion, c. The effect of the soil's spatial variability and site understanding on the geotechnical resistance factor has been investigated via both simulation and theory, by considering various soil statistics and sampling locations. The simulation involved 10 000 realizations for each set of parameters, and the results of the Monte Carlo simulation were compared to the proposed theory. The agreement between theory and simulation was found to be very good, except at very small failure probabilities where the simulation estimator error is large. Optimal geotechnical resistance factors were recommended for the design of deep foundations for four maximum acceptable failure probabilities (10^{-2}, 10^{-3}, 10^{-4}, and 10^{-5}).

The suggested design procedure using the proposed LRFD method is summarized as follows:

1. Decide on a maximum acceptable failure probability, p_m for the pile. The choice of p_m depends on the severity of failure consequences and the level of pile redundancy.
2. Sample the soil and estimate the characteristic soil property using eq. [8]. The characteristic ultimate resistance is calculated using eq. [9].
3. Determine load factors from structural design codes as described in the section "Random load model".
4. Select a geotechnical resistance factor corresponding to the maximum acceptable failure probability, p_m, and sampling distance from Table 3. Note that sampling distance has been used here as a proxy for level of site understanding. The actual geotechnical resistance factor used in design may be reduced somewhat, depending on the magnitude of model and measurement errors, as discussed below.
5. Compute the required pile length, given load factors, α_L and α_D, geotechnical resistance factor, φ_{gu}, and the effective pile perimeter, p, using eq. [19].

The evaluation of geotechnical resistance factors for pile design involves the soil field's uncertainty level (e.g., coefficient of variation, v_c), correlation level (e.g., correlation length, θ), and sampling location and accuracy. Since coefficient of variation, v_c and correlation length, θ, are usually unknown for a given site, various v_c are considered in this study for deep foundation limit state design, along with a worse-case value of θ, i.e., the intermediate value of θ corresponding to the higher probabilities of failure.

Three sampling schemes have been considered in this study. Better estimates of conditions at the pile can be obtained when samples are taken at the pile location (r = 0 m). Specifically, lower probability of failure and larger geotechnical resistance factor values are obtained by sampling at the pile location.

No attempt is made in this study to include the effects of measurement error, nor of correlation errors in translating actual observations (e.g., from a CPT sounding) to engineering properties such as cohesion. Thus, the predicted failure probability (either from theory or simulation) will be somewhat unconservative (failure probability increases as measurement error increases). However, both theory and the simulation treat all uncertainty in the same way, allowing a consistent comparison between the two.

The recommended geotechnical resistance factors for ultimate limit state design of deep foundations should be considered to be upper bounds because the measurement and model errors are not considered in this study. The statistic of measurement errors are very difficult to determine, since the true values need to be known. Similarly, model errors, which relate both the errors associated with translating measured values and the errors associated with predicting cohesive resistance by an equation, such as eq. [2], to the actual cohesive resistance, are very difficult to measure simply because the true cohesive resistance along with the true soil properties, are rarely, if ever, known. When confidence in the measured soil properties or in the model used is low, the results presented here can still be employed by assuming that the soil samples were taken further away from the the pile centerline than they actually were (e.g., if low-quality soil samples are taken at the pile location, $r = 0$, the geotechnical resistance factor corresponding to a larger value of r, say $r = 4.5$ m should be used) and (or) by using a larger v_c value.

The generally good agreement between the geotechnical resistance factors shown in Table 4 with current literature and LRFD code recommendations suggests that the theory is in reasonable agreement with past experience. The current study now provides a rigorous basis for the determination of geotechnical resistance factors for pile design in soils under total stress conditions, allowing code developers to go beyond calibration with the past.

Acknowledgements

The authors would like to thank the Natural Sciences and Engineering Research Council of Canada (NSERC), under Discovery Grant OPG0105445, for their essential support of this research. The authors would also like to thank Farzaneh Naghibi for her help with the tables and with reviewing the work.

