PROBABILISTIC ANALYSIS OF A SPATIALLY VARIABLE $c'-\phi'$ SLOPE

Y.H. Chok[1], M.B. Jaksa[2], D.V. Griffths[3], G.A. Fenton[4] and W.S. Kaggwa[2]

[1]*AECOM, Brisbane, Australia*

[2]*School of Civil, Environmental and Mining Engineering, University of Adelaide, Australia*

[3]*Department of Civil and Environmental Engineering, Colorado School of Mines, USA*

[4]*Department of Engineering Mathematics and Internetworking, Dalhousie University*

ABSTRACT

In practice, inherent soil variability is not commonly considered in routine slope stability analysis. This is due mainly to the fact that the effect of soil variability is complex and difficult to quantify. Furthermore, the majority of available slope stability analysis computer programs used in practice, which adopt conventional limit equilibrium methods, are unable to consider this aspect explicitly. To predict the stability of a slope more accurately, especially the marginally stable ones, the effect of soil variability needs to be accounted for. In this paper, an advanced probabilistic analysis method called the random finite element method (RFEM), developed by Griffiths and Fenton in the 1990s, is used to investigate the effect of soil variability on the reliability of a c'-ϕ' soil slope. The results from the probabilistic study demonstrate that soil variability has a significant effect on the reliability of a slope. It is concluded that the deterministic factor of safety (FOS) is not a reliable measure of the true safety of a slope with spatially variable soils.

1 INTRODUCTION

It is well recognised that the underlying soil profiles of a natural slope are unlikely to be completely uniform and homogenous, due largely to the complex deposition and alteration processes which influence soil materials. Even within a so-called 'homogenous' soil layer, soil properties tend to vary from one location to another (Vanmarcke, 1977). This inherent variation of soil properties in distance or space is known as *spatial variability*. In routine practice, the stability of a slope is usually assessed using conventional limit equilibrium methods, and the soil profiles are often assumed to be uniform and homogenous. The conventional slope stability analyses are usually performed within a deterministic analysis framework where single best estimates or characteristic values for soil parameters are used. To account for the variability and uncertainty in soil properties, a higher factor of safety (FOS) is usually adopted. As a result, the conventional slope stability analysis approach may give a poor estimate of the potential failure of a slope because the effect of soil variability is not properly modelled and accounted for.

Probabilistic analysis is a more realistic approach to assess slope stability because the uncertainty and variability in soil properties can be explicitly accounted for. Unlike a deterministic analysis, which is based on assumed characteristic values of soil properties, a probabilistic analysis considers the variable nature of soil properties, based on their statistical characteristics. The latter approach leads to a more realistic measure of the stability of a slope, which is usually characterised by the *probability of failure*, P_f, or *reliability index*. One of the available probabilistic analysis methods is the random finite element method (RFEM) (Griffiths and Fenton, 2000, 2004; Fenton and Griffiths, 2008; Griffiths *et al.* 2009), which combines random field simulation with finite element analysis. RFEM is a powerful probabilistic method for slope stability analysis because the spatial correlation of soil properties is modelled explicitly and no assumption about the shape or location of the failure surface is required to be made in advance. Failure occurs through soil elements whose shear strength is lower than the applied shear stresses and is associated with excessive distortion of these elements. This paper investigates the effect of soil variability on the stability of a cohesive-frictional (c'-ϕ') soil slope using a freely available[1] RFEM computer program called `rslope2d`.

2 OVERVIEW OF THE PROBABILISTIC ANALYSIS METHODOLOGY

The procedures for probabilistic slope stability analysis adopted in the computer program `rslope2d`, which is based on the RFEM, can be summarised as follows:

1. Simulate a 2-dimensional (2-D) spatially random soil profile based on the prescribed statistical parameters of the chosen soil properties,
2. Perform finite element slope stability analysis on the simulated soil profile to determine whether the slope 'fails' under specific convergence criteria and

[1] http://courses.engmath.dal.ca/rfem/

3.Repeat Steps 1 and 2 many times as part of the Monte Carlo simulation process to establish the probability of failure, P_f.

Descriptions of each process are presented in the following sections.

2.1 SIMULATION OF SPATIALLY RANDOM SOIL PROFILE

To incorporate soil variability in slope stability analysis, it is essential to generate a soil profile that can represent the variability and spatial correlation of the properties of real soil deposits. In `rslope2d`, a 2-D spatially random soil profile is generated based on random field theory (Vanmarcke, 1977; 1983), which makes use of three statistical properties: the mean, μ, the variance, σ^2, or standard deviation, σ, and the scale of fluctuation, θ. The standard deviation, σ, can be expressed in terms of the dimensionless coefficient of variation, COV, defined as:

$$\mathrm{COV} = \sigma / \mu \tag{1}$$

The scale of fluctuation, θ, is a parameter describing the spatial correlation of soil properties with distance. A small value of θ implies rapid fluctuation of the soil property in space about the mean, whereas a large value of θ implies a smoothly varying field.

Prior to the random field generation process, it is important to identify the soil parameters that are required to be treated as random variables. The soil constitutive model used in the elastoplastic finite element slope stability analysis algorithm in `rslope2d` consists of the following input parameters: (1) effective cohesion, c'; (2) effective friction angle, ϕ'; (3) unit weight, γ; (4) dilation angle, ψ; (5) Young's Modulus, E_s; and (6) Poisson's ratio, ν.

