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ARTICLE

Calibration of resistance factors for geotechnical seismic design

Farzaneh Naghibi and Gordon A. Fenton

Abstract: The next edition of the Canadian Highway Bridge Design Code will contain a table of geotechnical resistance factors to be used for seismic design. This paper will estimate the geotechnical resistance factors for shallow foundations required to achieve various target maximum acceptable failure probabilities, which in turn may depend on the assumed design earthquake return period. The investigation will include consideration of design lifetime, uncertainty in the magnitude of the maximum lifetime earthquake event, and the uncertainty in ground properties. The results suggest resistance factors that are lower than commonly used at the moment in Canada and that the failure probability is not greatly dependent on the return period of the design earthquake. The paper will present recommendations on geotechnical resistance factors for seismic design that can be used to guide and calibrate future editions of civil design codes in Canada.

Key words: reliability-based design, seismic design, earthquake design, resistance factor, shallow foundations, bearing capacity.

Résumé : La prochaine édition du Code canadien sur le calcul des ponts routiers contiendra un tableau des facteurs de résistance géotechnique à utiliser pour la conception sismique. Le présent document estimera les facteurs de résistance géotechnique pour les fondations superficielles nécessaires pour atteindre différentes probabilités de défaillance maximum acceptables cibles, qui peuvent à leur tour dépendre de la période de retour d'un tremblement de terre type. L'enquête tiendra compte de la durée de vie théorique, de l'incertitude quant à la magnitude du séisme ayant une durée de vie maximale et de l'incertitude des propriétés du sol. Les résultats suggèrent que les facteurs de résistance sont inférieurs à ceux couramment utilisés actuellement au Canada et que la probabilité de défaillance ne dépend pas beaucoup de la période de retour d'un tremblement de terre type. Le document présentera des recommandations sur les facteurs de résistance géotechnique pour la conception sismique qui peuvent être utilisées pour guider et calibrer les éditions futures des codes de conception civile au Canada. [Traduit par la Rédaction]

Mots-clés : conception basée sur la fiabilité, conception sismique, conception parasismique, facteur de résistance, fondations superficielles, capacité portante.

Introduction

The assessment of failure probability of geotechnical systems under seismic loading involves several factors. First of all, the probability of having a seismic event of a certain strength occurring during the system lifetime must be combined with the reduction in geotechnical strength as a result of the event. Given that seismic events are random, as is the geotechnical strength, the total probability of failure can be estimated using the total probability theorem as follows (see, e.g., Naghibi and Fenton 2018):

$$p_f = \mathrm{P}[F] = \mathrm{P}[F\,|\,R = r_1]\mathrm{P}[R = r_1] + \mathrm{P}[F\,|\,R = r_2]\mathrm{P}[R = r_2] + \ldots = \sum_{i=1}^{\infty}\mathrm{P}[F\,|\,R = r_i]\mathrm{P}[R = r_i] \le p_m \tag{1}$$

where F is the event that the footing fails, R is the random return period of the earthquake, r_i is a specific realization of R, and p_m is the maximum acceptable failure probability. Larger values of r_i imply stronger earthquakes.

In this study, the bearing capacity failure probability of a shallow foundation, in particular a strip footing, under seismic loading is estimated by a combination of Monte-Carlo simulation and theory. In particular, the conditional failure probability $\mathrm{P}[F|R = r_i]$ in eq. (1) is estimated via simulation, while the unconditional probability $\mathrm{P}[R = r_i]$ is estimated by theory. The goal of this work is to determine resistance factors required to achieve certain target failure probabilities for extreme limit state seismic design of shallow foundation by means of the load and resistance factor design (LRFD) approach.

The resistance factors required to achieve target failure probabilities are estimated as a function of the return period of the earthquake being designed against. For example, if the foundation is being designed to resist an earthquake with a return period 975 years, then seismic forces, or accelerations, are imposed on the foundation and the design aims to achieve a target failure probability consistent with the performance criteria of the Canadian Highway Bridge Design Code of Canada (CHBDC). This target failure probability will change as the return period changes, because the performance criteria changes as the return period changes, so that the required resistance factor may also change.

The LRFD of shallow foundations against bearing failure in the purely static case (no earthquake loading considered) has been studied previously by Fenton et al. (2008). The geotechnical design proceeds by ensuring that the factored geotechnical resistance at least equals the effect of factored loads, that is

$$\varphi_g \hat{R}_u \ge \sum \alpha_i \hat{F}_i = \hat{F}_T \tag{2}$$

in which φ_g is the geotechnical resistance factor at ultimate limit state (ULS), $\hat{R}_u$ is the characteristic ultimate resistance, and $\alpha_i\hat{F}_i$ is

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F. Naghibi and G.A. Fenton. Department of Engineering Mathematics and Internetworking, Dalhousie University, Halifax, NS B3J 2X4, Canada.
Corresponding author: Gordon A. Fenton (email: gordon.fenton@dal.ca).

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the ith factored load. The load factors, α_i, typically account for uncertainty in loads, and are usually greater than 1.0 for ULS design while the geotechnical resistance factor, φ_g, is typically less than 1.0 and accounts for uncertainties in the geotechnical parameters and prediction models used to estimate the characteristic geotechnical resistance. This paper extends the work by Fenton et al. (2008) by including the effects of uncertain maximum lifetime earthquake loading on failure probability.

The paper is organized as follows: in the next section, the random fields used to model the soil supporting the foundation are described along with the random load model. This is followed by a discussion of the reliability-based footing design approach. In the section "Estimation of actual footing resistance", the actual footing resistance is estimated. A theoretical model is then developed to estimate the probability of failure of a footing. The simulation model used to estimate conditional failure probabilities is described in the section titled "Simulation model". The results are presented in the "Results and discussion" section and conclusions are summarized in final section.

Random ground and load models

A random field is a collection of random variables whose values are associated with each spatial location. In this paper, two random fields are used to represent the soil properties cohesion and friction angle. The cohesion field, c, is assumed to be lognormally distributed while the friction angle field, ϕ, is assumed to be bounded between 10° and 30° using a bounded tanh distribution with $s = 3$, where s is the scale parameter (see Fenton and Griffiths 2008). A value of $s = 3$ corresponds to a standard deviation of $\sigma_\phi \approx 4°$ (coefficient of variation of $\nu_\phi \approx 0.2$). The cohesion and friction angle fields are assumed to be independent, which is slightly conservative (see Fenton and Griffiths 2003).

