# Use of local average subdivision to characterize marine mineral reserves —A conceptual framework

**T. Wambeke and M. Alvarez Grima**
MTI Holland (Member of IHC Merwede Group), Kinderdijk, Netherlands

**G. A. Fenton**
Department of Civil Engineering, Dalhousie University, Halifax, Nova Scotia, Canada

**J. Benndorf**
Department of Resource Engineering, Delft University of Technology, Delft, Netherlands

**A. Vervoort**
Department of Civil Engineering, KU Leuven, University of Leuven, Leuven, Belgium

**ABSTRACT** High-risk underwater mining operations necessitate characterizing spatial heterogeneity and resource uncertainty. Local average subdivision is a method of simulating spatial deposit variability inside a cell through a sequence of discretization stages, during which a cell is further subdivided into four or eight cells. New developments are presented to validate simulated deposit models and deal with point measurements of geotechnical and ore-grade properties. This paper illustrates the relevance of translating spatial deposit variability into financial or operational performance indicators during underwater resource exploitation. Two synthetic case studies highlight the challenges of underwater mining in relation to production plans and equipment selection.

**KEYWORDS** Geostatistical simulations, Local average subdivision (LAS), Marine deposit evaluation, Resource uncertainty, Spatial variability, Underwater mining

**RÉSUMÉ** Les opérations d'extraction minière subaquatique à risque élevé exigent une caractérisation de l'hétérogénéité spatiale et de l'incertitude de la ressource. La moyenne locale de subdivision est une méthode de simuler la variabilité spatiale du gisement dans une cellule par une séquence d'étapes de discrétisation durant lesquelles une cellule est subdivisée en quatre ou huit cellules. De nouveaux développements sont présentés afin de valider les modèles de simulation des gisements et de traiter des mesures ponctuelles des propriétés géotechniques et de teneur du minerai. Le présent article illustre la pertinence de traduire la variabilité spatiale du gisement en indicateurs de rendement financier ou opérationnel durant l'exploitation subaquatique de la ressource. Deux cas synthétiques soulignent les défis de l'exploitation minière subaquatique par rapport aux plans de production et de choix des équipements.

**MOTS CLÉS** Évaluation d'un gisement sous-marin, exploitation minière subaquatique, incertitude de la ressource, moyenne locale de subdivision, simulations géostatistiques, variabilité spatiale

## INTRODUCTION

A changing global economic and geopolitical climate has intensified exploration for unconventional resources such as seafloor mineral deposits in deep water. Whereas offshore diamond, tin, phosphate, iron sand, and gold projects—ranging from the exploration to the exploitation phase—are well established, marine mineral resources in deep water require assessment for potential exploitation. These deposits are located in active mineralization sites near tectonic plate boundaries, and are in the form of metalliferous sediments, seafloor massive sulphides, manganese nodules, manganese crust, and gas hydrates.

Field sampling programs are required to collect data to build deposit models outlining the spatial distribution of geotechnical and resource parameters; however, the sample coverage of marine exploration programs is limited by high capital and the operating and maintenance costs of diving support vessels and remotely operated vehicles. Additionally, data reliability is compromised by remote locations and the technical difficulties associated with sample collection in deep marine environments (van de Ketterij, 2010).

The sparsity of data to evaluate deep marine deposits leads to a high degree of uncertainty for block predictions. At most, conventional modelling techniques (e.g., ordinary kriging, inverse distance weighting) can indicate how well a given algorithm is performing (Caers, 2011) and they tend to smooth the spatial variability inherent to the deposit. The spatial variability can have a significant impact on downstream processing steps (i.e., the necessity of blending grades to stabilize the feed of the processing plant) and thus on the final realized value (Goovaerts, 1997; Vann et al., 2012; Benndorf, 2013; Benndorf & Dimitrakopoulos, 2013).

To minimize smoothing effects, geostatistical simulation techniques produce 20–100 possible deposit realizations, each of which emulates the spatial variability (variability within a realization) and attribute uncertainty (variability among realizations). Because decision-making is generally based on a block scale, realizations are often simulated on the selective mining unit (SMU) scale. Accurate quantification and propagation of deposit variability protects strategic investments and creates an operation that performs closer to its potential (Dimitrakopoulos, Farrelly, & Godoy, 2002), a statement that also applies to onshore operations.

This paper introduces an alternative algorithm to existing block simulation methods (e.g., generalized sequential Gaussian simulation; Dimitrakopoulos & Luo, 2004) to simulate the block properties of a deposit model. The local average subdivision (LAS) method comprises a sequence of calculation stages, during which a cell is subdivided into four (2D) or eight (3D) equal-sized cells, a process repeated to obtain increasingly smaller cell sizes. The final value of each simulated cell (SMU) represents the local average of the modelled property over the area/volume delineated by the cell. The proposed simulation method is part of a general framework that is systematic and robust. Accompanying validation guidelines help reduce the likelihood of costly mistakes and ensure that simulation results are representative.

Once developed, code can be used to generate an entire collection of block realizations, each with the correct spatial variability. Ultimately, the entire set of realizations is propagated through transfer functions to integrate the deposit variability into financial or technical project risk. Two synthetic case studies illustrate the application of a probabilistic evaluation approach. The first demonstrates the economic performance of ore/waste classification decisions made after integrating block model uncertainty with the particular cost structure of an underwater operation. The second case study illustrates how the uncertainty of geotechnical parameters can be translated into performance indicators, thereby enabling better selection of seafloor excavation equipment.

## LAS TO CHARACTERIZE SPATIAL VARIABILITY AND BLOCK MODEL UNCERTAINTY

### Necessity of geostatistical simulations

Geostatistical estimation theory includes a set of algorithms to calculate the "best" estimate at a single location, with the objective of providing an estimate as close as possible to the true but unknown grade at a specific SMU. The quality of each estimate is measured independently of neighbouring estimates, in terms of unbiasedness (average difference between the expected value of the estimator and the true value) and error variance (Journel & Huijbregts, 1978). The best estimator refers to a linear estimate that minimizes the error variance (i.e., kriging).

A deposit model of such best estimates, however, might not represent the best theoretical or practical model. In general, minimizing the estimation variance tends to smooth out the natural spatial variability inherent in the deposit (Figure 1). In other words, the estimated values of two neighbouring points tend to be more similar than what is observed in nature. Typically, small values are overestimated, whereas large values are underestimated (extreme values are filtered and the variation is reduced). Another drawback related to this smoothing effect is that the degree of spatial variability is inversely related to sample spacing (Webster & Oliver, 2004). The larger the kriging variance on average, the more variance is lost. On the other hand, the simulation approach generates a more realistic representation of the in-situ variability (compare Figure 1a, d). Note that the volume of the estimation, simulation, and data points is the same in the example shown in Figure 1.

Problems associated with geostatistical estimation are not only caused by its inability to represent the correct spatial variability, but also by the generation of only a single estimated orebody model, which inevitably leads to incorrect decisions (Dimitrakopoulos, 1998; Savage, 2003; Martinez, 2009). These incorrect decisions might result from Jensen's inequality, which states that because the value of a project, $v$, is an unknown and therefore a random variable, and the option value of the project, $OV$, is a convex function, then $OV(E[v]) \neq E[OV(v)]$. In other words, average input does not always yield average output when dealing with uncertainty and nonlinear transfer functions. This flaw of averages further justifies the use of simulated orebody models.