The parameters c' and ϕ' commonly represent the soil shear strength behaviour, and the Mohr-Coulomb failure criterion was adopted. The dilation angle, ψ, expresses the volume change of the soil during yielding while the elastic parameters (i.e. Young's modulus, E_s, and Poisson's ratio, ν) which are used to compute deformations prior to yielding of the soil element in slope stability analysis, have little influence on the predicted factor of safety (Griffiths and Lane, 1999). Griffiths and Lane (1999) also concluded that the most important parameters in finite element slope stability analysis are the same as those used in the traditional limit equilibrium approach, namely, the strength parameters c' and ϕ', unit weight, γ, and the geometry of the slope. Hence, it is logical to assume that, in a probabilistic analysis, only the variability of the cohesion, friction angle and unit weight, influence the probability of failure of a slope.

In addition, a previous probabilistic study conducted by Alonso (1976) concluded that the influence of the soil density or unit weight on the probability of failure of a clay slope is relatively small compared with the shear strength parameters. This is due to the fact that the variability of soil unit weight is usually small, as published in the literature (e.g. Lee *et al.*, 1983; Phoon and Kulhawy 1999; Duncan, 2000; Baecher and Christian, 2003). Therefore, throughout this study, only the shear strength parameters c' and ϕ' are modelled as random fields, while the other parameters are held constant and treated deterministically. The variability of the shear strength parameters c' and ϕ' is characterised by a lognormal distribution. This is because the lognormal distribution avoids the generation of negative values of strength parameters c' and ϕ' that a normal distribution allows. Furthermore, available field data indicate that some soil properties are well represented by a lognormal distribution (e.g. Hoeksema and Kitanidis, 1985; Sudicky, 1986; Cherubini, 2000).

To generate random fields of a soil property (i.e. c' or ϕ'), random field theory is implemented in `rslope2d` using the local average subdivision (LAS) method developed by Fenton and Vanmarcke (1990). This method produces correlated local averages of the soil property based on a standard normal distribution function (i.e. having zero mean and unit variance) and a spatial correlation function. An isotropic exponentially decaying (Markovian) correlation function is assumed in this study, and it can be expressed as:

$$\rho(\tau) = \exp\left(-\frac{2|\tau|}{\theta}\right) \tag{2}$$

where ρ = correlation coefficient between the underlying random field values at any two points separated by a lag distance τ.

If a soil property X (i.e. c' or ϕ') is assumed to be characterised statistically by a lognormal distribution defined by a mean, μ_X, and a standard deviation, σ_X, the local averages of a standard normal random field, $G(x)$, generated by the LAS method is then necessary to be transformed into a lognormal distribution using the following relationship:

$$X_i = \exp\{\mu_{\ln X} + \sigma_{\ln X} G(x_i)\} \tag{3}$$

where x_i is the vector containing the coordinates of the centre of the ith element; X_i is the soil property value assigned to that element; $\mu_{\ln X}$ and $\sigma_{\ln X}$ are the mean and standard deviation, respectively, of the underlying normally distributed $\ln X$. $\mu_{\ln X}$ and $\sigma_{\ln X}$ can be computed using Equations (4) and (5), respectively, as shown below:

$$\mu_{\ln X} = \ln \mu_X - \tfrac{1}{2}\sigma_{\ln X}^2 \quad (4)$$

$$\sigma_{\ln X} = \sqrt{\ln\left(1 + \mathrm{COV}_X^2\right)} \quad (5)$$

Once the random field is transformed into the desired lognormal field, it is then mapped onto the finite element mesh, which is established according to the user-defined slope geometry. Each element within the slope geometry is assigned a random variable of the particular soil property (i.e. c' or ϕ').

The relationship or cross-correlation between the strength parameters c' and ϕ' is poorly understood and no consensus is provided in the literature. In addition, it is strongly dependent on the soil being studied (Fenton and Griffiths, 2003). However, Cherubini (2000) reported values of cross-correlation between c' and ϕ' ranging from –0.24 to –0.70. In `rslope2d`, cross-correlation between c' and ϕ' is implemented using the covariance matrix decomposition approach (Fenton, 1994).

2.2 FINITE ELEMENT SLOPE STABILITY ANALYSIS

The finite element slope stability analysis algorithm in `rslope2d` assumes 2-D and plain strain conditions. It uses an elastic-perfectly plastic stress-strain law with a Mohr-Coulomb failure criterion. It utilises 8-node quadrilateral elements with reduced integration in the gravity loads generation, stiffness matrix generation and stress redistribution phases of the algorithm. The theoretical basis of the method is described fully by Smith and Griffiths (1998; 2004) and the application of the finite element method in slope stability problems is described by Griffiths and Lane (1999).

In brief, the analyses involve the application of gravity loading and the monitoring of stresses at all Gauss points. The forces generated by the self-weight of the soil are modelled by a standard gravity 'turn-on' procedure. This procedure generates normal and shear stresses at all Gauss points within the mesh and the soil is initially assumed to be elastic. These stresses are then compared with the Mohr-Coulomb failure criterion, which can be written in terms of principal stresses as follows:

$$F = \frac{\sigma_1 + \sigma_3}{2}\sin\phi' - \frac{\sigma_1 - \sigma_3}{2} - c'\cos\phi' \quad (6)$$

where σ_1 and σ_3 are the major and minor principal stresses.