Values within each random field are correlated with one another as a function of the distance between them. In this paper, an isotropic exponentially decaying Markov correlation function is used, defined by

$$\rho(\tau_{ij}) = \exp\left(\frac{-2\tau_{ij}}{\theta}\right) \tag{3}$$

where $\tau_{ij} = |\underset{\sim}{x}_i - \underset{\sim}{x}_j|$ is the distance between any two points, $\underset{\sim}{x}_i$ and $\underset{\sim}{x}_j$, in the random field; and θ is the correlation length (Fenton and Griffiths 2008). The same correlation length is used for both cohesion and friction angle fields. The use of isotropic random fields to represent the ground, rather than perhaps more realistic anisotropic fields, does not particularly affect the results of this paper because resistance factors are primarily dependent on overall variability and averaging details are not so important so long as the averaging domain remains approximately the same.

As the cohesion field is lognormally distributed with mean and standard deviation μ_c and σ_c, then ln$c$ is normally distributed with parameters

$$\begin{aligned}\sigma_{\ln c}^2 &= \ln\left(1 + \nu_c^2\right)\\ \mu_{\ln c} &= \ln(\mu_c) - \frac{1}{2}\sigma_{\ln c}^2\end{aligned} \tag{4}$$

where v_c is the coefficient of variation of c (= σ_c/μ_c).

The random load applied to the footing is equal to the sum of the maximum lifetime live load, F_L, and the relatively static dead load, F_D, i.e.,

$$F_T = F_L + F_D \tag{5}$$

where F_L and F_D are each assumed to be lognormally distributed, with means μ_L and μ_D, and standard deviations σ_L and σ_D, respectively. The mean and variance of the total load, F_T, assuming live and dead loads are independent, are thus given by

$$\begin{aligned}\mu_T &= \mu_L + \mu_D\\ \sigma_T^2 &= \sigma_L^2 + \sigma_D^2\end{aligned} \tag{6}$$

Footing design

In general, the seismic design of a footing involves consideration of both seismic effects and the imposed loads that are likely to be present during the seismic event. The load factors for seismic design are given by the CHBDC (CSA 2014) to be $\alpha_L = 0$ and $\alpha_D = 1.25$. In other words, the CHBDC assumes that no live load will be acting at the time of the earthquake. The authors consider this to be somewhat unrealistic and will conservatively use $\alpha_L = 1$ to account for both vertical seismic loading as well as the component of live load that is present during an earthquake event. In this paper, the vertical design load during an earthquake event is taken to be equal to

$$\hat{F}_T = \alpha_L\hat{F}_L + \alpha_D\hat{F}_D \tag{7}$$

where $\hat{F}_L$ is the characteristic live load; $\hat{F}_D$ is the characteristic dead load; and α_L and α_D are the live and dead load factors, respectively. The characteristic loads $\hat{F}_L$ and $\hat{F}_D$ are obtained by applying bias factors to the means of the load distribution: $\hat{F}_L = \mu_L/0.9$, $\hat{F}_D = \mu_D/1.05$ (Fenton et al. 2016), where μ_L and μ_D are the means of the maximum lifetime dead and live loads, respectively.

The bearing capacity of a strip footing subjected to static loading was given by Meyerhof (1963) to be

$$q_u = cN_c s_c d_c i_c + qN_q s_q d_q i_q + 0.5\gamma BN_\gamma s_\gamma d_\gamma i_\gamma \tag{8}$$

where q_u denotes the ultimate bearing capacity of the foundation under a vertical centered load; c is the soil's cohesion; q is the total pressure on the unit length of the bearing surface (= γD); γ is the soil's unit weight; s_c, s_q, s_γ are the shape factors; d_c, d_q, d_γ are the depth factors; i_c, i_q, i_γ are the load inclination factors; B is the footing width; D is the foundation depth; and N_c, N_q, N_γ are the bearing capacity factors that only depend on the soil's friction angle, ϕ, defined as follows:

$$\begin{aligned}N_q &= e^{\pi\tan\phi}\tan^2\left(\frac{\pi}{4} + \frac{\phi}{2}\right)\\ N_c &= (N_q - 1)/\tan\phi\\ N_\gamma &= 2(N_q + 1)\tan\phi\end{aligned} \tag{9}$$

Budhu and Al-karni (1993) introduce seismic factors into eq. (8) to account for seismic effects as follows:

$$q_{uE} = cN_c s_c d_c e_c + qN_q s_q d_q e_q + 0.5\gamma BN_\gamma s_\gamma d_\gamma e_\gamma \tag{10}$$

where the seismic factors are defined as

$$\begin{aligned}e_c &= \exp\left(-4.3k_h^{1+\frac{c}{\gamma H}}\right)\\ e_q &= (1 - k_v)\exp\left(\frac{-5.3k_h^{1.2}}{1 - k_v}\right)\\ e_\gamma &= \left(1 - \frac{2}{3}k_v\right)\exp\left(\frac{-9k_h^{1.1}}{1 - k_v}\right)\end{aligned} \tag{11}$$

and k_h and k_v are horizontal and vertical acceleration coefficients, respectively, in units of gravitational acceleration (g). This paper distinguishes between the lifetime maximum acceleration coefficients, k_h and k_v, which are realizations of the random variables K_h and K_v, and the acceleration coefficients used for design, $\hat{k}_h$ and $\hat{k}_v$, which are nonrandom.

The parameter

$$H = \frac{0.5B}{\cos[(\pi/4)+(\phi/2)]}\exp\left(\frac{\pi}{2}\tan\phi\right)+D \tag{12}$$

is the depth of the soil's failure zone from the ground surface during the seismic event.

It is recognized that eqs. (8) and (10) are regarded as being quite conservative, but the same equations will be used subsequently to assess the reliability of the design so that the design conservatism largely cancels out of this study. This implies that this study concentrates on the uncertainties due to actual lifetime maximum seismic hazard level, which is unknown, and uncertainty in the ground properties.

To the best of the authors' knowledge, the seismic factors developed by Budhu and Al-karni (1993) include the effects of load inclination arising from the seismic inertial forces, so that the horizontal seismic loads do not need to be explicitly considered in the seismic design process. As a result, the bearing capacity predicted by eq. (10) is to be compared only to the vertical component of the applied load on the footing, F_T, in evaluating the design, and the horizontal seismic load is ignored.

