Computers and Geotechnics 60 (2014) 29–32

Contents lists available at ScienceDirect

Computers and Geotechnics

journal homepage: www.elsevier.com/locate/compgeo

Technical Communication

Prediction of pile settlement in an elastic soil

Farzaneh Naghibi [a], Gordon A. Fenton [a,*], D.V. Griffiths [b,c]

[a] Department of Engineering Mathematics, Dalhousie University, Halifax, NS B3J 2X4, Canada
[b] Department of Civil and Environmental Engineering, Colorado School of Mines, Golden, CO 80401, United States
[c] Australian Research Council Centre of Excellence for Geotechnical Science and Engineering, University of Newcastle, Callaghan, NSW 2308, Australia

ARTICLE INFO

Article history:
Received 28 February 2014
Received in revised form 25 March 2014
Accepted 25 March 2014
Available online 14 April 2014

Keywords:
Foundation settlement
Deep foundations
Finite element method
Design pile length

ABSTRACT

This paper presents a simple regression to predict settlement of a single floating pile supported by a homogeneous elastic soil and subjected to a vertical load. The regression, which is calibrated by a finite element model, allows the direct computation of the pile length required for serviceability limit state design of deep foundations.

1. Introduction

Deep foundations, or piles, are structural members, usually made of steel, concrete, and/or timber, which transmit some or the entire applied load to ground below the surface. Piles resting on a stiffer stratum are called end-bearing. If not end-bearing, they are called floating, in which case most of the resistance is derived from skin friction and/or cohesion. In this paper, only floating piles are considered and end-bearing is ignored.

As load is applied to the pile, the pile settles due to both deformation of the pile itself and deformation of the surrounding soil. Assuming that the surrounding soil is perfectly bonded to the pile shaft through friction and/or adhesion, any displacement of the pile is associated with an equivalent displacement of the adjacent soil. Following the classic work of authors such as [6,7,10], the soil is assumed to be linearly elastic, so that this displacement is resisted by a force which is proportional to the soil's elastic modulus and the magnitude of the displacement. Thus, the support provided by the soil to the pile depends on the elastic properties of the surrounding soil. As stated by Vesic [10], the pile settlement is a constant (dependent on Poisson's ratio and pile geometry) times F/E_s, where F is the applied load and E_s is the soil's elastic modulus.

The main objective of this note is to present a simple formula predicting the settlement of a single floating pile within an elastic soil. To accomplish this, a regression model is developed based on finite element results. One of the primary benefits of the model is that it is easily inverted to allow a direct computation of the pile length, H, required for the serviceability limit state design of floating piles.

The pile is assumed to be placed in a three-dimensional uniform (spatially constant) elastic soil. A vertical load is applied to the pile and the settlement of the pile is calculated using a linear elastic finite element model [9,4,3]. The pile itself is assumed to be square, for reasons to be discussed later, with fixed cross-sectional dimension $d \times d$.

The remainder of this paper is organized as follows: In Section 2, an analytical result is presented for the settlement of a pile under a given load, and a regression is developed and calibrated using a linear elastic finite element model, which can be used to easily compute pile length required for serviceability limit state design. Conclusions are presented in Section 3.

2. Methodology

Prediction of elastic pile settlement has been studied previously by various authors [6,7,2,1]. The settlement prediction used in this paper has the same form as suggested by Poulos and Davis [6] and Randolph and Wroth [7], for a single cylindrical pile embedded in a homogeneous elastic soil,

$$\delta = \frac{F}{E_s d} I_p \quad (1)$$

* Corresponding author. Tel.: +1 9024946002.
E-mail addresses: farzaneh.naghibi@dal.ca (F. Naghibi), gordon.fenton@dal.ca (G.A. Fenton), D.V.Griffiths@mines.edu (D.V. Griffiths).