### Local average theory and change of support

Model application should guide the choice of method to simulate a given orebody. For industrial scale applications, computationally efficient methods have been developed and successfully applied on a point scale (e.g., Benndorf & Dimitrakopoulos, 2007). For mine planning and scheduling applications, decisions are generally made on a block scale. An appropriate choice, therefore, is a computationally efficient simulation method, which directly generates realizations on a scale of interest related to the SMU.

The selected simulation technique consists of a simulation phase and a conditioning phase. During the simulation phase, an unconditioned random field is generated by LAS, at a resolution consistent with the desired SMU scale (Fenton & Vanmarcke, 1990). During a sequence of calculation stages, each cell is subdivided into four equal-sized cells (Figure 2), a process that is repeated to further reduce the

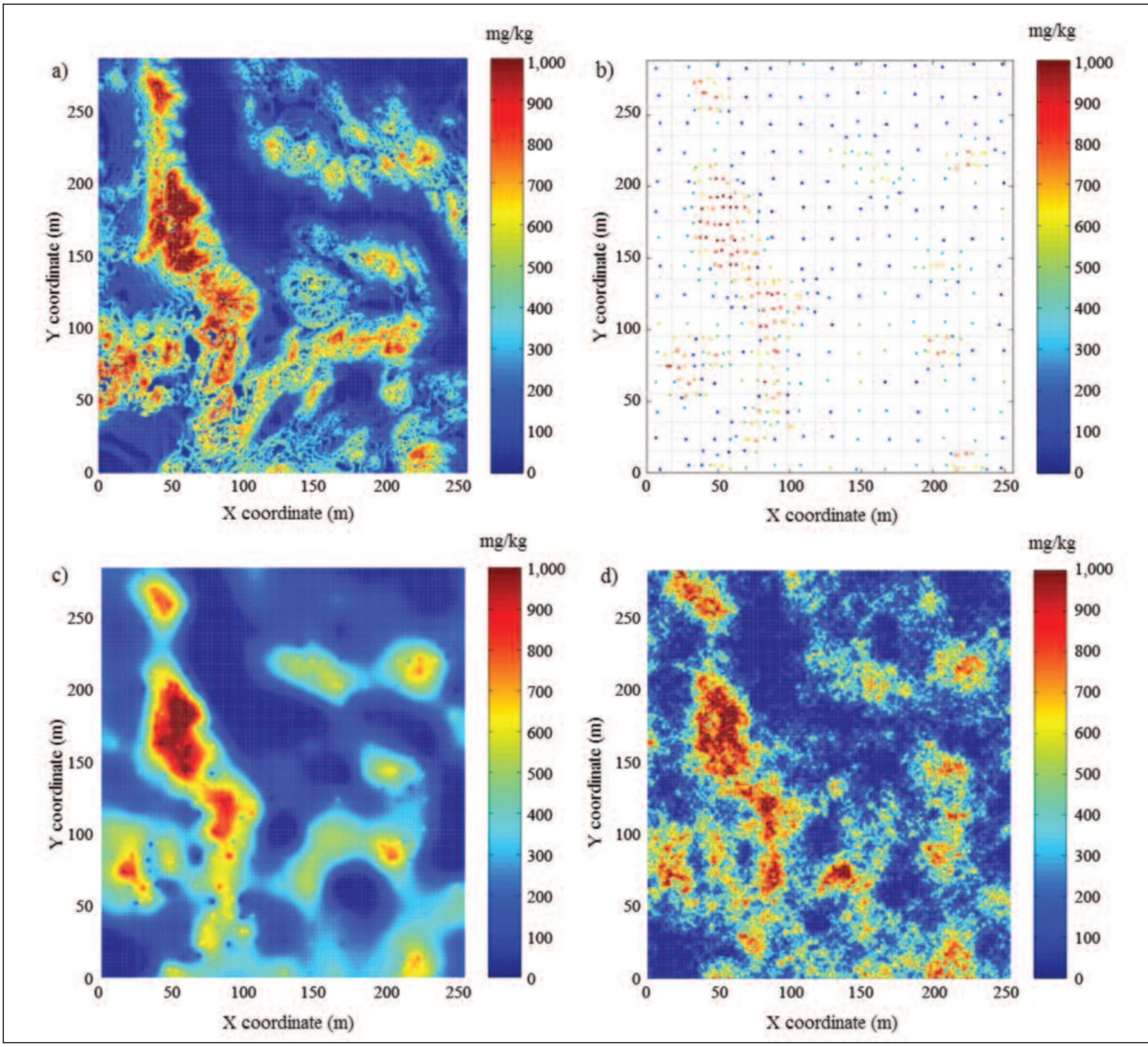


Figure 1. Estimated and simulated metal concentrations, based on the Walker Lake dataset (Isaaks & Srivastava, 1989): a) known field concentration; b) sampled field concentration; c) estimated 1 m × 1 m block concentration; d) simulated 1 m × 1 m block concentration

cell size. The final value of each simulated pixel represents the local average of the modelled property. This focusing operation eventually results in a picture (realization) of the random process, whose statistics are consistent with the desired field resolution. During the conditioning phase, a conditioning operation is performed to ensure that each simulated random field honours the locations of rich and poor zones, as observed at the sampling location.

In mining geostatistics, the "support" of the sample is the volume to be estimated/simulated, with its particular size, shape, and orientation. It is important to note that the sampling size has a considerable influence on the distribution of the values obtained in a simulation, in particular, the variance. For example, attributes measured on rock-chip–sized samples can be highly variable relative to attributes obtained on the basis of large truckloads. The mixing of high- and low-grade values in large volumes results in less variable average values (Isaaks & Srivastava, 1989). Changes in the support effect can be described by a variance reduction formula or computed based on a Gaussian quadrature approximation (Press,

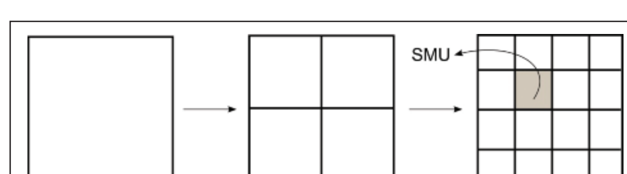


Figure 2. Resolution improvement during an LAS simulation of a local average random process. The initial coarse resolution of a generated random field is improved through a sequence of refinement stages, during which cell sizes get smaller and smaller.

Teukolsky, Vetterling, & Flannery, 2007). A detailed study of the underlying mathematics reveals that averaging not only reduces variance, but also smoothes results. The amount of variance reduction is proportional to the short-range variability of the attribute under consideration; however, the mean of a local average remains constant with a change of support (Fenton & Griffiths, 2008).

The LAS method explicitly models this important change of support effect. This advantage strongly differs from some conventional methods that implicitly model the change of support by reblocking point simulations (averaging point values inside a given block).