The choice of design $\hat{k}_h$ and $\hat{k}_v$, which can be substituted into eq. (11) in place of k_h and k_v in the design process, depends on the peak ground acceleration (PGA) for the site. Melo and Sharma (2004) provide the following estimates:

$$\begin{aligned}\hat{k}_h &= 0.5a_p\\ \hat{k}_v &= 0.25\hat{k}_h\end{aligned} \tag{13}$$

where a_p is the PGA in units of gravitational acceleration (g) (= PGA/g). The following regressions were fit to the a_p values, estimated for three Canadian cities — Halifax, Ottawa, and Vancouver — by the Natural Resources Canada (NR Can) earthquake hazard website over the four earthquake return periods 100, 475, 975, and 2475 years (http://www.earthquakescanada.nrcan.gc.ca/hazard-alea/interpolat/index_2015-en.php):

$$a_p = \begin{cases} 0.0952190 - 0.0398830\ln(r_i) + 0.0045917\ln^2(r_i) & \text{(Halifax)}\\ 0.4184263 - 0.1783138\ln(r_i) + 0.0205590\ln^2(r_i) & \text{(Ottawa)}\\ 0.1161445 - 0.0642386\ln(r_i) + 0.0122857\ln^2(r_i) & \text{(Vancouver)}\end{cases} \tag{14}$$

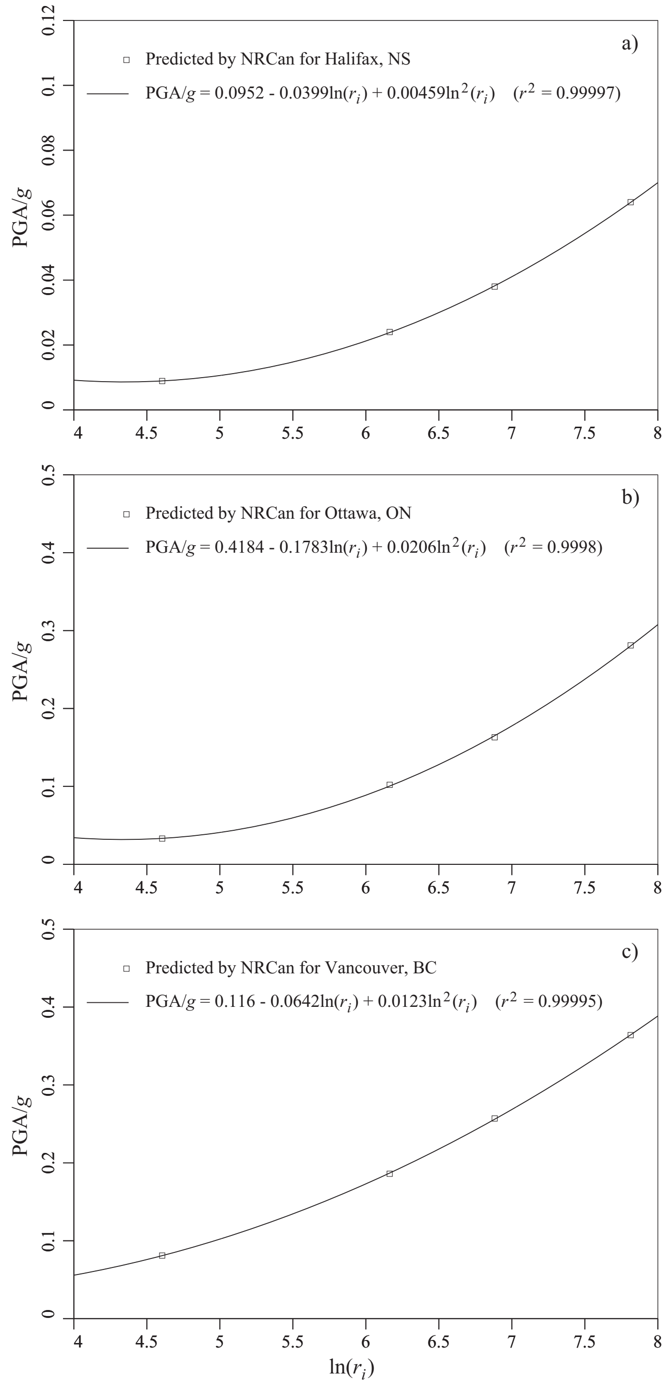
where r_i is the return period of an earthquake having magnitude m_i. The above regressions are plotted in Fig. 1 and can be seen to fit the seismic predictions very accurately.

In this paper, the embedment depth, D, of the footing used in eq. (12) is assumed to be zero for simplicity, and all shape and depth factors are set to 1. Thus, the simplified equation

$$\hat{q}_{uE} = \hat{c}\hat{N}_c\hat{e}_c + 0.5\gamma B\hat{N}_\gamma\hat{e}_\gamma \tag{15}$$

will be used here for the design of a strip footing under seismic events. The earthquake parameters $\hat{e}_c$ and $\hat{e}_\gamma$ used in eq. (15) are obtained substituting $\hat{k}_h$ and $\hat{k}_v$ into eq. (11) in place of k_h and k_v using the design earthquake coefficients $\hat{k}_h$ and $\hat{k}_v$ from eq. (13). The characteristic ultimate geotechnical resistance of the strip footing now becomes

Fig. 1. Peak ground acceleration (PGA) values for 100, 475, 975, and 2475 year earthquake return periods along with their corresponding regressions for (*a*) Halifax, (*b*) Ottawa, and (*c*) Vancouver.

