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Research Paper

Optimal geotechnical site investigations for slope design

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ABSTRACT

Site investigation in combination with field and laboratory testing, plays a vital role in characterizing the soil profile for geotechnical design in order to reduce uncertainty. In spite of this, site investigations are often limited in scope due to high costs. In this paper, conditional random fields are used to examine the influence of soil strength mean, standard deviation and spatial correlation length on the risk of slope design failure for different levels of site investigation scope. An undrained slope example is used to illustrate how the proposed approach can be used to assess the risk reduction that can be obtained as the scope of a site investigation is increased. By combining the cost of site investigation with the cost of slope failure, the results indicate that there exists an optimal site investigation scope, beyond which the cost of additional boreholes does not justify the cost savings due to reduced slope failure risk.

1. Introduction

Almost all natural soil and rock deposits are highly variable in their properties. Soil properties can vary by orders of magnitude from site to site, and even within a single site [32]. As a result, the soil profiles cannot be identified with certainty, even if an extensive subsurface exploration program is executed. In most cases, measurements are only obtained from a limited number of site investigation tests at scattered locations over a construction site (e.g., [6]). The site investigation phase of any geotechnical design plays a vital role, where inadequate characterization of the subsurface conditions may contribute to either a significantly over design that is not cost-effective, or an under design, which may lead to potential failures which can be even more serious when continued into full-life costing. It is not realistic to expect a site investigation to reveal ground conditions in their entirety, but increasing the scope of site investigation (i.e., additional sampling) the risk of a design should reduce but initial costs go up. The benefit of a detailed site investigation has not been recognised nor rewarded in most geotechnical design codes. These codes specify a factor of safety regardless of site investigation effort. However, is it really worth spending additional money to perform additional samples? Most of the time, the scope of the site investigation is governed by how much the client and project manager are willing to spend, rather than by what is needed to characterize the subsurface conditions. Therefore, it is of great significance to quantify the risk associated with different site investigation scopes (e.g., [23,34;35]).

Probabilistic methods have been applied in geotechnical engineering to assess the effectiveness of site investigation strategies. Goldsworthy et al. [9] investigated the effect of a single sample location on the design of a pad footing. Jiang et al. [17] and Jiang et al. [19] discussed the optimal borehole location which can gain maximum amount of prior knowledge based on conditional random field simulation. Yang et al. [36] proposed a framework which can quantify the error probabilities for different sampling locations by Monte Carlo simulation. The optimal sampling location near the slope crest is found by minimizing the probability of making the wrong decisions while giving the most information.

A well-designed geotechnical investigation and appropriate in situ and/or laboratory testing should involve optimal sampling locations and adequate sampling tests. The best sampling location of a site investigation is only part of the story. The scope of site investigation is also important for reduce the uncertainty as far as possible. Alhalaby and Whyte [1] concluded that 90% of risk to projects originate from unforeseen ground conditions which could often have been avoided by adequate and full site investigation (e.g., [21,24;30]). It is difficult to assess the effectiveness of a site investigation. The only practical means is to have complete information about the site which can act as a benchmark. However, gaining complete knowledge is infeasible. Jaksa

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Nomenclature

b	covariances vector between the unconditional random field values at the known points and the prediction point
b_s	component of **b**
$C_{\text{false safe}}$	consequence of false safe
$C_{\text{false unsafe}}$	consequence of false unsafe
$\boldsymbol{C}$	covariances matrix between the unconditional random field values at the known points
C_{jk}	covariance between known observations j and k
C_{SI}	cost of the site investigation
D	foundation depth ratio
FS	factor of safety
H	slope height
i	slope counter
I_F	real stability index
I_{F_c}	estimated stability index
j	known point
k	known point
m	total number of real slopes
n_s	number of observations
N	total number of points in the field
n_{sim}	number of conditional simulations
p	unknown values
$p_{\text{false safe}}$	probability of false safe
$p_{\text{false unsafe}}$	probability of false unsafe
p_{f_c}	conditional probability of failure
p_f	unconditional probability of failure
R_i	risk associated with site investigation
s	known points
x_p	spatial position of unknown points in the random field
X_s	observations
x_s	spatial position of observation
$X(\mathbf{x})$	normally distribution random field
$\widehat{X}(\mathbf{x})$	Kriging estimator based on measurements
$X_c(\mathbf{x})$	normally distribution conditional random field
$\widehat{X}_{uc}(\mathbf{x})$	unconditional random field
$\widehat{X}_{uc}(\mathbf{x})$	Kriging estimator based on unconditionally simulated values at the same measurement locations
$Z(\mathbf{x})$	lognormally distribution random field
$Z_c(\mathbf{x})$	lognormally conditional random field
α	slope angle
β	vector of weighting coefficient Simple Kriging weight matrix
β_s	weighting coefficient
γ	soil unit weight
γ_{sat}	saturated soil unit weight
θ_{c_u}	spatial correlation length of undrained cohesion
μ_{c_u}	mean undrained cohesion
$\mu_{p_{\text{false safe}}}$	mean probability of false safe
$\mu_{p_{\text{false unsafe}}}$	mean probability of unfalse safe
μ_R	mean risk associate with site investigation
μ_X	normal mean strength
μ_Z	lognormal mean strength
σ_{cu}	standard deviation undrained cohesion
σ_E	standard deviation of estimation error
σ_X	normal standard deviation
σ_Z	lognormal standard deviation