## Two-dimensional LAS

The local average theory can extend the simulation principles introduced in Figure 2 to formulate a simulation technique. Mathematical details of the simulation method can be found in Fenton and Vanmarcke (1990). Briefly, in two dimensions, the subdivision procedure described above continues until the desired field resolution is obtained. Figure 3 illustrates a parent cell, $Z_5^i$, subdivided into four child cells $Z_j^{i+1}, j = 1,2,3,4$. Although all parent cells undergo subdivision, to retain simplicity only one example is shown. Single element notation results in a cluttered collection of equations; therefore, a vector-matrix notation is used. The four child variables of parent cell $Z_5^i$ are stored in one column vector, $\boldsymbol{Z}^{i+1} = \{Z_1^{i+1}, Z_2^{i+1}, Z_3^{i+1}, Z_4^{i+1}\}$; the nine conditioning parent values are stored in another column vector, $\boldsymbol{Z}^i = \{Z_1^i, \ldots, Z_9^i\}$.

The unknown local averages of the four child cells are subsequently modelled as normally distributed random

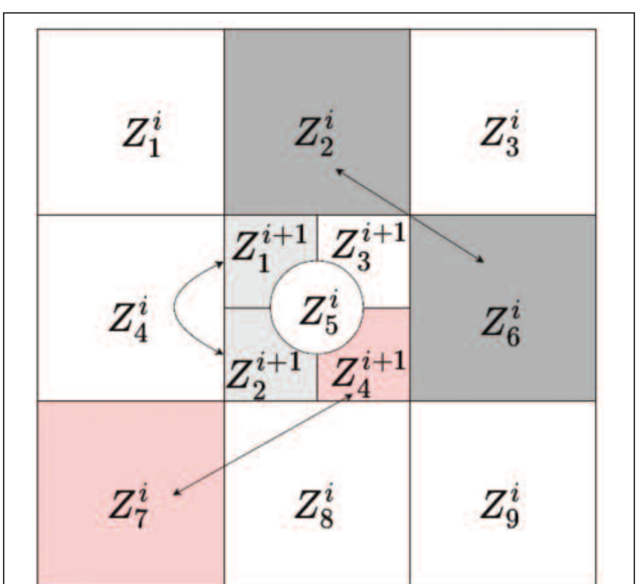


Figure 3. Cell subdivision in a two-dimensional LAS procedure. Arrows between the coloured squares represent three types of block-to-block covariance that need to be calculated (after Fenton & Vanmarcke, 1990); $Z$ is the variable to be estimated. In the LAS procedure, all cells are subdivided as shown for the central cell $Z_5^i$.

variables, with a mean and variance selected to satisfy the following criteria:

- The four local variables average to the parent value, so the global average remains constant throughout the sequence of subdivisions.
- The four local variables show a correct variance according to the local average theory.
- The four local variables are properly correlated with one another, according to the child-child covariance relations calculated based on Gaussian quadrature approximation.
- The four local variables are properly correlated with the neighbouring child values across the parent boundaries, according to the child-parent covariance relations calculated based on Gaussian quadrature approximation.

## Conditioning of the random field

The following formula is used to perform conditioning calculations:

$$Z_c(\boldsymbol{x}) = Z_u(\boldsymbol{x}) + [Z_k(\boldsymbol{x}) - Z_s(\boldsymbol{x})] \quad (1)$$

where $Z_c(\boldsymbol{x})$ represents the conditioned field, $Z_u(\boldsymbol{x})$ is an unconditional simulation, $Z_k(\boldsymbol{x})$ denotes the block kriging estimate based on known measured values at the sampling location, and $Z_s(\boldsymbol{x})$ denotes the block kriging estimate based on the unconditional simulated values at the sampling location.

## New integrated framework for producing reliable results

To increase the applicability of the proposed simulation technique, a solution was developed to overcome two model constraints: the data must be normally distributed, and a data support must be similar to the pixel size of the simulated images. Moreover, the proposed solution is integrated in a general simulation procedure that is systematic, robust, and easy to follow. This procedure can reduce the possibility of costly mistakes and help to ensure the validity and representativeness of the simulation results (Nowak & Verly, 2004). The LAS simulation technique described above is the core of the process. Other steps needed to complete the geostatistical analysis constitute three main types of components (Figure 4):

- Main operations (seven steps) are numbered and located in a vertical sequential path.
- Data and area statistics boxes (six steps) are indicated by a capital letter, and include the same set of tools used to summarize the results of the operations in global and spatial statistics.
- Validation procedures (two steps) are indicated by diamonds; they control and safeguard the representativeness of the simulation results.

The support and projection spaces (blue ovals) change during analysis.