If the stresses at a particular Gauss point lie within the Mohr-Coulomb failure envelope ($F < 0$), then that location is assumed to remain elastic. If the Mohr-Coulomb failure criterion is violated ($F \geq 0$), then that location is assumed to be yielding. Yielding stresses are redistributed to neighbouring elements that still have reserves of strength. The plastic stress redistribution is accomplished by using a visco-plastic algorithm (Zienkiewicz and Cormeau, 1974). This is an iterative process which continues until the Mohr-Coulomb failure criterion and global equilibrium are both satisfied at all Gauss points within the mesh.

The finite element algorithm in `rslope2d` computes a deterministic factor of safety (FOS) based on the mean values of the shear strength parameters using the strength reduction method (Matsui and Sun, 1992). The FOS of a slope is defined as the factor that the original shear strength parameters must be divided by in order to bring the slope to the point of failure. The strength parameters at the point of failure, c'_f and ϕ'_f, are therefore given by:

$$c'_f = c'/\mathrm{FOS} \quad (7)$$

$$\phi'_f = \tan^{-1}\left(\tan\phi'/\mathrm{FOS}\right) \quad (8)$$

This definition of the factor of safety is essentially the same as that used in limit equilibrium methods, which is defined as the ratio of shear strength of soil to shear stress required for equilibrium (Duncan, 1996). Validation studies conducted by Griffiths and Lane (1999) indicate good agreement between the FOS computed by the finite element method and that obtained from the stability charts developed by Taylor (1937) and Bishop and Morgenstern (1960).

In `rslope2d`, non-convergence of the algorithm within a user-specified maximum number of iterations or iteration limit is used as an indicator of slope failure. A slope is considered to have 'failed' when no stress distribution can be found that simultaneously satisfies both the Mohr-Coulomb failure criteria and global equilibrium (Griffiths and Lane, 1999). This is usually accompanied by a dramatic increase in the nodal displacements within the mesh. Griffiths and Fenton (2004) reported that an iteration limit of 500 was adequate to ensure convergence of solutions for a case study of a 2H:1V cohesive slope problem. The iteration limit required for the c'-ϕ' slope problem considered in this paper is investigated and discussed later.

2.3 MONTE CARLO SIMULATION

Based on a given set of soil property statistics (mean, standard deviation and scale of fluctuation), there are an infinite number of possible random fields that can be generated. Although these random fields have the same statistics, the arrangement of the 'strong' and 'weak' soil elements is different in each random field realisation, which in turn yields different outcomes in the finite element analyses. Hence, probabilistic analysis involves the repeated finite element examination of every single realisation of the generated random fields, as part of the Monte Carlo simulation process. The probability of failure, P_f, of a slope is estimated by the following relationship:

$$P_f \approx \frac{n_f}{n_{sim}} \tag{9}$$

where n_f = number of realisations reaching failure; and n_{sim} = total number of realisations in the simulation process.

The accuracy of the estimated probability of failure depends on the number of realisations in the Monte Carlo simulation process. In general, the accuracy increases as the number of realisations increases. However, it is important to determine the minimum number of realisations to produce a reliable and reproducible result. The reason is that repetitive finite element analysis is very time consuming and the estimation of P_f usually converges within a certain number of realisations. Any further increase in the number of realisations will not improve the estimation greatly, but will adversely affect the computational time and effort. Griffiths and Fenton (2004) reported that 1,000 realisations of the Monte Carlo simulation process was adequate for a 2H:1V cohesive slope problem in order to produce converged estimations of the probability of failure. The minimum number of realisations required for the c'-ϕ' slope problem considered in this paper is discussed later.

3 DESCRIPTION OF THE NUMERICAL STUDIES UNDERTAKEN

This paper deals with the c'-ϕ' slope problem, involving an effective stress analysis. Effective or drained cohesion, c', and friction angle, ϕ', are the shear strength parameters. The soil shear strength is defined by the Mohr-Coulomb failure criterion. The slope geometry, together with the finite element mesh used, is presented in Figure 1. The slope has a height, H, of 10 m, and a gradient of 1H:1V. A fixed base is assumed at the lower boundary and rollers are assumed at the two vertical boundaries. The element size was fixed at 0.5 m × 0.5 m, for the purpose of modelling soil with small scales of fluctuation, θ. A deep water table was assumed in this study, hence, the effect of pore water pressure was not considered in the analysis.

The shear strength parameters c' and ϕ' were modelled as random variables and both were described by a lognormal distribution. Other parameters were held constant, e.g. slope height, H = 10 m; unit weight, γ = 20 kN/m^3; Young's modulus, $E_s = 1 \times 10^5$ kPa; Poisson's ratio, $\nu = 0.3$; and dilation angle, $\psi = 0°$.