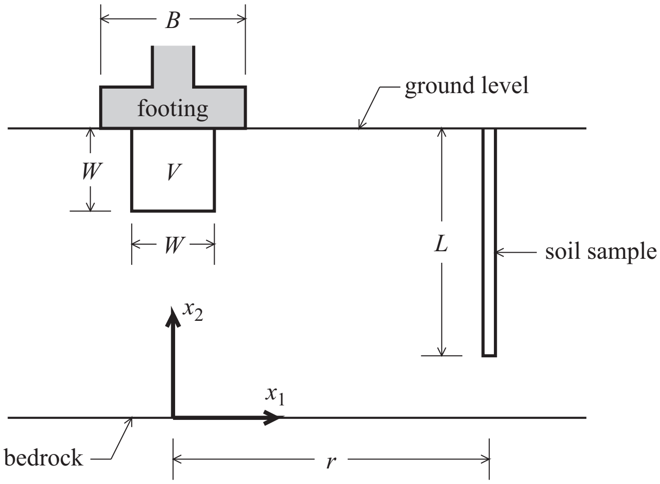


$$\hat{R}_{uE} = B\hat{q}_{uE} \tag{16}$$

The other hat parameters, $\hat{c}$, $\hat{N}_c$, and $\hat{N}_\gamma$, in eq. (15) are obtained by sampling the soil in the vicinity of the footing, as shown in Fig. 2. In particular, $\hat{c}$ is estimated here as the geometric average of n observations $\hat{c}_1, \hat{c}_2, ..., \hat{c}_n$ of soil cohesion taken at the sample location

$$\hat{c} = \left(\prod_{i=1}^{n}\hat{c}_i\right)^{1/n} = \exp\left(\frac{1}{n}\sum_{i=1}^{n}\ln\hat{c}_i\right) \tag{17}$$

Fig. 2. Averaging regions used to predict probability of bearing capacity failure.



while $\hat{\phi}$ is computed as the arithmetic average of n observed friction angle values, $\hat{\phi}_1$, $\hat{\phi}_2$, ..., $\hat{\phi}_n$, according to

$$\hat{\phi} = \frac{1}{n}\sum_{i=1}^{n}\hat{\phi}_i \tag{18}$$

The "observations" are extracted from the random field simulation and so are actually values associated with each random field cell having dimensions Δx and Δy. If the sample is over depth L, then the number of "observations" is $n = L/\Delta y$.

The goal of the design is to determine the footing width, $\hat{B}$, which satisfies the LRFD eq. (2), using $\hat{R}_{\mathrm{u}}$ replaced by $\hat{R}_{\mathrm{uE}}$. As the seismic factors in eq. (11) involve the footing width (see eq. (12)), the determination of $\hat{B}$ requires an iteration. The one-point iteration method was found to converge very quickly. The basic idea of one-point iteration is to start with an initial guess of $\hat{B}$, compute seismic coefficients in eq. (11), then solve for an updated $\hat{B}$ using the LRFD equation, and repeat until $\hat{B}$ remains stable. A trial design footing width of

$$B_{\mathrm{o}} = \hat{F}_{\mathrm{T}}/\varphi_{\mathrm{g}}\mu_{\mathrm{c}}\mu_{\mathrm{Nc}} \tag{19}$$

with a moderate resistance factor of $\varphi_{\mathrm{g}} = 0.7$ was used as the initial guess, where μ_{Nc} is approximated by using mean soil properties (μ_ϕ) in eq. (9)

$$\mu_{\mathrm{Nc}} = \frac{\mathrm{e}^{\pi\tan\mu_\phi}\tan^2\left(\frac{\pi}{4} + \frac{\mu_\phi}{2}\right) - 1}{\tan\mu_\phi} \tag{20}$$

The design footing width, $\hat{B}$, is then obtained by substituting eqs. (7) and (16) into the LRFD eq. (2), and solving at the equality. This leads to solving the following quadratic equation:

$$0.5\gamma\hat{N}_\gamma\hat{e}_\gamma\varphi_{\mathrm{g}}\hat{B}^2 + \hat{c}\hat{N}_{\mathrm{c}}\hat{e}_{\mathrm{c}}\varphi_{\mathrm{g}}\hat{B} - \hat{F} = 0 \tag{21}$$

for $\hat{B}$, giving the following solution:

$$\hat{B} = \frac{-\hat{c}\hat{N}_{\mathrm{c}}\hat{e}_{\mathrm{c}} + \sqrt{c^2\hat{N}_{\mathrm{c}}^2\hat{e}_{\mathrm{c}}^2 + (2\hat{F}\gamma\hat{N}_\gamma\hat{e}_\gamma/\varphi_{\mathrm{g}})}}{\gamma\hat{N}_\gamma\hat{e}_\gamma} \tag{22}$$

Estimation of actual footing resistance

Fenton et al. (2008) found that the static bearing capacity of a footing was well approximated by making use of a suitable geometric average of soil strength properties under the footing. They suggested an effective averaging domain of size $V = W \times W$, centered directly under the footing as shown in Fig. 2. The dimension W was taken to be 80% of the average mean depth of the wedge zone which, according to the classical Prandtl (1921) failure mechanism, is

$$W = \frac{0.8}{2}\mu_{\mathrm{B}}\tan\left(\frac{\pi}{4} + \frac{\mu_\phi}{2}\right) \tag{23}$$

where μ_ϕ is the mean friction angle (in radians) within the zone of influence of the footing and μ_{B} is an estimate of the mean footing width obtained by using mean soil properties (μ_{c} and μ_ϕ) rather than characteristic values in eq. (22). The assumption of a fixed averaging domain, V, is essentially a first order approximation, which assumes that the mean of the failure domains for each realization is approximated by the failure domain of the mean soil field. Although the use of the actual failure domain would be superior, the approach would require a complete random finite element method (RFEM) simulation. The work by Fenton et al. (2008) showed that a complete RFEM analysis was acceptably replaced by a fixed averaging domain using eq. (23), and this fixed averaging domain is believed to give reasonable probabilistic results even in the presence of earthquake loading.