**Main operations** Seven main operations constitute the spine of the simulation framework (refer to Figure 4):

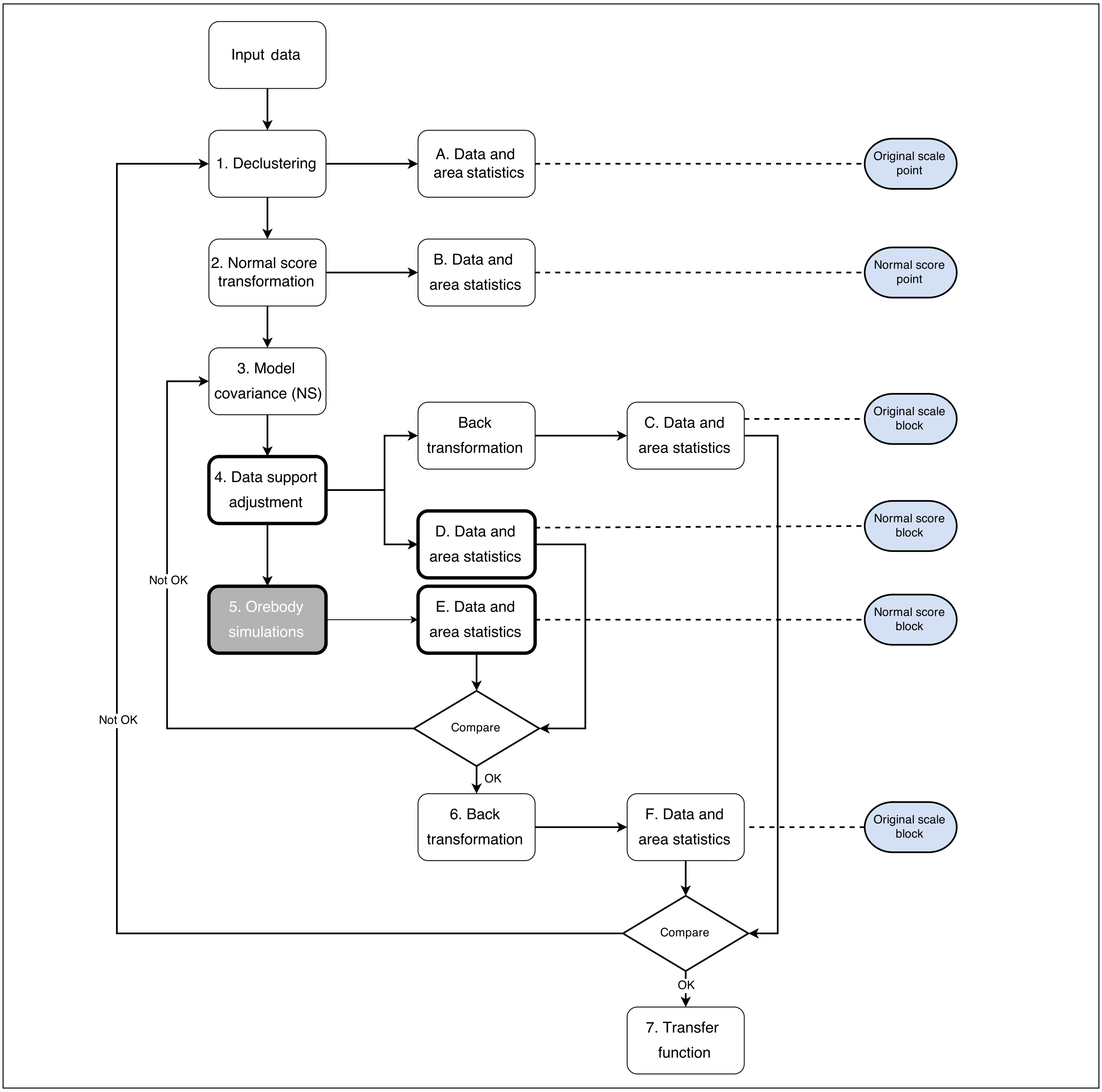


Figure 4. Flowchart of the LAS simulation process. Seven main operations are numbered and located in a vertical sequential path. Note that the support and projection spaces (blue ovals) change during the analysis. The internal part of the simulation framework, which operates in the normal score domain, is indicated by thick lined boxes. Validation procedures (diamonds) depend on calculating summary statistics to approve simulation results.

1. A declustering algorithm removes the bias associated with preferential sampling and makes the data more spatially representative of the area or volume under investigation. The input for the algorithm is the original scale point values; the output is original scale point values and the declustering weights.
2. Normal score transformation removes the normal distribution constraint on the original data. The input is the original scale point values and the declustering weights; the output is normal score (NS) point values.
3. Using a model covariance function, a theoretical model is fitted to the computed experimental covariances of the NS values. The input is the NS point values; the output is the analytical covariance function.
4. The data support adjustment is an affine correction that overcomes the constraint that the support of the data must approximate the pixel size of the simulated images. The input is NS point values; the output is NS block values.
5. Orebody simulation generates equally probable representations of the in-situ orebody variability from a

combination of random field simulation and conditioning. The input is NS block values (conditioning data); the output is simulated NS block values.

6. Back transformation is responsible for representing the orebody models in the original data space. The input is simulated NS block values; the output is simulated original scale block values (operation in sequential path), or the input is NS block values and the output is original scale block values (box connected with data support adjustment operation).
7. Transfer functions are economical and technical transfer functions that translate the simulated geological and geotechnical models into financial and operational performance indicators. The input is original scale simulated block values and the output is performance indicators.

**Data and area statistics** The inference process embedded in each data and area statistics box (Figure 4) estimates the same relevant summary statistics for each inserted collection of data values. Each box contains the same set of tools to compare the outcomes of relevant operations. As the name suggests, data and area statistics can be subdivided into global and spatial (area) statistics groups.

Global statistics characterizing general data properties are computed using formulas for the following conventional statistical measures: mean, standard deviation, interquantile range, frequency, and cumulative distribution. The conventional formulas are adjusted to correct for the bias introduced by preferential sampling. For example, the mean is calculated with variable declustering weights that give more importance to isolated locations. Spatial statistics estimate spatial correlation structures inherent within the area under investigation, frequently using variograms, covariance, and correlation functions. The lag mean, lag variance, and moving window statistics are also calculated.

**Validation procedures** The embedded validation procedures safeguard the quality and representativeness of the simulation results. Without a validation procedure, even the most sophisticated technique can yield unreliable results. The proposed practical process contains two main validation steps (upper and lower diamonds in Figure 4):

- The first validation step checks the simulation results in the NS domain and approves them if their frequency distribution, cumulative distribution, and variograms match the conditioning NS block values. If the validation statistics do not match, the entire set of realizations is rejected and the simulation is repeated with adjusted parameters.
- The second validation procedure tests the complete set of geostatistical simulation results by checking if the final simulation results (after back transformation) reproduce the global and spatial statistics of the conditioning block data.

This double validation procedure was designed to help detect where the calibration parameters (e.g., range, nugget, and sill of the variogram model, neighbourhood size) must be adjusted in the practical process. A full description of the proposed practical process is described by Wambeke (2013).

## QUANTIFYING BLOCK MODEL UNCERTAINTY AT THE OCEAN FLOOR TO IMPROVE DECISION-MAKING

### Concept of the transfer function

To identify the (financial) project risk or to optimize the design of deep-sea extraction equipment, the block model uncertainty must be propagated through the complete value chain. Thus, the derived spatial distributions (spatial stochastic models) have to be inserted into physical models, which translate the block parameters (e.g., uniaxial compressive strength [UCS], Brazilian tensile strength [BTS], and friction angle) into financial or operational

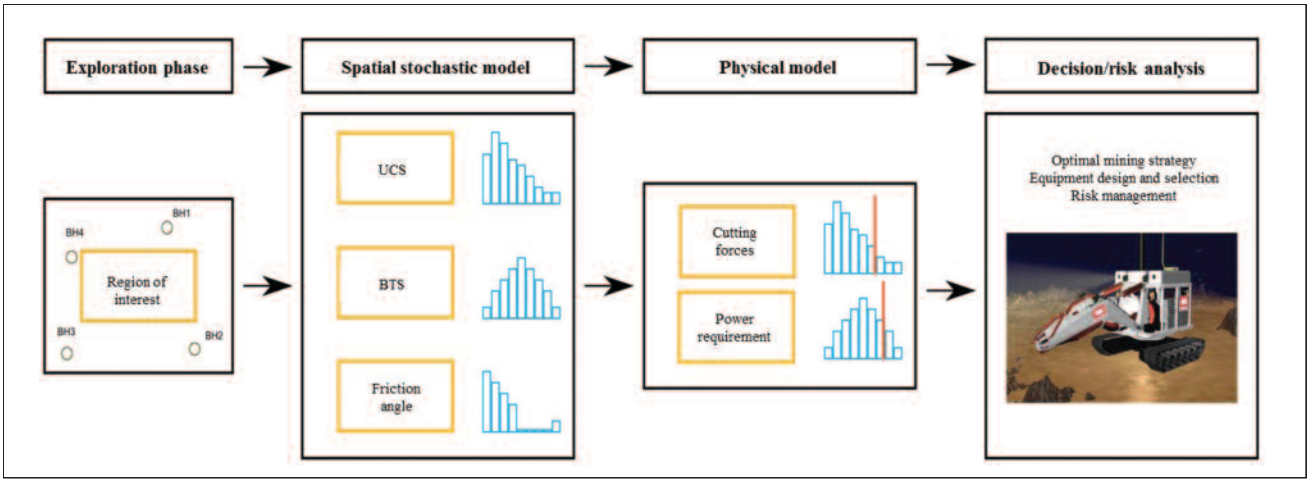


Figure 5. Conceptualization of the added value of the combination between geostatistical simulations and transfer functions. To identify the (financial) project risk or to optimize the design of deep-sea extraction equipment, it is necessary to propagate the characterized block model uncertainty through the complete chain. The spatial distributions of geotechnical parameters (spatial stochastic models) are inserted into physical models to translate them into operational performance indicators (e.g., cutting forces and power requirements).

performance indicators (Figure 5). These models—so-called utility or transfer functions (Dimitrakopoulos, 1998)—can be used to calculate indicators such as the cutting force, power requirement, bearing capacity, cash flow, grade tonnage curve, or net present value (NPV).