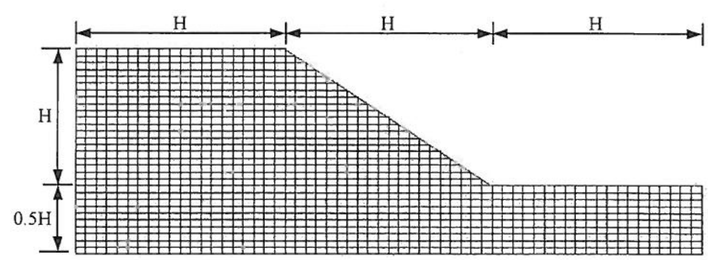


Figure 1: Mesh and slope geometry used for the c'-ϕ' slope problem.

In the process of simulating the soil profiles, random fields of c' and ϕ' were generated independently, based on their prescribed statistical parameters (i.e. mean, standard deviation, and scale of fluctuation). The standard deviation, σ, were expressed in terms of the coefficient of variation, COV, while the scale of fluctuation was expressed in the dimensionless form of θ/H.

In the first part of the parametric studies, the mean values of cohesion and friction angle were held constant at 10 kPa and 30°, respectively. The COV of c' and ϕ', and θ/H were varied systematically according to Table 1. It is noted that the COV of ϕ' is assumed to be half of the COV of c'. This is due to the fact that the variability of the friction angle is generally smaller than that of cohesion, based on the published data from the literature (e.g. Lee *et al.*, 1983; Phoon and Kulhawy, 1999; Baecher and Christian, 2003). These data indicate that the COV of cohesion is in the range of 0.1 – 0.5, while the friction angle is in the range of 0.05 – 0.15. No cross-correlation between c' and ϕ' was assumed in the first part of analysis. Cross-correlation between c' and ϕ' is investigated and discussed later. An isotropic scale of fluctuation was assumed throughout the analysis.

Table 1: Input values of COV and θ/H used in the c'-ϕ' slope problem.

Parameters	*Input values*
$COV_{c'}$	0.1, 0.2, 0.3, 0.4, 0.5
$COV_{\phi'}$	0.05, 0.1, 0.15, 0.2, 0.25
θ/H	0.1, 0.5, 1, 5, 10

4 RESULTS AND DISCUSSIONS

4.1 DETERMINISTIC SOLUTIONS

Deterministic analyses were conducted using both the finite element method and the limit equilibrium methods, based on the mean values of the shear strength parameters, i.e. $c' = 10$ kPa and $\phi' = 30°$. The limit equilibrium solution was obtained by using the commercial slope stability analysis software SLOPE/W (GEO-SLOPE International Ltd., 2008). The computed factor of safety (FOS), based on a simplified Bishop's method, is 1.22. The critical slip surface of the slope is shown in Figure 2, which indicates a 'toe' failure. Toe failure is generally expected in a c'-ϕ' slope problem due to the low value of effective cohesion. The FOS computed by the finite element method using `rslope2d` is 1.12, which is comparable with that obtained from the limit equilibrium method.

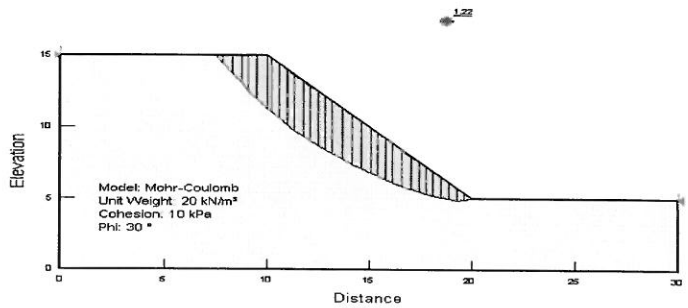


Figure 2: Critical slip surface obtained from SLOPE/W using simplified Bishop's method (FOS = 1.22).

4.2 ITERATION LIMIT AND NUMBER OF REALISATIONS

Preliminary analyses were conducted to determine the iteration limit, *maxit*, and number of realisations, n_{sim}, required to produce a reliable estimate of the probability of failure, P_f. Figure 3 indicates that an iteration limit of 1,000 is required for a c'-ϕ' slope problem in order to obtain a stable estimation of P_f. Figure 4 indicates that 4,000 realisations would give a reliable and reproducible estimate of P_f. As a result, *maxit* = 1,000 and n_{sim} = 4,000 were adopted in this study.

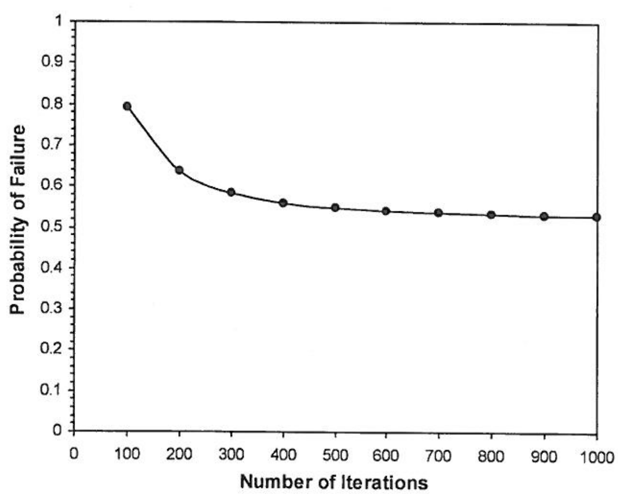


Figure 3: Effect of number of iterations on P_f.