http://dx.doi.org/10.1016/j.compgeo.2014.03.015
0266-352X/

List of Symbols

a_i	settlement prediction parameter
d	pile diameter
E_s	soil elastic modulus
E_p	pile elastic modulus
F	applied load
H	pile length
I_p	settlement influence factor
k	pile to soil stiffness ratio $= E_p/E_s$
l_x	pile location measured from the left edge of the soil mass ($x = 0$)
δ	pile settlement, positive downwards
ζ	variable in Randolph's prediction for I_p
λ	variable in Randolph's prediction for I_p
ν_s	Poisson's ratio

where δ is the settlement at the top of the pile, F is the applied load, E_s is the elastic modulus of the soil, and d is the diameter of the pile shaft. I_p is a settlement influence factor which depends on a number of parameters such as Poisson's ratio of the soil, ν_s, the pile slenderness ratio, H/d, where H is the pile length, and the pile to soil stiffness ratio, $k = E_p/E_s$, E_p being the pile elastic modulus.

The following expression derived from closed-form solutions obtained by Randolph and Wroth [7], can be used to calculate I_p for a constant diameter cylindrical floating pile:

$$I_p = 4(1+\nu_s)\left[1+\frac{1}{\pi\lambda}\frac{8}{(1-\nu_s)}\frac{\tanh(\mu H)}{\mu H}\frac{H}{d}\right]\Big/\left[\frac{4}{(1-\nu_s)}+\frac{4\pi}{\zeta}\frac{\tanh(\mu H)}{\mu H}\frac{H}{d}\right] \tag{2}$$

with

$$\zeta = \ln[5(1-\nu_s)H/d]$$

$$\mu H = (2H/d)\sqrt{2/(\zeta\lambda)}$$

$$\lambda = 2(1+\nu_s)E_p/E_s$$

For piles shorter than $0.25d\sqrt{2(1+\nu_s)E_p/E_s}$, Fleming et al. [5] suggest that the settlement coefficient, I_p, should become

$$I_p = \frac{(1+\nu_s)}{\frac{1}{(1-\nu_s)}+\frac{\pi}{\zeta}\frac{H}{d}} \tag{3}$$

However, for piles longer than $1.5d\sqrt{2(1+\nu_s)E_p/E_s}$, Fleming et al. [5] suggest that the value of $\tanh(\mu H)$ approaches unity and hence Eq. (2) reduces to

$$I_p = \frac{2(1+\nu_s)\sqrt{2\zeta/\lambda}}{\pi} \tag{4}$$

Furthermore, Fleming et al. [5] state that, for long piles, the pile response should become independent of pile length, since very little load reaches the base of the pile which is reasonable. However, both Eqs. (2) and (4) show a slow increase in settlement with pile length when the pile length exceeds $1.5d\sqrt{2(1+\nu_s)E_p/E_s}$, which is unexpected. It is believed that this slow increase is merely an artifact of approximations made in the settlement predictions and should be ignored.

Eq. (2) is not easily inverted to solve for the pile length, H. In this paper a simpler power function of the form

$$I_p = a_0 + \frac{1}{(H/d+a_1)^{a_2}} \tag{5}$$

has been found to fit the I_p values estimated using 3-D finite element analysis for various values of k and H/d. The calibration of I_p will be discussed shortly.

Substituting Eq. (5) into Eq. (1) results in the following settlement prediction

$$\delta = \frac{F}{E_s d}\left(a_0 + \frac{1}{(H/d+a_1)^{a_2}}\right) \tag{6}$$

The primary motivation of the functional form assumed in Eq. (5) is that it is easily inverted and solved for H. For a given I_p value, inverting Eq. (5) and solving for H gives,

$$H = d\left[\left(\frac{1}{I_p - a_0}\right)^{1/a_2} - a_1\right] = d\left[\left(\frac{1}{(\delta E_s d/F) - a_0}\right)^{1/a_2} - a_1\right] \tag{7}$$

The calibration of the settlement influence factor, I_p, given by Eq. (5) can be achieved either by fitting Eq. (5) to Eq. (2), or by fitting Eq. (5) to elastic FE results. The latter was selected in this study because Eq. (2) is not asymptotic to a minimum value and begins to erroneously increase for piles longer than $1.5d\sqrt{2(1+\nu_s)E_p/E_s}$. The FE results, on the other hand, do tend to a minimum value as the pile length increases.