The actual ultimate resistance is thus estimated to be

$$\bar{R}_{\mathrm{uE}} = \hat{B}\bar{q}_{\mathrm{uE}} = \hat{B}(\bar{c}\bar{N}_{\mathrm{c}}\bar{e}_{\mathrm{c}} + 0.5\gamma\hat{B}\bar{N}_\gamma\bar{e}_\gamma) \tag{24}$$

where the bar parameters in eq. (24) are obtained by averaging the soil properties $\bar{c}$ and $\bar{\phi}$ over the region V underneath the footing, as illustrated in Fig. 2. In particular, $\bar{c}$ is estimated as the geometric average of soil cohesion over V according to

$$\bar{c} = \exp\left[\frac{1}{V}\int_V \ln c(\underset{\sim}{x})\,d\underset{\sim}{x}\right] \tag{25}$$

while $\bar{\phi}$ is computed as the arithmetic average over the same region

$$\bar{\phi} = \frac{1}{V}\int_V \phi(\underset{\sim}{x})\,d\underset{\sim}{x} \tag{26}$$

The bar parameters are now defined as

$$\begin{aligned} \bar{N}_{\mathrm{q}} &= \mathrm{e}^{\pi\tan\bar{\phi}}\tan^2\left(\frac{\pi}{4} + \frac{\bar{\phi}}{2}\right) \\ \bar{N}_{\mathrm{c}} &= (\bar{N}_{\mathrm{q}} - 1)/\tan\bar{\phi} \\ \bar{N}_\gamma &= 2(\bar{N}_{\mathrm{q}} + 1)\tan\bar{\phi} \end{aligned} \tag{27}$$

The earthquake parameters $\bar{e}_{\mathrm{c}}$ and $\bar{e}_\gamma$ used in eq. (24) are obtained by eq. (11) using the actual lifetime maximum earthquake coefficients K_{h} and K_{v}, which are uncertain and thus treated as random.

Estimation of failure probability

Given that the earthquake magnitude and its corresponding return period are unknown, we must make use of the total probability theorem to compute the failure probability of the designed foundation for a given resistance factor, φ_{g}, and design $\hat{k}_{\mathrm{h}}$

$$p_{\mathrm{f}} = \sum_{i=1}^{n_{\mathrm{r}}} \mathrm{P}[F_{\mathrm{T}} > \bar{R}_{\mathrm{uE}} \,|\, R = r_i]\mathrm{P}[R = r_i] \tag{28}$$

By comparing the actual load on the footing (random) to the actual resistance of the footing (also random) during a seismic event, the conditional failure probability of a footing for a given return period $R = r_i$ is

$$\mathrm{P}[F_{\mathrm{T}} > \bar{R}_{\mathrm{uE}} \,|\, R = r_i] = \mathrm{P}[F_{\mathrm{T}} > \bar{R}_{\mathrm{uE}} \,|\, K_{\mathrm{h}} = k_{\mathrm{h}_i}] \tag{29}$$

where k_{h_i} is the pseudo-static seismic coefficient corresponding to a return period r_i: $k_{\mathrm{h}_i} = 0.5a_{\mathrm{p}}$, $k_{\mathrm{v}_i} = 0.25k_{\mathrm{h}_i}$, where a_{p} is determined by eq. (14).

As $\bar{R}_{\mathrm{uE}}$ is a function of local averages of soil properties, as well as the footing dimension, $\hat{B}$, all of which are random and cross-correlated, eq. (29) has no analytical solution, so far as the authors are aware, and so is estimated by simulation as described in the next section.

The unconditional probability $\mathrm{P}[R = r_i]$ used in eq. (28) is obtained as follows:

$$\begin{aligned}\mathrm{P}[R = r_i] &= \mathrm{P}[K_{\mathrm{h}} = k_{\mathrm{h}_i}] = \mathrm{P}[M_{\max}(l) = m_i] \approx F_{M_{\max}}\left(m_i + \frac{\Delta m}{2}\right) \\ &\quad - F_{M_{\max}}\left(m_i - \frac{\Delta m}{2}\right) = \exp(-l/r_{i+0.5}) - \exp(-l/r_{i-0.5})\end{aligned} \tag{30}$$

where (see, e.g., Fenton and Naghibi 2017)

$$F_{M_{\max}}(m_i) = \exp(-l/r_i) \tag{31}$$

$M_{\max}(l)$ is the maximum earthquake magnitude experienced over lifetime l and $F_{M_{\max}}$ is the cumulative distribution function of $M_{\max}(l)$.

To compute the sum in eq. (28), the range in return periods must be discretized. For simplicity, n_{r} is selected to be 41 subdividing the range $\ln(r_i)$ from 4.0 to 8.0, corresponding to return periods ranging from 55 to 3000 years, into 40 intervals such that

$$\begin{aligned}r_i &= \exp[4.0 + (i-1)\Delta\ln(r_i)] \\ &= \exp\left[4.0 + (i-1)\frac{8.0 - 4.0}{n_{\mathrm{r}} - 1}\right] = \exp[4.0 + 0.1(i-1)]\end{aligned} \tag{32}$$

so that the following are used in eq. (30):

$$\begin{aligned}r_{i+0.5} &= \exp[4.0 + 0.1(i - 0.5)] \\ r_{i-0.5} &= \exp[4.0 + 0.1(i - 1.5)]\end{aligned} \tag{33}$$

Once the probability of failure is computed via eq. (28), it can be compared to the maximum acceptable failure probability, $p_{\mathrm{m}} = \Phi(-\beta)$, where β is the target reliability index corresponding to p_{m}, and Φ is the standard normal cumulative distribution function. If p_{f} exceeds p_{m}, then the resistance factor needs to be reduced. Conversely, if $p_{\mathrm{f}} < p_{\mathrm{m}}$, then the resistance factor needs to be increased.

Simulation model

The simulation involves n_{sim} = 100 000 realizations. The standard deviation of the failure probability estimate is thus $\sqrt{p_{\mathrm{f}}(1-p_{\mathrm{f}})/n_{\mathrm{sim}}}$, which is approximately $0.003\sqrt{p_{\mathrm{f}}}$ for small failure probabilities. This means that if $p_{\mathrm{f}} = 1 \times 10^{-4}$, then the standard deviation of its estimate is about 3×10^{-5}. In other words, n_{sim} = 100 000 can reasonably resolve probabilities down to about 10^{-4}.

Table 1. Input parameters used in simulation.