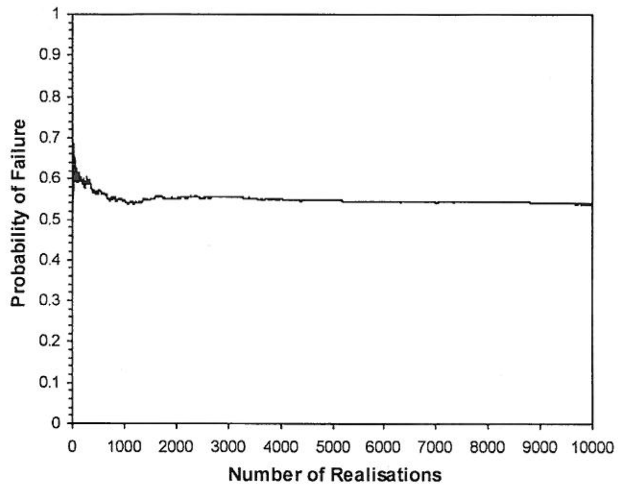
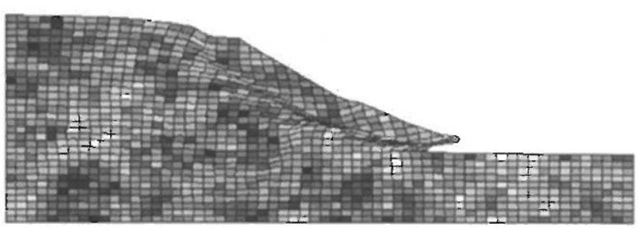


Figure 4: Effect of number of realisations on P_f.

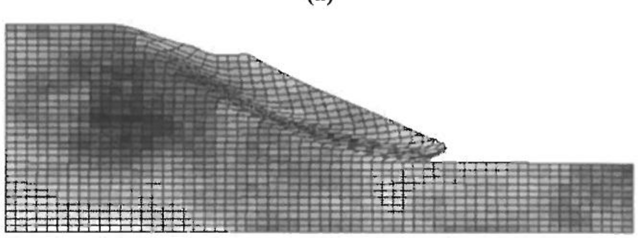
4.3 EFFECT OF COV AND θ/H ON PROBABILITY OF FAILURE

As mentioned above, the deterministic FOS computed by the finite element and limit equilibrium methods are 1.12 and 1.22, respectively, which suggest that the slope is marginally stable. The deterministic solution is based on the assumption that there is no variability of soil properties (i.e. COV $\rightarrow$ 0), as well as the soil profile is uniform and homogenous (i.e. $\theta \rightarrow \infty$). This section deals with the influence of incorporating soil variability on the stability of a c'-ϕ' slope.

Figure 5 shows the typical deformed meshes for the c'-ϕ' slope being considered, with θ/H of 0.1 and 10, respectively. The COVs of c' and ϕ' are fixed at 0.3 and 0.15, respectively. Dark and light regions indicate 'strong' and 'weak' soil elements, respectively. It can be observed from Figure 5 that a smaller value of θ generates a more rapidly varying soil profile with distance (i.e. a more spatially random soil profile), while a larger value of θ generates a more continuously varying soil profile (i.e. a more uniform soil profile). It is noted that the finite element method predicted a similar failure mechanism as that obtained from the limit equilibrium method. No noticeable difference in the failure mode, between soils with small and large scales of fluctuation, is observed in this case.



(a)



(b)

Figure 5: Typical deformed mesh at slope failure for the c'-ϕ' slope with (a) θ/H = 0.1 (b) θ/H = 10, ($COV_{c'} = 2COV_{\phi'} = 0.3$).

Figure 6 shows the variations of P_f with COV of c'. As discussed earlier, COV of ϕ' was assumed to be equal to half of that of c'. It is observed that, P_f increases as COV increases, for all cases of θ/H. To achieve practically no failure in the slope, the COV of c' and ϕ' must be smaller than 0.1 and 0.05, respectively. When the COV of c' and ϕ' is varied within the range suggested in the literature, P_f as high as 0.38 is obtained, which indicates a high likelihood for slope failure. It is noted that, as the COV increases, lower values of shear strength parameters (i.e. c' and ϕ') are likely to be encountered, and more often, in any single realisation. These low values tend to control the stability of the slope and the chances of a 'failed' slope (i.e. having a FOS < 1.0) to occur increase accordingly.

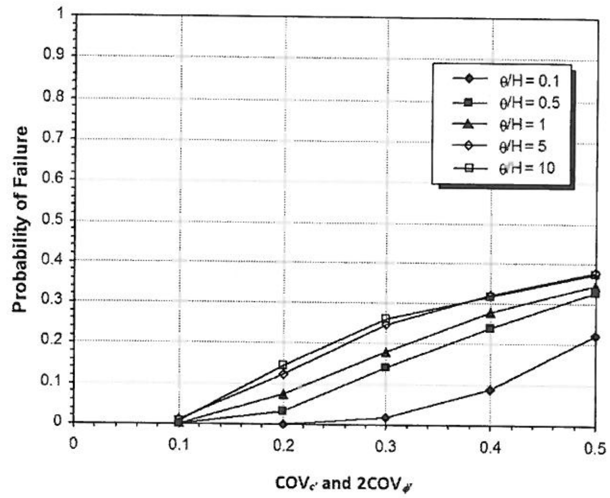


Figure 6: Effect of varying COVs on probability of failure for different values of θ/H.