This work looks specifically at the cases where the pile to soil stiffness ratio $k = E_p/E_s$ ranges between 200 and 1000. The calibration of I_p is done here by calculating the settlement of a pile of length H surrounded by a soil with uniform elastic modulus $E_s = 30$ MPa, Poisson's ratio $\nu_s = 0.3$, and supporting load $F = 1.6$ MN using the finite element method. Note that the settlement coefficient, I_p, depends on $k = E_p/E_s$ and not on E_s directly nor does it depend on F. The dependence on ν_s is only slight, showing changes of no more than about 5% for ν_s ranging from 0.1 to 0.4, with higher values showing slightly less settlement. The pile is founded in a three-dimensional linearly elastic soil mass. The mesh selected is 50 elements by 30 elements in plan by 30 elements in depth, as shown in Fig. 1. Eight-node brick elements are used with dimensions: 0.3 m by 0.3 m in the X, Y (plan) and by 0.5 m in the Z (vertical) directions. Within this mesh, the pile is modeled as a column of elements having depth H and elastic modulus ranging from 6 GPa to 30 GPa (which are several orders of magnitude higher than that of the surrounding soil). Thus, the pile is assumed here to be of square cross-section with dimension $d = 0.3$ m and depth ranging from 1 to 10 m (a maximum pile length of 10 m was selected to avoid boundary effects with the base). The pile is placed in the middle of the mesh where the pile settlement is computed more accurately due to the minimized influence of boundary conditions on pile settlement (to be discussed later). Because the stiffness matrix of a 50 by 30 by 30 element mesh requires about 2 GBytes of memory, a conjugate gradient iterative solver is employed in the finite element model to avoid the need to assemble the entire stiffness matrix in the finite element analysis.

The proposed methodology to develop a simple relationship to predict elastic pile settlement proceeds as follows:

(1) A finite element prediction of pile settlement is performed for each of a range of pile lengths $H = 1, \ldots, 10$ m, for given $F = 1.6$ MN, $k = E_p/E_s$, $d = 0.3$, and $\nu_s = 0.3$, resulting in a set of settlement values, δ. Fig. 2 illustrates one such analysis.

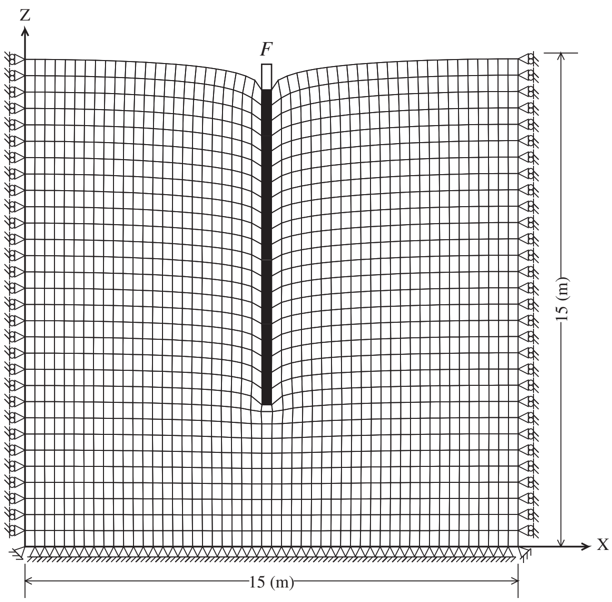

Fig. 1. Typical mesh of 8-node square elements (50 by 30 by 30 elements).

(2) I_p is computed for each H/d value by inverting Eq. (1). A function of the form given by Eq. (5) is then fit by regression to the finite element results, leading to a set of a_i values, $i = 0, 1, 2$.
(3) Steps 1 through 2 are repeated for various values of $k = 200, \ldots, 1000$, producing a set of a_i values for each k.
(4) Once the a_i values are obtained for each k, the following power functions are fit by a subsequent regression to each set of a_i values,

$$\begin{aligned} a_0 &= 2069.4633(k+350)^{-1.6054} \\ a_1 &= 0.07 + (0.2934k^{0.3108}) \\ a_2 &= 0.6903 + (8.2464k^{-0.5268}) \end{aligned} \tag{8}$$

(5) Substituting Eq. (8) into Eq. (6) results in a relationship for computing pile settlement as a function of the pile slenderness ratio, H/d, and the pile stiffness ratio, $k = E_p/E_s$.