Parameters	Values considered
μ_c, σ_c (kN/m^2)	70, 21
μ_ϕ, σ_ϕ (°)	20, ≈4
μ_{L}, σ_{L} (kN)	200, 60
μ_{D}, σ_{D} (kN)	600, 90
α_{L}, α_{D}	1.0, 1.25
$\hat{k}_{\mathrm{h}}$	
Halifax	0.012, 0.019, 0.032
Ottawa	0.051, 0.082, 0.141
Vancouver	0.093, 0.129, 0.182
$\hat{F}_{\mathrm{T}}$ (kN)	936.5
$r = \theta$ (m)	5
L (m)	5
$\Delta x = \Delta y$ (m)	0.15
$n = L/\Delta y$	33
γ (kN/m^3)	15
n_r	41
n_{sim}	100 000

The steps involved in the simulation are as follows:

1. Simulate the c and ϕ random fields using local average subdivision (LAS, Fenton and Griffiths 2008).
2. Sample the soil at a distance r from the footing center line to obtain $\hat{c}$ and $\hat{\phi}$.
3. Obtain the design footing width, $\hat{B}$, for a given resistance factor, φ_{g}, and design $\hat{k}_{\mathrm{h}}$. The design footing width changes from realization to realization because the c and ϕ fields change.
4. Average the c and ϕ fields over the domain V to obtain $\bar{c}$ and $\bar{\phi}$ and estimate the actual footing resistance, $\bar{R}_{\mathrm{uE}}$.
5. Simulate $F_{\mathrm{T}} = F_{\mathrm{L}} + F_{\mathrm{D}}$. The footing fails if $F_{\mathrm{T}} > \bar{R}_{\mathrm{uE}}$. If so, increment the number of failures counter, n_{fail}.
6. Repeat, from step i, n_{sim} times.
7. Estimate failure probability, given φ_{g}, $\hat{k}_{\mathrm{h}}$, and k_{h}, as $\mathrm{P}[F_{\mathrm{T}} > \bar{R}_{\mathrm{uE}}|K_{\mathrm{h}} = k_{\mathrm{h}_i}] \approx n_{\mathrm{fail}}/n_{\mathrm{sim}}$.

Results and discussion

The objective of this section is to determine resistance factors required to achieve a maximum tolerable lifetime failure probability, p_{m}, corresponding to a target reliability index of $\beta = -\Phi^{-1}(p_{\mathrm{m}})$. As failure probability of a design is largely independent of mean values (Fenton and Griffiths 2008), the results to follow are mostly influenced by the choice of coefficients of variation, sampling distance, and correlation length. For simplicity, attention is restricted to a particular case study whose parameters are listed in Table 1, where coefficients of variation ranging from 0.15 to 0.3 are used for load and soil properties. In addition, sampling distance and correlation length are selected to be the same and equal to 5 m.

The design values, $\hat{k}_{\mathrm{h}}$, are obtained for Halifax, Ottawa, and Vancouver using eqs. (13) and (14) for return periods r_i = 475, 975, and 2475 years.

Figure 3 depicts the conditional failure probabilities of eq. (29) as a function of lifetime maximum k_{h} for $\varphi_{\mathrm{g}} = 0.5$ considering the case $\hat{k}_{\mathrm{h}} = 0$ (no seismic design) as well as three design $\hat{k}_{\mathrm{h}}$ values corresponding to the three earthquake return periods 475, 975, and 2475 years. In the $\hat{k}_{\mathrm{h}} = 0$ case, the design earthquake loading has been assumed to be zero even though the foundation will almost certainly be subjected to one or more earthquakes over its design life. This means that the failure probability for this case is the largest because the foundation has not been designed to resist an earthquake.

Notice that sampling error is clearly evident in Fig. 3, particularly for Halifax. However, for Halifax, for example, the standard deviation of the probability estimate is approximately $0.003\sqrt{0.024} \approx 0.0005$. The error seen in Fig. 3*a* appears to be about 0.001, which is

Fig. 3. Plot of conditional failure probability (eq. (29)), estimated via simulation, given lifetime maximum k_h, for $\varphi_g = 0.5$ and four design $\hat{k}_h$ values for (*a*) Halifax, (*b*) Ottawa, and (*c*) Vancouver.