Figure 7 shows the effect of varying θ/H on P_f, for different COVs of c' and ϕ'. It can be seen that, for all cases of COV of c' and ϕ', P_f increases as θ/H increases. This observation is similar to that found in the cohesive slope problem (Griffiths and Fenton, 2004; Chok *et al.*, 2007a, 2007b). This is because, by increasing the θ/H, the variation in the output statistic is also increased due to the effects of local averaging and variance reduction along the failure surface and this consequently increases the chances of a 'failed' slope to occur. It should be noted that assuming a perfectly correlated soil profile (i.e. θ/H → ∞) and completely ignoring the spatial correlation in probabilistic slope stability analysis could overestimate the probability of failure of a slope, as observed in Figure 7. In real soil deposits, the scale of fluctuation of soil properties will lie between the two extreme cases: completely random soils (i.e. θ/H → 0) and perfectly correlated soil profiles (i.e. θ/H → ∞).

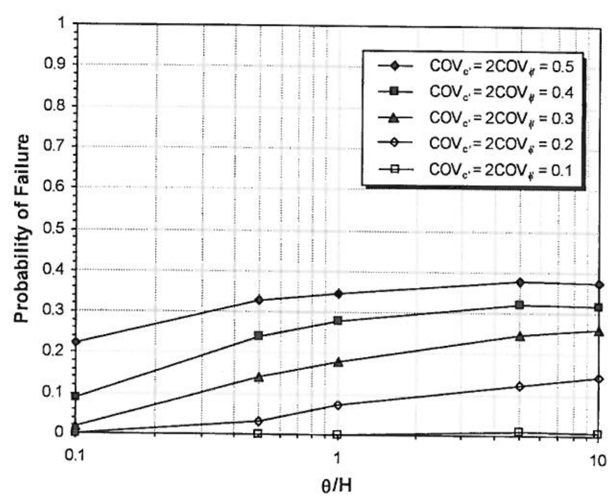


Figure 7: Effect of varying θ/H on probability of failure for different COVs of c' and ϕ'.

4.4 COMPARISON OF PROBABILITY OF FAILURE AND FACTOR OF SAFETY

In this section, the mean values of c' and ϕ' are varied, and the P_f is compared directly with the deterministic FOS. The mean values of c' and ϕ' that were considered in the analysis, and their corresponding values of deterministic FOS, are summarised in Table 2. Figure 8 shows the direct comparison between P_f and FOS, for different values of COV of c' and ϕ'. The value of θ/H was fixed at 1 in this case. It is observed that, P_f decreases as FOS increases, as expected. The curves intersect at the point $P_f = 0.75$ and FOS = 0.95. When FOS < 0.95, a larger COV leads to a lower value of P_f. In contrast, when FOS > 0.95, a larger COV leads to a higher value of P_f. It is also noted that, for the case with $COV_{c'} = 2COV_{\phi'} = 0.5$, FOS greater than 1.6 is required to reduce P_f to insignificant levels (i.e. below 1/4000). In practice, any slope with a FOS = 1.5 would generally be regarded as a stable slope. The results shown in Figure 8 suggest that the deterministic FOS is not a reliable measure of the true safety of a slope. In fact, the FOS is meaningful only when the COV of the strength parameters is very small (i.e. $COV_{c'} = 2COV_{\phi'} < 0.1$).

Figure 9 shows plots of P_f versus FOS for different values of θ/H, with $COV_{c'} = 2COV_{\phi'} = 0.5$. In this case, the intersection point occurs approximately at $P_f = 0.6$ and FOS = 1.0. When the FOS < 1.0, a larger value of θ/H leads to a lower value of P_f, which indicates that the θ/H = ∞ case is unconservative. On the other hand, when the FOS > 1.0, a larger value of θ/H leads to a higher value of P_f, which indicates that the θ/H = ∞ case is conservative.

Table 2: FOS for c'–ϕ' slope with different mean values of c' and ϕ'.

$\mu_{c'}$ (kPa)	$\mu_{\phi'}$ (degrees)	FOS
0	20	0.38
0	30	0.60
10	20	0.88
10	30	1.12
20	20	1.25
20	30	1.55

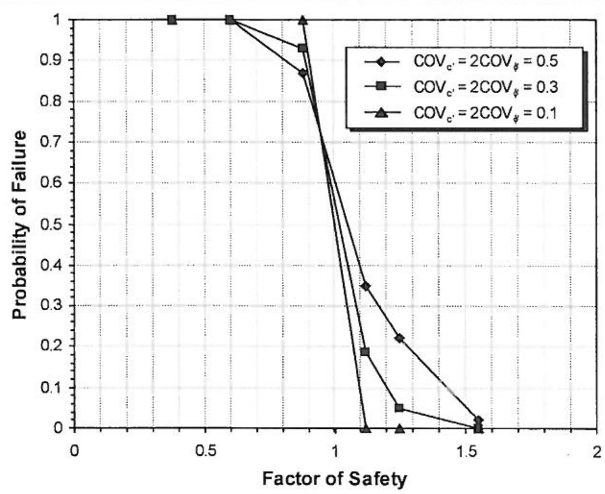


Figure 8: Probability of failure versus factor of safety for different COVs of c' and ϕ' (θ/H = 1).