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List of symbols

c cohesion
$\hat{c}$ arithmetic average of observed (sampled) cohesion values
$\bar{c}$ arithmetic average of cohesion field along pile surface
$\hat{c}_i$ observed (sampled) cohesion value ($i = 1, 2, \ldots, m$)
$\bar{c}_i$ local average of cohesion over ith element along pile surface
D depth of soil sample
E expectation operator
F total true (random) load
$\hat{F}$ sum of characteristic live and dead loads ($\hat{F} = \hat{F}_L + \hat{F}_D$)
F_D true (random) dead load
$\hat{F}_D$ characteristic dead load ($\hat{F}_D = k_D\mu_D$)
$\hat{F}_i$ ith characteristic load effect
F_L true (random) live load
$\hat{F}_L$ characteristic live load ($\hat{F}_L = k_L\mu_L$)
$G_{\ln c}$ standard normal random field (log cohesion)
H designed pile length
I_i importance factor corresponding to the ith characteristic load effect

k_D dead load bias factor
k_L live load bias factor
m number of soil observations
n_f number of times Monte Carlo simulation steps result in pile failure
n_{sim} number of times Monte Carlo simulation steps are repeated
P probability operator
P_a standard atmospheric pressure (P_a = 101.325 kPa)
p pile perimeter length
p_f probability of failure
p_m maximum acceptable probability of failure
$\hat{Q}$ total factored characteristic load ($\hat{Q} = \alpha_L \hat{F}_L + \alpha_D \hat{F}_D$)
$R_{D/L}$ ratio of dead to live load means
$\hat{R}_{D/L}$ characteristic dead to live load ratio
R_u true ultimate resistance (random)
$\hat{R}_u$ ultimate characteristic resistance (based on characteristic soil properties)
r distance between soil sample and pile centerline
s_u undrained shear strength
t distance
v coefficient of variation
v_c coefficient of variation of cohesion
$v_{\hat{c}}$ coefficient of variation of $\hat{c}$
$v_{\bar{c}}$ coefficient of variation of $\bar{c}$
W true load times ratio of characteristic to equivalent cohesive resistance in soils under total stress conditions
X horizontal direction in random cohesion field
Y vertical direction in random cohesion field
z depth from ground surface
Δz vertical dimension of soil samples
z_1 depth of point 1
z_2 depth of point 2
z_i^o depth of the center of the ith soil sample (i = 1, 2, …, m)
α adhesion coefficient
α_D dead load factor
α_i load factor corresponding to the ith load effect
α_L live load factor
α_T total load factor
β reliability index
γ_{HD} average correlation coefficient between the cohesion samples over length D and the cohesion along the pile of length H
$\gamma(D)$ variance function giving variance reduction due to local averaging over sample domain D
$\gamma(H)$ variance function giving variance reduction due to averaging over pile length H
θ correlation length of the random cohesion field
μ mean
μ_c cohesion mean
μ_D mean dead load
μ_F mean total load on pile
μ_L mean live load
$\mu_{\ln c}$ log cohesion mean
$\mu_{\ln \hat{c}}$ mean of the characteristic log cohesion (based on an arithmetic average of cohesion observations)
$\mu_{\ln \bar{c}}$ mean of the equivalent log cohesion (based on an arithmetic average of cohesion over pile length H)
$\mu_{\ln F}$ mean total log load on pile
$\mu_{\ln W}$ mean of $\ln W$
ρ correlation function
σ standard deviation
σ_c cohesion standard deviation
$\sigma_{\hat{c}}$ standard deviation of the soil sample average
$\sigma_{\bar{c}}$ standard deviation of the equivalent soil cohesion along the pile surface
σ_D dead load standard deviation
σ_F total load standard deviation
σ_L live load standard deviation
$\sigma_{\ln c}$ log cohesion standard deviation
$\sigma_{\ln \hat{c}}$ standard deviation of $\ln \hat{c}$
$\sigma_{\ln \bar{c}}$ standard deviation of $\ln \bar{c}$
$\sigma_{\ln F}$ standard deviation of total log load
$\sigma_{\ln W}$ standard deviation of $\ln W$
$\sigma_{\hat{p}_f}$ standard deviation of failure probability estimate
τ ultimate shear stress acting on the surface of the pile
Φ standard normal cumulative distribution function
ϕ friction angle
φ_{gu} ultimate geotechnical resistance factor