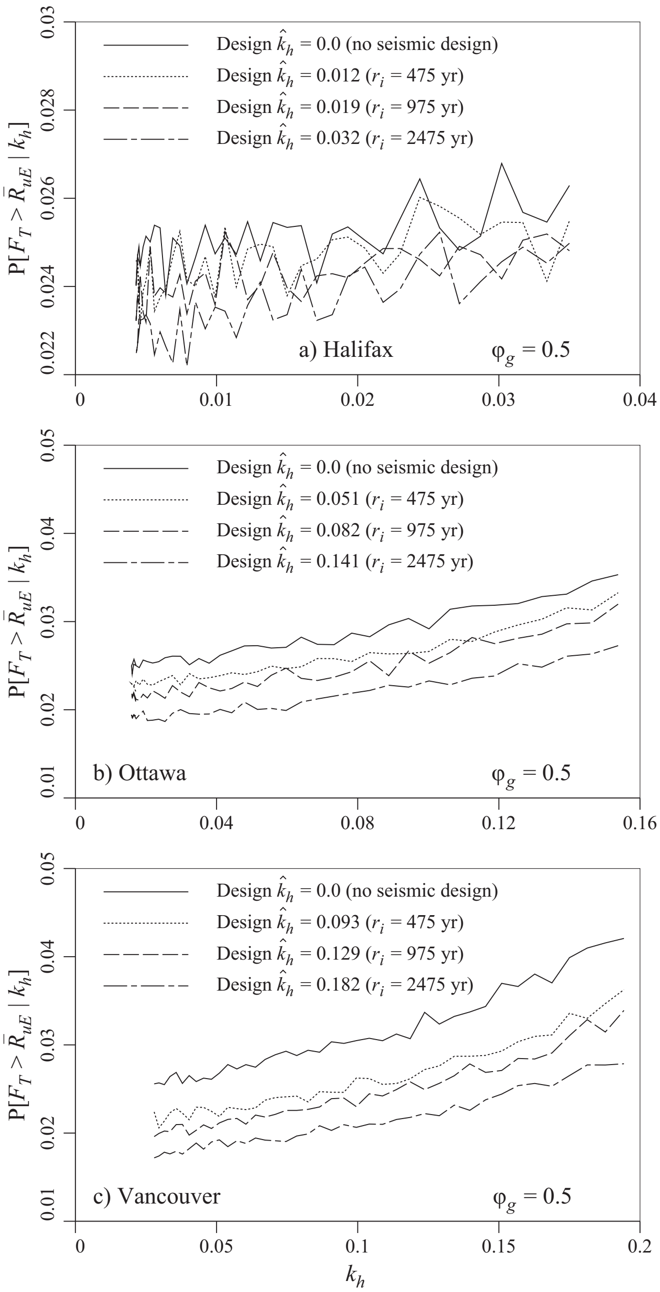
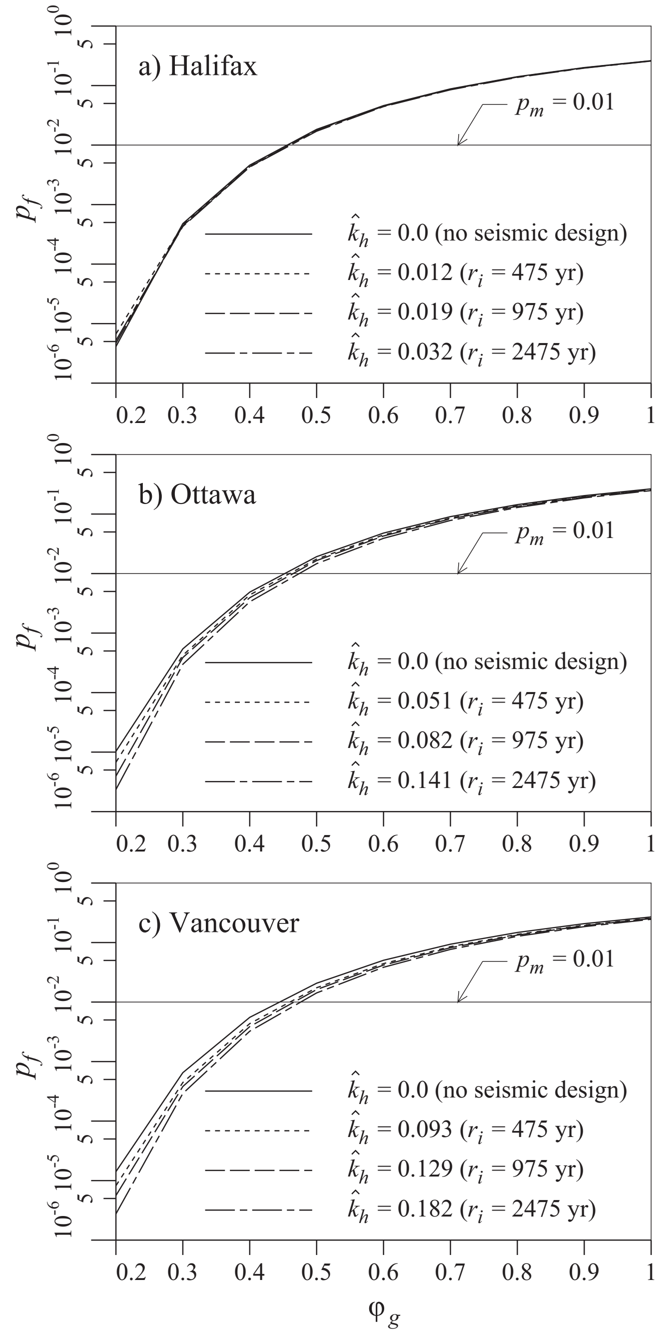


Fig. 4. Plot of failure probability versus resistance factor for four design $\hat{k}_h$ values estimated via eq. (28) for (*a*) Halifax, (*b*) Ottawa, and (*c*) Vancouver.



about two standard deviations, as expected. Despite the sampling error, the use of the conditional probabilities estimated by simulation in the total probability theorem (eq. (28)) leads to smooth estimates of p_f, as seen in Fig. 4.

Figure 4 shows the total failure probability of the footing as a function of resistance factor for various design $\hat{k}_h$ values at the three cities considered. The $\hat{k}_h = 0.0$ case agrees with the work by Fenton et al. (2008) when k_h is also taken to be zero. Figure 4 can be used for design purposes by drawing a horizontal line across the plot at the maximum tolerable failure probability, p_m, and then reading off the required resistance factor for a given design $\hat{k}_h$. For example, if $p_m = 0.01$, it can be seen that the resistance factor is about 0.5 in all three cities regardless of the return period or the geographical location.

As evident in Fig. 4, the failure probabilities are largely independent of location and thus of the design seismic hazard level. This is because the design process yields a smaller or larger foundation for smaller or larger expected seismic effects and the final foundation for small seismicity regions will have about the same probability of failure as the final foundation at high seismicity regions. What this means is that, fortunately, the resistance factor is largely independent of the seismic hazard level and Fig. 4 can be used to determine the resistance factors required for various p_m values. For example, if the target reliability for a return period of 475 years is $p_m = 0.001$, then a resistance factor of about 0.35 is required. At the other extreme, if the target reliability for a

2475 year return period is $p_m = 0.1$, then a resistance factor of around 0.75 appears reasonable.

As can also be seen in Fig. 4, the probability of failure has a small dependence on the design $\hat{k}_h$. In general, the probability of failure decreases as design $\hat{k}_h$ increases, especially for small φ_g values. The only exception is for Halifax where $\hat{k}_h = 0.012$ has a slightly higher failure probability than the $\hat{k}_h = 0.0$ case for small φ_g values. The dependence on $\hat{k}_h$ is apparent in Fig. 3 where the conditional failure probability also decreases as the design $\hat{k}_h$ value increases. To explain this behaviour, one must remember that the lifetime maximum k_h values considered in this paper range from return periods 55 to 3000 years. The largest design $\hat{k}_h$ corresponds to a return period of 2475 years at which point there are not that many stronger earthquakes considered in the simulation. In other words, the probability of failure for large design values is smaller because there is less chance of a stronger earthquake occurring than designed for. It is possible that this dependence on design $\hat{k}_h$ would reduce if the range in actual k_h values were to be increased. However, as the dependence is in any case only slight, the authors feel that the current range is reasonable.

Conclusions

This paper presents an investigation into the relationship between geotechnical resistance factor, lifetime, and uncertain extreme future events. In principle, the resistance factor should not be involved in the consideration of extreme loads. The load factor could be used to account for the uncertainty in extreme loads. However, current practice in Canada involves adjusting the resistance factor to account for the rare nature of seismic loads, which is justified in this study, at least, because the seismic effects are in fact modifying the geotechnical resistance in eq. (10). In other words, the seismic events result in increased uncertainty in the geotechnical resistance simply because the magnitude of the seismic event is random (see eq. (30)).