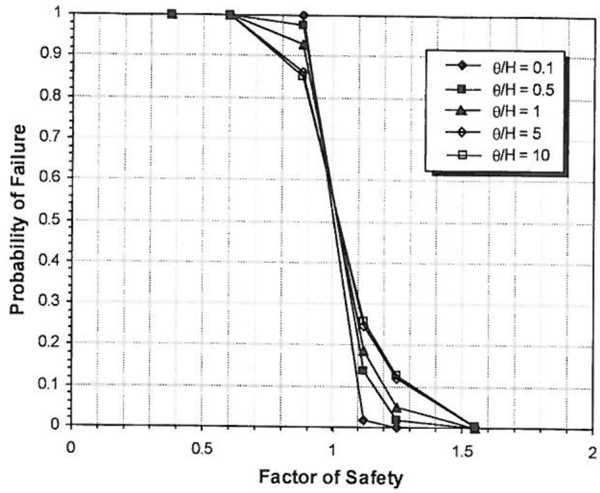


Figure 9: Probability of failure versus factor of safety for different values of θ/H ($COV_{c'} = 2COV_{\phi'} = 0.5$).

4.5 EFFECT OF $c' - \phi'$ CORRELATION ON PROBABILITY OF FAILURE

The results discussed so far are based on the assumption of no cross-correlation between the parameters c' and ϕ'. Analyses were conducted to investigate the influence of the c'–ϕ' correlation. The cross-correlation between c' and ϕ' is defined by the correlation coefficient, ρ, as previously discussed. Values of $\rho = -1$, 0, and 1, correspond to a completely negatively correlated, uncorrelated, and completely positively correlated soil, respectively. In this study, values of $\rho = -1, -0.5, 0, 0.5$ and 1 were considered. Cherubini (2000) reported that c' and ϕ' are negatively correlated, with values ranging from –0.24 to –0.70, as mentioned previously.

Figure 10 shows the variations of P_f with respect to c'–ϕ' correlation, ρ, for slopes with different values of COVs of c' and ϕ', and θ/H. The results indicate that, for all cases of COV and θ/H, negative correlation between c' and ϕ' leads to a lower estimate of P_f, while positive correlation leads to a higher estimate of P_f, so that if the actual correlation is negative, as is commonly thought, then the assumption of independence is conservative as it gives higher failure probabilities.

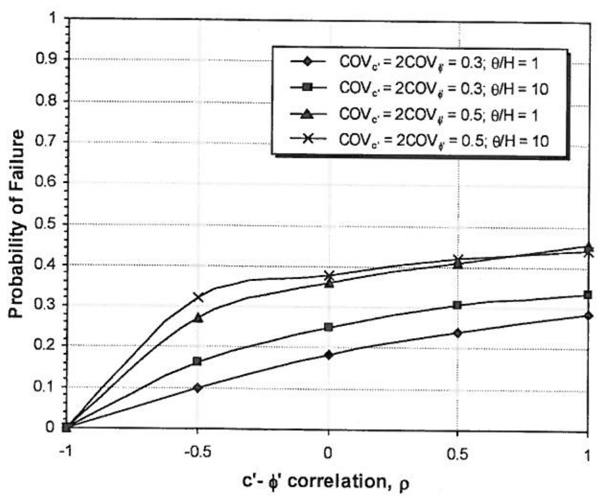


Figure 10: Probability of failure versus ρ for different values of COVs of c' and ϕ', and θ/H.

5 SUMMARY AND CONCLUSIONS

The random finite element method (RFEM) was used to investigate the influence of soil variability on the reliability of a spatially variable c'–ϕ' soil slope. The spatial variability of soil properties was modelled by the coefficient of variation, COV, and the scale of fluctuation, θ. Parametric studies were conducted to investigate the effect of varying COV and θ on the probability of failure, P_f, of a slope. The effective shear strength parameters (i.e. c' and ϕ') were treated as spatially random variables, which were assumed to be lognomally distributed. The probability of failure, P_f, of a slope was computed via the Monte Carlo simulation process.

The results of numerical studies indicated that both COV and θ/H have a significant effect on the estimated P_f. It was generally found that, P_f increased as COV and θ/H increased. It can be concluded that ignoring the spatial correlation in probabilistic slope stability analysis (i.e. θ/H → ∞) could overestimate the probability of failure of a slope. Direct comparison between the probability of failure, P_f, and the deterministic factor of safety (FOS) was made and the results indicated that values of FOS as high as 1.5 were associated with significant probabilities of failure when the COV was varied within the range suggested in the literature. It can be concluded that the deterministic FOS becomes unreliable when the variability in soil properties is significant. It was also determined that, assuming negative correlation between c' and ϕ' leads to a lower estimate of P_f, while positive correlation between c' and ϕ' leads to a higher estimate of P_f.

6 REFERENCES

Alonso, EE 1976, 'Risk analysis of slopes and its application to slopes in Canadian sensitive clays', *Géotechnique*, vol. 26, no. 3, pp. 453-72.