To further assist the decision-making process, the likelihood and corresponding (economic) consequences of different scenarios can be compared and evaluated. Often the entire analysis can be summarized, through a decision or forecast model, into a single monetary value that estimates the expected profit, loss, or cost associated with each scenario. Typically, the scenario that maximizes the monetary value of the mining project is preferred for implementation. The main advantage of this simulation framework is that the spatial variability and uncertainty is propagated through the entire equipment design and mine-planning process. Consequently, the complete procedure results in a "risk-robust" decision that adds value to the project (e.g., Figure 5).

## Economic evaluation of a synthetic marine gold deposit

A case study illustrates how the economic value of a marine gold mining operation can be significantly improved by considering block model uncertainty during the planning phase. For each excavated block, the production engineer must decide whether to classify the mined material as ore or waste. Because the economic consequence of misclassifying a block of ore as waste might result in a significant loss, and misclassifying a block of waste as ore might result in a manageable additional cost, it is prudent to implement a risk-based selection strategy to minimize classification errors. Thus, decisions are made by comparing the expected economic consequences of both classification scenarios. The proposed strategy is more capable of exploiting the full potential of each single block. The risk-based selection strategy will be benchmarked against the conventional cut-off grade approach.

An economic model of a deep-sea mining operation considers three main cost components: excavation, transportation, and processing. The costs used here are very rough estimates based on a technical report by Nautilus Minerals (Lipton, 2012). The excavation cost (US$15/t) is considered to be the same for waste and ore; however, transportation costs might differ substantially. Waste must be pumped horizontally, possibly over a large distance of the ocean floor, to a disposal area (US$10/t). Ore, on the other hand, must be pumped horizontally to the vertical riser assembly, lifted to the surface, transported by barge to the shore, and then transported to the processing plant (US$30/t). Processing costs are considered only for ore (US$55/t).

To illustrate the effectiveness of the risk-based selection strategy, the case study was carried out using data from a completely known area. A dataset containing 470 samples was originally derived from a satellite image depicting surface elevation around Walker Lake (Isaaks & Srivastava, 1989). Data were modified to represent a horizontal 1 m thick rich top layer of a larger deposit on the ocean floor. After declustering, the distribution remained positively skewed with the following statistics: mean gold grade = 2.9 g/t; median gold grade = 2.8 g/t; standard deviation = 2 g/t; minimum gold grade = 0 g/t; maximum gold grade = 10.2 g/t. The average density of the material was 2.7 t/m$^3$.

The 1 m thick slab of the deposit was subsequently subdivided into 288 $16 \times 16\,m^2$ blocks with a unit thickness (one realization shown in Figure 6a). To keep the case study simple, one can assume that every block in the investigated layer will be excavated row by row, starting from the lower

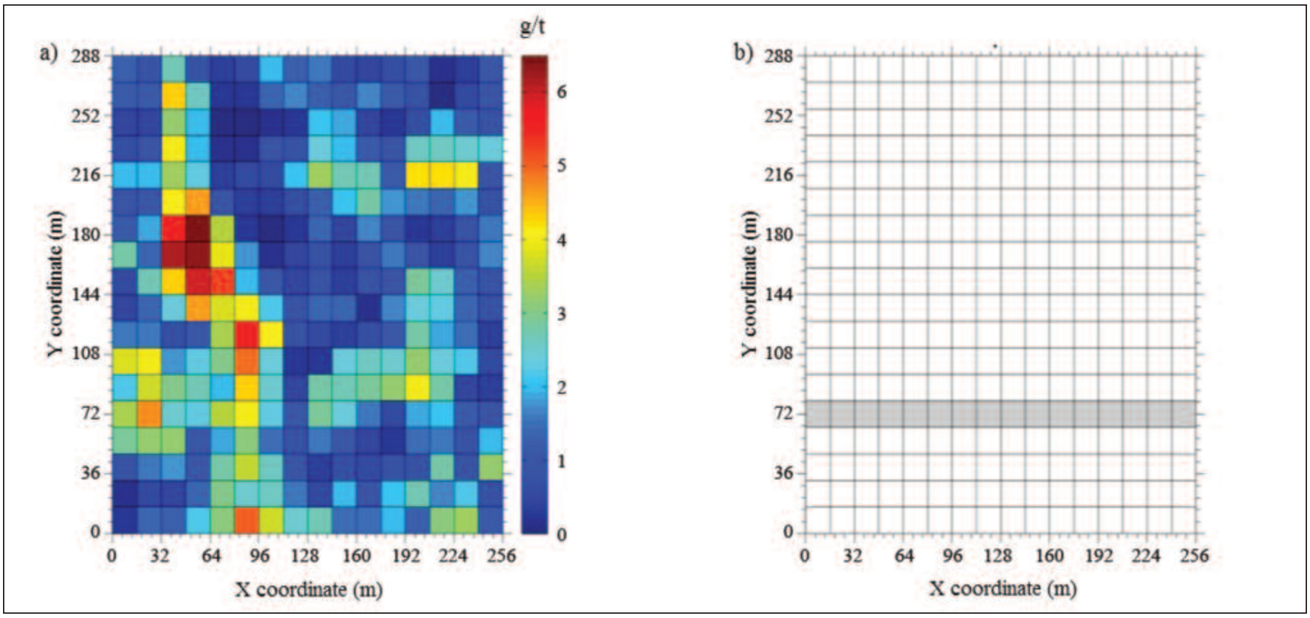


Figure 6. Preliminary planning of a gold mining operation: a) simulated gold concentrations; b) planned extraction sequence during the fifth mining period shown in the grey row

left corner and ending in the upper right (Figure 6b). One row containing 16 blocks will be completely mined during a single mining period. The displayed area will be mined over a total of 18 periods.

Before excavation, blocks need to be classified as either ore or waste (this is generally known well ahead of excavation):

- Scenario 1: Mined block is classified as waste and transported to the waste dump at the ocean floor. Costs include excavation (US\$15/t), horizontal transportation, and disposal (US\$10/t).
- Scenario 2: Mined material is classified as ore, pumped to the surface by means of a vertical riser, transported by barge to the shore, and treated in a processing plant. Costs include excavation (US\$15/t), transportation (US\$30/t), and processing (US\$55/t).

A mined block typically moves through the second logistical scenario if the price of the estimated recovered metal exceeds the sum of the mining, transportation, and processing costs. This minimum amount of metal required is generally linked with the economic cut-off grade, which is calculated as

$$z_{cut\text{-}off} = \frac{15 + 30 + 55}{p.r} \tag{2}$$

where $p$ is the metal price (US\$53/g) and $r$ is the percentage of contained metal that can subsequently be recovered (95%). Traditionally, the cut-off grade (2 g/t) is compared with estimated block grades to classify the mined material. Applying conventional classification to one estimated block model, 113 blocks are classified as ore (grey) because their estimated grade exceeds the cut-off grade (Figure 7a).