By using the same equation to perform the design as to assess the reliability of the design, the conservatism of the design equation is largely canceled out. In other words, eq. (10) is widely considered to be very conservative and its use means that the designed foundations are overly substantial. However, using the same bearing capacity equation, with improved soil parameter estimates (see eq. (24)), means that the probabilistic analysis concentrates on the effects of uncertainty in the actual lifetime maximum earthquake magnitude (i.e., in K_h) and uncertainty in soil parameters (i.e., cohesion and friction angle through sampling distance, r) as it should.

The results of this paper are somewhat surprising in that the suggested resistance factors are lower than currently used in Canadian practice. If, for example, it is desired to achieve the same reliability level as static design, which is typically around $p_m = 0.001$ ($\beta \approx 3.1$), a resistance factor of around $\varphi_g \approx 0.35$ would be required. These results suggest that current codes of practice need further scrutiny to justify their use of relatively large resistance factors in seismic design.

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List of symbols

a_p	peak ground acceleration, in units of gravitational acceleration, g
B	footing width
B_o	initial footing width
$\hat{B}$	design footing width
c	cohesion
$\bar{c}$	geometric average of cohesion field over the domain V
$\hat{c}$	geometric average of observed (sampled) cohesion values
$\hat{c}_i$	observed (sampled) cohesion value
D	depth of foundation
d_c, d_q, d_γ	depth factors
F	footing failure
F_D	dead load (random) (kN/m)
$\hat{F}_D$	characteristic dead load (kN/m)
$\hat{F}_i$	ith characteristic load effect
F_L	live load (random) (kN/m)
$\hat{F}_L$	characteristic live load (kN/m)
$F_{M_{max}}$	cumulative distribution function of $M_{max}(l)$
F_T	true total load (random)
$\hat{F}_T$	characteristic total load
g	gravitational acceleration
e_c, e_q, e_γ	seismic factors
$\hat{e}_c$, $\hat{e}_\gamma$	characteristic (design) seismic factors
$\bar{e}_c$, $\bar{e}_q$, $\bar{e}_\gamma$	estimated actual seismic factors
H	depth of soil's failure zone from ground surface under seismic loading
i_c, i_q, i_γ	load inclination factors
K_h, K_v	actual lifetime maximum horizontal and vertical acceleration coefficients, respectively (random)
k_h, k_v	realizations of actual lifetime maximum horizontal and vertical acceleration coefficients, respectively
$\hat{k}_h$, $\hat{k}_v$	design horizontal and vertical acceleration coefficients (= $0.5a_p$, $0.25\hat{k}_h$, respectively)
k_{h_i}	pseudo-static seismic coefficient corresponding to return period r_i
L	sample depth
l	target lifetime (in years)
$M_{max}(l)$	maximum earthquake magnitude experienced over lifetime l
m_i	earthquake magnitude corresponding to return period r_i
N_c, N_q, N_γ	bearing capacity factors
$\hat{N}_c$, $\hat{N}_\gamma$	characteristic (design) bearing capacity factors
$\bar{N}_c$, $\bar{N}_q$, $\bar{N}_\gamma$	estimated actual bearing capacity factors
n	number of soil samples
n_{fail}	number of failures in simulations
n_r	number of intervals subdividing range $\ln(r_m)$ from 4.0 to 8.0 (= 41)
n_{sim}	number of simulations
P[.]	probability operator
PGA	peak ground acceleration

p_f failure probability of bearing capacity (= n_{fail}/n_{sim} in simulation or $P[F_T > \bar{R}_{uE} | K_h = k_{h_i}]$ in theory)
p_m maximum acceptable failure probability
q total pressure on unit length of bearing surface (= γD)
q_u ultimate bearing capacity of foundation under vertical centered load
q_{uE} seismic ultimate bearing capacity of foundation under vertical centered load
$\hat{q}_{uE}$ characteristic seismic ultimate bearing capacity of foundation
$\bar{q}_{uE}$ estimated actual seismic ultimate bearing capacity of foundation
R return period of earthquake
$\hat{R}_u$ characteristic ultimate geotechnical resistance
$\hat{R}_{uE}$ characteristic seismic ultimate geotechnical resistance
$\bar{R}_{uE}$ estimated actual seismic ultimate geotechnical resistance
r distance between soil sample and footing center and coefficient of determination
r_i specific realization of R, which is return period of earthquake having magnitude at least m_i
s_c, s_q, s_γ shape factors
V effective soil property averaging domain centered under footing of size $W \times W$
W side dimension of effective averaging domain D
$\underset{\sim}{x}_i$ spatial coordinate (x_i, y_i) in two dimensions
α_i load factor corresponding to the ith load effect
α_L live load factor
α_D dead load factor
β reliability index
γ soil's unit weight
Δm distance between the midpoint of the previous magnitude interval and the midpoint of the next magnitude interval (= $m_{i+0.5} - m_{i-0.5}$)
Δx, Δy random field cell size in x- and y-directions, respectively
θ correlation length of random field
μ_B mean design width approximated by using mean soil properties (μ_c and μ_ϕ)
μ_c mean cohesion
μ_D mean dead load
μ_L mean live load
μ_T mean total load
$\mu_{\ln c}$ mean of $\ln c$
μ_{Nc} mean N_c approximated by using mean soil properties (μ_c and μ_ϕ)
μ_ϕ mean friction angle
v_c coefficient of variation of cohesion (= σ_c/μ_c)
v_ϕ coefficient of variation of friction angle (= σ_ϕ/μ_ϕ)
$\rho(\tau)$ correlation function between any two points separated by distance τ
σ_c standard deviation of cohesion
σ_D standard deviation of dead load
σ_L standard deviation of live load
$\sigma_{\ln c}$ standard deviation of $\ln c$
σ_T standard deviation of total load
σ_ϕ standard deviation of friction angle
τ_{ij} distance between any two points, $\underset{\sim}{x}_i$ and $\underset{\sim}{x}_j$ (= $|\underset{\sim}{x}_i - \underset{\sim}{x}_j|$)
Φ standard normal cumulative distribution function
φ_g geotechnical resistance factor
ϕ friction angle (radians unless otherwise stated)
$\bar{\phi}$ arithmetic average of friction angle field over domain V
$\hat{\phi}$ arithmetic average of observed (sampled) friction angle values
$\hat{\phi}_i$ observed (sampled) friction angle value