Baecher, GB & Christian, JT 2003, *Reliability and statistics in geotechnical engineering*, John Wiley & Sons Ltd, Chichester.

Bishop, AW & Morgenstern, NR 1960, 'Stability coefficients for earth slopes', *Géotechnique*, vol. 10, pp. 129-50.

Cherubini, C 2000, 'Reliability evaluation of shallow foundation bearing capacity on c', ϕ' soils', *Canadian Geotechnical Journal*, vol. 37, pp. 264-9.

Chok, YH, Jaksa, MB, Griffiths, DV, Fenton, GA and Kaggwa, WS 2007a, 'A parametric study on reliability of spatially random cohesive slopes', *Australian Geomechanics*, Vol. 42, No. 2, pp. 79-85.

Chok, YH, Jaksa, MB, Griffiths, DV, Fenton, GA and Kaggwa, WS 2007b, 'Effect of spatial variability on reliability of soil slopes', *Proceedings of the 10th Australia New Zealand Conference on Geomechanics*, Brisbane, Vol. 2, pp. 584-89.

Duncan, JM 1996, 'State of the art: Limit equilibrium and finite-element analysis of slopes', *Journal of Geotechnical Engineering, ASCE*, vol. 122, no. 7, pp. 577-96.

Duncan, JM 2000, 'Factors of safety and reliability in geotechnical engineering', *Journal of Geotechnical and Geoenvironmental Engineering, ASCE*, vol. 126, vol. 4, pp. 307-16.

Fenton, GA 1994, 'Error evaluation of three random field generators', *Journal of Engineering Mechanics, ASCE*, vol. 120, no. 12, pp. 2478-97.

Fenton, GA & Griffiths, DV 2003, 'Bearing capacity prediction of spatially random c-ϕ soils', *Canadian Geotechnical Journal*, vol. 40, pp. 54-65.

Fenton, GA & Griffiths, DV 2008, *Risk assessment in geotechnical engineering*, John Wiley & Sons, Hoboken, NJ.

Fenton, GA & Vanmarcke, EH 1990, 'Simulation of random fields via local average subdivision', *Journal of Engineering Mechanics, ASCE*, vol. 116, no. 8, pp. 1733-49.

GEO-SLOPE International Ltd. 2008, *Stability modelling with SLOPE/W 2007 version: an engineering methodology*, 3rd edn, Calgary, Alberta.

Griffiths, DV & Fenton, GA 2000, 'Influence of soil strength spatial variability on the stability of an undrained clay slope by finite elements'. In *Slope Stability 2000*, GSP No. 101. Proc GeoDenver 2000 Symposium. (eds. D.V. Griffiths *et al.*), Pub. ASCE, pp.184-193.

Griffiths, DV & Fenton, GA 2004, 'Probabilistic slope stability analysis by finite elements', *Journal of Geotechnical and Geoenvironmental Engineering, ASCE*, vol. 130, no. 5, pp. 507-18.

Griffiths, DV, Huang, J, & Fenton, GA 2009. 'Influence of spatial variability on slope reliability using 2-D random fields'. *Journal of Geotechnical and Geoenvironmental Engineering, ASCE*, vol. 135, pp. 1367-1378.

Griffiths, DV & Lane, PA 1999, 'Slope stability analysis by finite elements', *Géotechnique*, vol. 49, no. 3, pp. 387-403.

Hoeksema, RJ & Kitanidis, PK 1985, 'Analysis of the spatial structure of properties of selected aquifers', *Water Resources Research.*, vol. 21, no.4, pp. 563-72.

Lee, IK, White, W & Ingles, OG 1983, *Geotechnical engineering*, Pitman, London.

Matsui, T & Sun, KC 1992, 'Finite element slope stability analysis by shear strength reduction technique', *Soils and Foundations*, vol. 32, no. 1, pp. 59-70.

Phoon, KK & Kulhawy, FH 1999, 'Characterization of geotechnical variability', *Canadian Geotechnical Journal*, vol. 36, pp. 612-24.

Smith, IM & Griffiths, DV 1998, *Programming the finite element method*, 3rd edn, John Wiley and Sons, Chichester, New York.

Smith, IM & Griffiths, DV 2004, *Programming the finite element method*, 4th edn, John Wiley and Sons, Chichester, New York.

Sudicky, EA 1986, 'A natural gradient experiment on solute transport in a sand aquifer: spatial variability of hydraulic conductivity and its role in the dispersion process', *Water Resources Research*, vol. 23, no. 13, pp. 2069-83.

Taylor, DW 1937, 'Stability of earth slopes', *Journal of the Boston Society of Civil Engineers*, vol. 24, no. 3, pp. 197-246.

Vanmarcke, EH 1977, 'Probabilistic modelling of soil properties', *Journal of Geotechnical Engineering Division, ASCE*, vol. 103, no. 11, pp. 1227-46.

Vanmarcke, EH 1983, *Random fields: analysis and synthesis*, MIT Press, Cambridge, Massachusetts.

Zienkiewicz, OC & Cormeau, IC 1974, 'Viscoplasticity, plasticity and creep in elastic solids: a unified numerical solution approach', *International Journal for Numerical Methods in Engineering*, vol. 8, pp. 821-45.