Fig. 2 shows the plot of I_p for $k = 200$, and 1000 along with their corresponding regressions. The agreement is excellent. Fig. 3 demonstrates the plot of pile settlements, δ, for $k = 200, 300, 500$, and 1000, along with their very good matches produced by substituting Eq. (8) into Eq. (6).

In order to make a direct comparison between the FE model and Randolph's analytical solution [7], Eq. (2) is substituted into Eq. (1), which is then used to compute pile settlement for the problem described previously in this Section. As mentioned earlier, Randolph's solution is developed for cylindrical piles only, but it can be extended to non-cylindrical piles (e.g. square or H piles) by choosing a reasonable value of d. The pile considered in the FE model is of square cross-section with dimension 0.3, thus $d = 0.3(4/\pi)$ is used

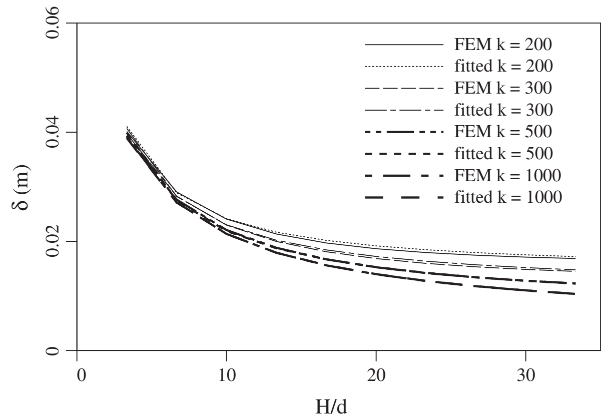

Fig. 3. Calibration of δ using FE model for k = 200, 300, 500, and 1000, produced by substituting Eq. (8) into Eq. (6).

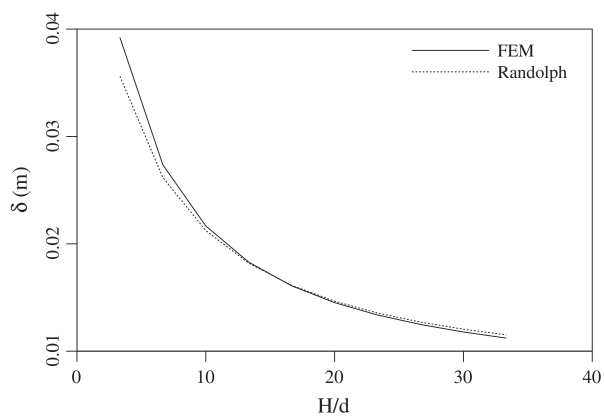

Fig. 4. Comparison of pile settlement, δ, obtained by FE model and Randolph's analytical solution [7] for $k = 700$.

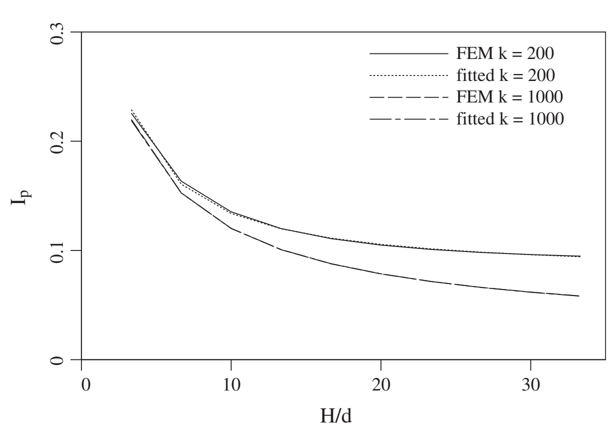

Fig. 2. Calibration of I_p using FE model for k = 200, and 1000.