This conventional classification strategy, however, does not account for uncertainty and risk. Given simulated probability distributions, the expected profit associated with each economic scenario of classification can be assessed and used to derive an economically optimal ore selection (Glacken, 1996). The expected "profit" of classifying a block as waste is given by

$$E[Pr_{waste}] = -C_m - P_o\,[p.r.m^+ - C_p] \tag{3}$$

where $P_o$ is the probability that the true grade of the block exceeds the cut-off grade, $m^+$ is the mean grade if the block is classified as ore, and $[p.r.m^+ - C_p]$ is the lost opportunity cost. The formula indicates that mining costs must be paid and a possible loss is associated with the disposal of profitable material.

The expected profit of sending a block to the processing plant is

$$E[Pr_{ore}] = -C_m + P_o\,[p.r.m^+ - C_p] + P_w\,[p.r.m^- - C_p] \tag{4}$$

where $P_w$ is the probability that the true grade of the block is lower than the cut-off grade, $m^-$ is the mean grade if the block is classified as waste, $[p.r.m^+ - C_p]$ is the possible profit if the classification is correct, and $[p.r.m^- - C_p]$ is a possible additional cost if the classification is incorrect. Finally, a block is selected as ore if the expected profit for classifying it as such is greater than the expected profit of classifying it as waste. The risk-based selection strategy results in a much greater volume of ore (166 blocks, Figure 7b) than the conventional selection protocol (113 blocks, Figure 7a). Probabilities and expected profits were calculated from 100 simulated deposit realizations.

A cash flow analysis was then computed using the same mining sequence (1 row = 1 mining period). Assuming a discount rate of 5%, the cash flow can ultimately be

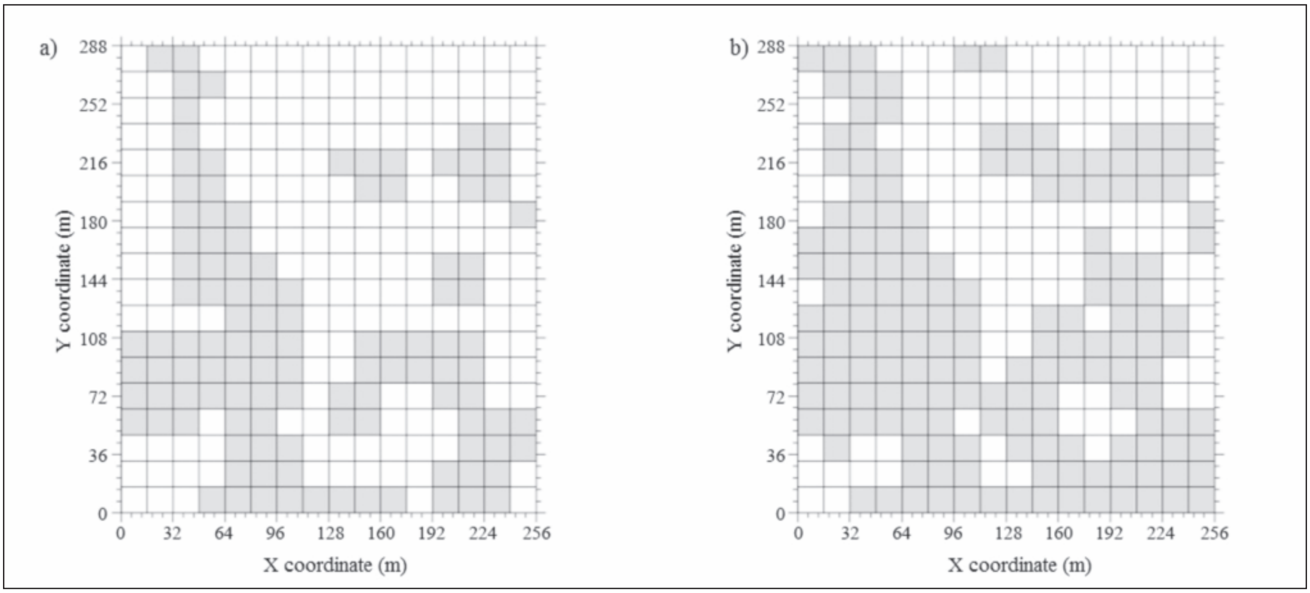


Figure 7. Improved decision making under the face of block model uncertainty for a) conventional ore/waste selection based on the estimated block grades and b) risk-based selection based on simulated probability distribution; grey = ore, white = waste

summarized as NPV. Figure 8 compares the real (possible because the case study is based on an area that is completely known) and predicted economic performance of the conventional and risk-based ore/waste classification strategies from Figure 7. The economic performance of the conventional and risk-based classification strategies are depicted by green and purple bars, respectively, in Figure 8. The predicted performance of the conventional classification strategy applied to the estimated block model is depicted by the red bar in Figure 8. The predicted performance of the risk-based classification strategy applied to each individual simulation (for a total of 100) is depicted by the grey bars in Figure 8. Besides a most expected NPV, the simulation approach yields an indication of the uncertainty of the entire project value. This uncertainty is a direct consequence of the limited amount of information available (470 data points).

The case study shows that a risk-based selection strategy can increase the project NPV from \$8.4 million to \$10.4 million (25%). Further research needs to be carried out to better understand the impact of a change in cut-off grade and other economic parameters on the final decision and the corresponding change in NPV.

## Propagation of block model uncertainty into performance indicators relevant for equipment selection

The second synthetic case study illustrates how a simulation-based geostatistical analysis can be used to support decisions regarding equipment selection. Assume that a decision needs to be made regarding the size and installed power of a deep-sea crawler with a drum cutter. The strength of the rock can significantly limit the amount of material that can be excavated, depending on the equipment. The case study illustrates how a collection of technical transfer functions is used to transform an array of rock-strength parameters into operational performance indicators. Evans and Pomeroy (1996) explained how to calculate the required forces on the cutting teeth and how to translate these forces into energy and required cutting power. A total of 73,728 1 $m^2$ blocks with a unit thickness were simulated in an area of 288 × 256 $m^2$. The analysis is performed as follows:

1. A geostatistical simulation is performed to assess and characterize the spatial variability and block model uncertainty in rock strength (BTS). The BTS of the rock ranges from 0.5 to 3.2 MPa.
2. The simulated deposit models are inserted into the Evans and Pomeroy cutting formulas to calculate the forces required to excavate the material. The required cutting forces range from 0.70 to 4.41 kN (Evans & Pomeroy, 1996).
3. A second transfer function computes the specific energy requirement (based on the previously obtained cutting forces) or the amount of energy needed to cut 1 $m^3$ of rock. The specific energies range from 417 to 2,635 kJ (Evans & Pomeroy, 1996).
4. Considering a desired production rate of 600 $m^3$/h, a third and final transfer function can be used to compute the required cutting power. The final calculated values vary between 69.43 and 437.71 kW (Evans & Pomeroy, 1996).

Because the collection of technical transfer functions is evaluated over the entire set of simulated realizations, uncertainty is automatically propagated through the calculations (i.e., the collection of transfer functions is evaluated over each individual realization); therefore, each cubic metre of material is connected with a specific distribution of required cutting power, with the width of each distribution representing the inherent uncertainty.