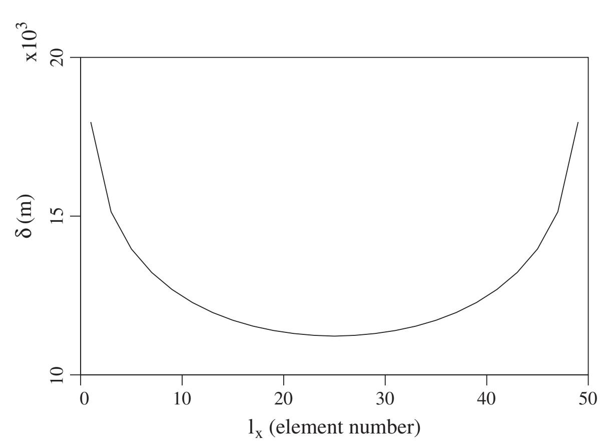

Fig. 5. Influence of side distance on pile settlement, using FE model for $k = 700$.

(adjusted by the ratio of square to circle perimeters) in Eqs. (1) and (2) resulting in δ values illustrated in Fig. 4 along with the FE results. As shown in Fig. 4, the agreement between the two methods is considered good for $H/d > 10$, however, an error of up to about 10% is evident for $H/d < 10$. This level of accuracy is considered reasonable since Randolph's solution shows errors of 20–30% compared to numerical analyses for H/d values of about 2 [8]. A study was performed to assess the influence of side distance on pile settlement. Fig. 5 gives a plot of pile settlement versus l_x where l_x is pile location measured from the left edge of the soil mass ($x = 0$). The results indicate a steep reduction in settlement in the range $1 < l_x < 10$ elements (corresponding to relative error >10%) and reasonably constant settlement values for $11 < l_x < 25$ (corresponding to relative error <10%), where $l_x = 25$ denotes the center along the x-direction. For this study, thus, the pile is fixed at the center of the mesh to minimize the edge effects on pile settlement.

3. Conclusions

In this work, an elastic prediction of pile settlement was investigated employing a linear finite element program to derive the settlement prediction of a vertically loaded single floating pile founded in a homogeneous soil. A regression model is developed and a simple mathematical expression is found for pile settlement, δ, that fits the FE results well, and shows reasonably good agreement with the analytical solution derived by Randolph and Wroth [7]. The advantage of the simplified expression developed in this study is that it is easily inverted and solved for pile length, H, given δ, which can be used in the design of piles for serviceability limit states.

Acknowledgement

The authors would like to thank the Ministry of Transportation of Ontario, under a Special Research Project Grant, for their essential support of this research.

References

[1] Coduto Donald P. Foundation design: principles and practices. 2nd ed. New Jersey: Prentice Hall; 2001.

[2] Das BM. Fundamentals of geotechnical engineering. Pacific Grove, California: Brooks/Cole; 2000.

[3] Fenton GA, Griffiths DV. Reliability-based deep foundation design. Probabilistic applications in geotechnical engineering, GSP No. 170, ASCE. Proc geo-denver 2007 symposium. Denver, Colorado. p. 1–12.

[4] Fenton GA, Griffiths DV. Three-dimensional probabilistic foundation settlement. ASCE J Geotech Geoenviron Eng 2005;131(2):232–9.

[5] Fleming WGK, Weltman AJ, Randolph MF, Elson WK. Piling engineering. 3rd ed. New York: Taylor & Francis; 2009.

[6] Poulos HG, Davis EH. Pile foundation analysis and design. New York: John Wiley & Sons; 1980.

[7] Randolph MF, Wroth CP. Analysis of deformation of vertically loaded piles. ASCE J Geotech Geoenviron Eng 1978;104(GT12):1465–88.

[8] Randolph MF. Design methods for pile groups and pile rafts. In: Proceedings, thirteenth international conference on soil mechanics and foundation engineering. New Delhi; 1994. p. 61–82.

[9] Smith IM, Griffiths DV. Programming the finite element method. 4th ed. New York (NY): John Wiley & Sons; 2004.

[10] Vesic, AF. Design of Pile Foundations, 1977, National Cooperative Highway Research Program, Synthesis of Practice No. 42, Transportation Research Board, Washington, D.C.