To optimize equipment selection and identify areas for additional drilling, the workability is compared among excavation equipment with an installed power of 200, 300, or 350 kW. Because each cubic metre is characterized by its own distribution of required cutting power, it is possible to calculate the probability that a tool with a given installed power is able to cut the rock. Probabilities were calculated from

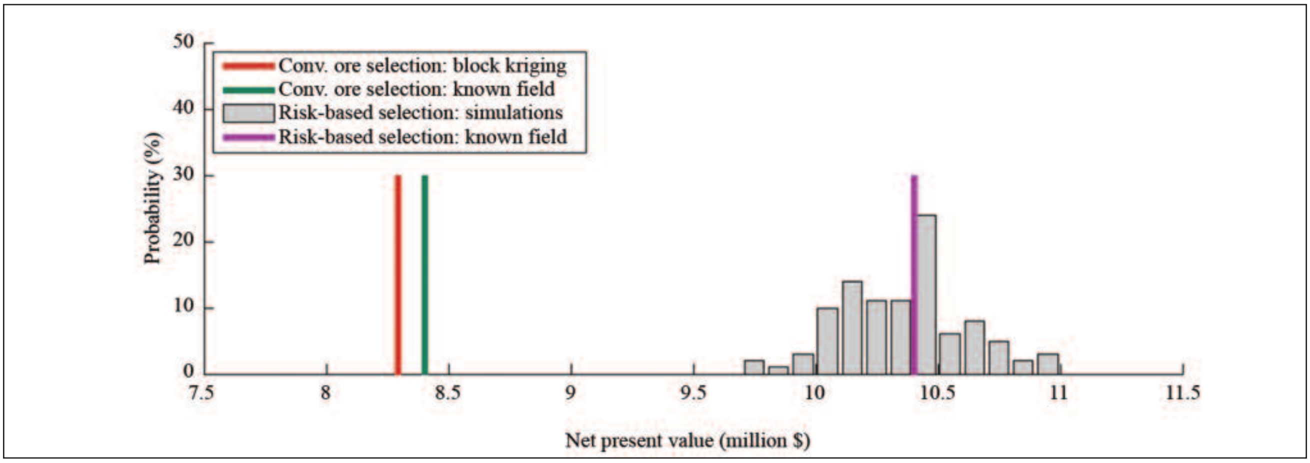


Figure 8. The net present value (NPV) of the investigated part of the gold mining project. The results from a risk-based ore/waste selection strategy are compared with those of a conventional selection strategy. Note that the risk-based selection strategy resulted in a much greater volume of ore (166 blocks) than the conventional selection protocol (113 blocks).

100 deposit realizations. To facilitate comparison, the calculated probabilities are divided into three categories (Figure 9):

- Event 1: Grey cells indicate regions with a greater than 80% chance of the required cutting power being below the limit of the selected mining tool. There is a high probability that the excavation equipment is able to cut the rock.
- Event 2: Orange cells indicate regions with a greater than 80% chance of the required cutting power exceeding the limit of the selected mining tool. During mining operations, cutting in these regions could be associated with low production rates, high wear, high maintenance requirements, or even breakdown.
- Event 3: Purple cells correspond to the intermediate scenario. Due to local uncertainty, it is difficult to make reliable statements regarding the cuttability of the rock with respect to the selected mining tool. Purple cells indicate locations where additional investigations would likely provide valuable information.

Only approximately 25% of the area can likely be excavated efficiently with the 200 kW excavation equipment (Figure 9a), whereas this value can be increased to approximately 85 or 95% by selecting a more powerful mining tool (Figure 9b, c). In addition, a more detailed exploration campaign could increase the area likely to be excavated efficiently. To further optimize equipment selection, a detailed risk assessment should be performed, comparing the likelihood and costs of breakdown, wear, and additional maintenance with the higher capital costs associated with more powerful equipment.

## CONCLUSIONS AND RECOMMENDATIONS

Conventional geostatistical estimation techniques cannot correctly characterize block model uncertainty. Such estimation models routinely smooth the spatial variability inherent in a deposit and thus are not suitable for investigating the risk associated with ore/waste decisions. To potentially overcome these restrictions, geostatistical simulation techniques can be used. Correct quantification and propagation of block model uncertainty protects strategic investments and creates an operation that performs closer to its potential (Dimitrakopoulos, Farrelly, & Godoy, 2002). This statement applies to conventional mining and even more so to deep-sea operations.

Generally, decisions are made on a block scale; therefore, it is optimal to generate realizations on the SMU scale of interest. This paper introduces the LAS method for generating simulated deposit models with average block concentrations. The proposed simulation method was further integrated into a systematic and robust framework. Accompanying validation guidelines were formulated to help reduce the likelihood of costly mistakes and ensure that the simulation results are representative.

The simulation package can be used to generate an entire collection of correct representations (20–100) of the spatial variability. The resulting realizations are propagated individually through a selection of transfer functions to translate the block model parameters into financial or technical project risk.

Two case studies demonstrate the application of a probabilistic evaluation approach. Integrating block model uncertainty into decision-making substantially improves the economic performance of classification decisions and the likelihood of optimal equipment selection to enhance the probability of a reliable and efficient operation. The simplified case studies were intended to illustrate the potential impact of a risk-based decision-making strategy for mining in general and for marine mining in particular. More research, however, is required to establish a better understanding of the selectivity of seafloor excavation equipment and its impact on the final mine design. The ore/waste decision might need to be made based on slurry volumes instead of a conventional mining block.

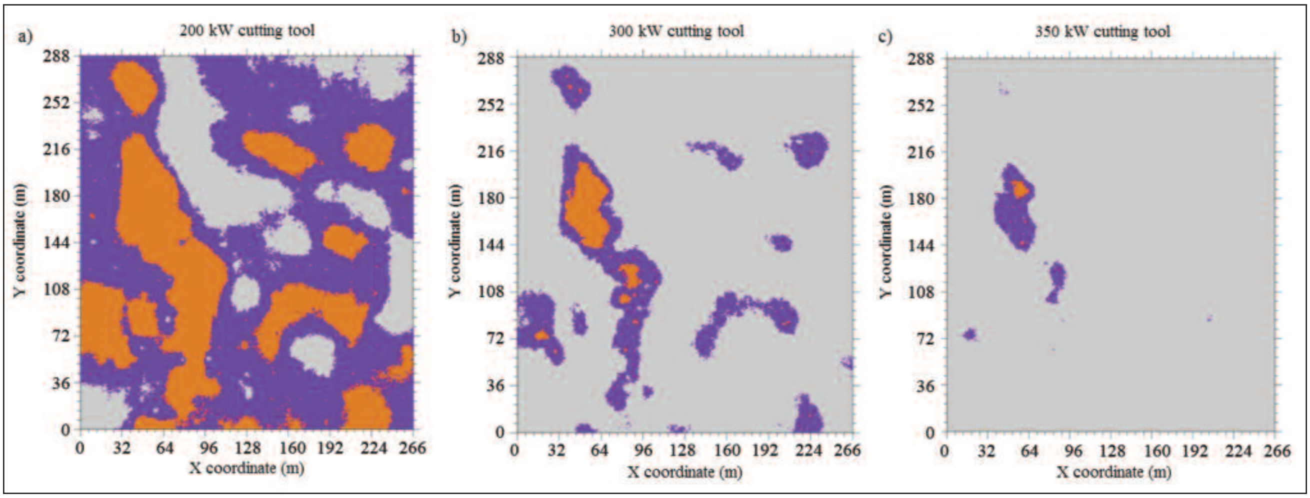


Figure 9. Synthetic example of a probabilistic analysis of the efficacy of equipment with a) 200 kW, b) 300 kW, and c) 350 kW installed cutting power. Grey indicates areas with a greater than 80% chance of the equipment being able to cut the rock. Orange indicates areas with a greater than 80% chance of equipment experiencing problems while cutting the rock. Purple indicates areas with intermediate probabilities.

## ACKNOWLEDGMENTS

Special thanks are due to Janice M. Burke and the anonymous reviewers for their comments, which contributed to improving earlier versions of the paper.

*An earlier draft of this paper was published in the World Mining Congress 2013 Conference Proceedings, prior to undergoing the* CIM Journal *peer-review process.*

Paper reviewed and approved for publication by the Geological Society of CIM.

**Tom Wambeke** graduated with a BSc in geotechnical engineering from the University of Leuven (Belgium) and obtained an MSc in resource engineering from Delft University of Technology (Netherlands). Currently, he works for IHC Merwede and is conducting a PhD research project at TU Delft. His research focuses on the characterization of resource uncertainty, real-time updating based on production data, and the optimization of operational decision variables.

**T.Wambeke@mtiholland.com**

**Gordon Fenton**, currently with Dalhousie University, is vice-chair of the International Society of Soil Mechanics and Geotechnical Engineering (ISSMGE TC304), chair of the Canadian Highway Bridge Design Code (CHBDC) Foundations Committee, and member of the National Building Code of Canada Standing Committee on Structural Design (NBC SCSD). He has received the Thomas C. Keefer Award, the George Stephenson Medal, the Gzowski Medal, and was elected a Fellow of the Canadian Academy of Engineering.

**Mario Alvarez Grima** is a geotechnical engineer with more than 28 years of experience in research and consultancy. He obtained his PhD from Delft University of Technology in the field of soft computing. He currently works at MTI Holland as senior geotechnical specialist, with a focus on dredging and wet mining.

**Joerg Benndorf** is a mining professional with 13 years of international experience in project management, mine planning, operations, and geological modelling in industry and academia. He has worked in Australia, United States, Jamaica, and Germany in different commodities. He is currently assistant professor of resource engineering at the Delft University of Technology.

**André Vervoort** graduated with an MSc in mining engineering from KU Leuven (Belgium) and obtained his PhD at the Faculté Polytechnique de Mons (Belgium). After working for several years at Chamber of Mines Research Organization (COMRO), South Africa, he became a professor at KU Leuven, where his teaching and research activities focus mainly on rock mechanics and geostatistics.

## REFERENCES

Benndorf, J. (2013). Application of efficient methods of conditional simulation for optimising coal blending strategies in large continuous open pit mining operations. *International Journal of Coal Geology, 112*(June), 141–153. http://dx.doi.org/10.1016/j.coal.2012.10.008

Benndorf, J., & Dimitrakopoulos, R. (2007). New efficient methods for conditional simulation of large orebodies. *Orebody and Strategic Mine Planning,* Spectrum Series, Vol. 14 (2nd ed., pp. 103–110). Carlton, Australia: The Australian Institute of Mining and Metallurgy.

Benndorf, J., & Dimitrakopoulos, R. (2013). Stochastic long-term production scheduling of iron ore deposits: Integrating joint multi-element geological uncertainty. *Journal of Mining Science, 49*(1), 68–81. http://dx.doi.org/10.1134/S1062739149010097

Caers, J. (2011). *Modeling uncertainty in the earth sciences*. Chichester, NY: John Wiley & Sons. http://dx.doi.org/10.1002/9781119995920

Dimitrakopoulos, R. (1998). Conditional simulation algorithms for modelling orebody uncertainty in open pit optimisation. *International Journal of Surface Mining, Reclamation and Environment, 12*(4), 173–179. http://dx.doi.org/10.1080/09208118908944041

Dimitrakopoulos, R., Farrelly, C. T., & Godoy, M. (2002). Moving forward from traditional optimization: Grade uncertainty and risk effect in open-pit design. *Mining Technology, 111*(1), 82–88. http://dx.doi.org/10.1179/mnt.2002.111.1.82

Dimitrakopoulos, R., & Luo, X. (2004). Generalized sequential Gaussian simulation on group size v and screen-effect approximations for large field simulations, *Mathematical Geology, 36*(5), 567–591. http://dx.doi.org/10.1023/B:MATG.0000037737.11615.df

Evans, A. A., & Pomeroy, C. D. (1996). *The strength and workability of coal.* New York, NY: Pergamon Press.

Fenton, G. A., & Griffiths, D. V. (2008). *Risk assessment in geotechnical engineering.* Hoboken, NJ: John Wiley & Sons. http://dx.doi.org/10.1002/9780470284704

Fenton, G. A., & Vanmarcke, E. H. (1990). Simulation of random fields via local average subdivision. *Journal of Engineering Mechanics, 116*(8), 1733–1749. http://dx.doi.org/10.1061/(ASCE)0733-9399(1990)116:8(1733)

Glacken, I. M. (1996). Change of support and use of economic parameters for block selection. E. D. Baafi & N. A. Schofield (Eds.), *Geostatistics Wollogong 1996*, 811–821.

Goovaerts, P. (1997). *Geostatistics for natural resources evaluation.* New York, NY: Oxford University Press.

Isaaks, E. H., & Srivastava, R. M. (1989). *An introduction to applied geostatistics.* New York, NY: Oxford University Press.

Journel, A. G., & Huijbregts, C. J. (1978). *Mining geostatistics.* London, UK: Blackburn Press.

Lipton, I. (2012). *Mineral resource estimate: Solwara project, Bismarck Sea, PNG* (Tech. rep.). Brisbane, Australia: Golder Associates.

Press, W. H., Teukolsky, S. A., Vetterling, W. T., & Flannery, B. P. (2007). *Numerical recipes in C: The art of scientific computing* (3rd ed.). New York, NY: Cambridge University Press.

Martinez, L. A. (2009). Why accounting for uncertainty and risk can improve final decision-making in strategic open pit mine evaluation. *Proceedings of Project Evaluation 2009* (pp. 113–124). Melbourne, Australia: The Australasian Institute of Mining and Metallurgy.

Nowak, M., & Verly. G. (2004). A practical process for simulation with emphasis on Gaussian simulation. *Proceedings of the Orebody Modelling and Strategic Mine Planning Conference,* Perth, Australia, 129–143.

Savage, S. (2003). *Decision making with insight.* Cincinnati, OH: South-Western College Publishers.

van de Ketterij, R. (2010). *Final conclusions on the FEMP workshop* (Tech. Rep.). Kinderdijk , Netherlands: IHC Merwede and Federation of European Mineral Programs (FEMP).

Vann, J., Jackson, S., Bye, A., Coward, S., Moayer, S., Nicholas G., & Wolff, R. (2012). Scenario thinking—A powerful tool for strategic planning and evaluation of mining projects and operations. *Proceedings of Project Evaluation 2012*, 5–14. Melbourne, Australia: The Australasian Institute of Mining and Metallurgy.

Wambeke, T. (2013). *Decision making on the ocean floor—A simulation framework to assess uncertainty and variability in deep sea mining* (Master's thesis). Delft University of Technology, Delft, Netherlands.

Webster, R., & Oliver, M. A. (2004). *Geostatistics for environmental scientists.* Chichester, NY: John Wiley & Sons.