et al. [16] proposed a framework which examined the site investigation effectiveness in a probabilistic manner using Monte Carlo simulation. The basis for the framework uses random field theory to generate spatially correlated properties of a “real” site where all subsurface information is known. The random field generation is based on the random finite element method which is described in detail in Fenton and Griffiths [5]. Full source code random finite element method downloads are available at the website www.mines.edu/~vgriffit. A virtual site investigation is then carried out numerically by sampling the “real” site at discrete locations. Using the limited information gained from the virtual site investigation, foundation design can then be performed. The efficiency of a site investigation scope can then be assessed by quantifying the discrepancy between the foundation design based on limited information and the design based on complete knowledge. Goldsworthy et al. [8] refined the approach by applying the risk to a foundation design. The foundation design based on limited measurements was performed using a deterministic analysis approach [31]. The foundation analysis based on partial information did not account for the spatial variability of soil properties.

There is currently no means available for determining the most appropriate scope of geotechnical investigation to quantify the risk of slope failure. When it comes to making use of the site investigation data, there arises the question: How should the samples be used in the design process? The idea is to use the data more effectively, so that it is worth the effort of cost spent in carrying out the investigation.

The present work is thus inspired by the limitations of previous works. Random fields are used in this paper to model spatially variable soil properties of a “real” site. The site investigation is then carried out numerically at discrete locations from the “real” site. A conditional random field approach is utilized to characterize the spatial variability of soil properties accounting for obtained limited investigation data. By comparing the difference in the designs based on complete and limited information, both Type I (the stability analysis of slope suggests that the slope is safe when it is not) and Type II errors (the stability analysis of a slope suggests that the slope is unstable when it actually is safe) can be identified based on a hypothesis test. The risk can be quantified by assigning different consequences for these two types of errors. As such, this paper proposes a framework to quantify the impact of varying the scope of a site investigation on the risk of a slope design. An undrained slope is evaluated as an example to illustrate that there is an optimal site investigation scope, which leads to the least risk, and where additional sampling becomes non-cost effective. The proposed framework can provide better insight into risk and provide a way to determine the optimal scope of site investigation based on the spatial variability of the ground conditions.

2. Methodology

It is never possible to know the geotechnical properties at every location. However, it is possible to generate soil profiles, by means of random field theory (e.g., [33]). Consider a realization of a two-dimensional domain. The soil profile is first discretised into a series of elements, e.g., 0.5 m× 0.5 m in size and geotechnical properties assigned to each element. Since the site has been simulated, its properties are “known” completely at every location and the finite element method can be used to assess whether the slope is stable or not.

Compare the above idealisation with the usual situation encountered in practice, where the soil properties are known only at a limited number of locations, as a result of in-situ or laboratory tests. Here we assume that the site investigation is performed based on continuous sampling, as would occur with a cone penetration test (CPT). If one then designs a slope based on the obtained undrained shear strengths, the design has uncertainty because the decision about whether a slope fails or not is made on the basis of a limited set of samples from the slope. The goal of this paper is to relate error probabilities with sampling strategies to determine the optimal sampling programme. It is obvious that more sampling locations yield better estimates of slope stability but is also more expensive.

Suppose the statistics of undrained shear strength of slopes are given, a slope sample is generated by unconditional random field simulation. This slope is treated as a “real” slope. And the stability of the slope can be analyzed by the finite element method. The factor of safety is normally used to check the level of slope stability. In slope stability analysis, failure can be checked without using strength reduction method (e.g., [13]), but by merely checking to see if the current realization fails to converge within a user-specified iteration ceiling, implying stability failure (e.g., [15]). This stability assessment is based on the assumption that the undrained shear strength of all the elements are known. So failure or non-failure of a given realization is defined by a stability index I_F as below

$$I_F = \begin{cases} 1 & \text{failure} \\ 0 & \text{non} - \text{failure} \end{cases} \tag{1}$$

Monte Carlo simulation can be used to estimate the unconditional probability of failure by randomly simulating a sequence of realizations of unconditional random field.

$$p_f \approx \frac{1}{n_{sim}} \sum^{n_{sim}} I_F \tag{2}$$

The probability of failure p_f is estimated as the number of realizations which failed divided by the total number of realizations (n_{sim}).

However, in reality slope stability is characterized by various uncertainties due to the inherent spatial variability and lack of knowledge of soil properties. The subsoil properties are often measured at a limited number of locations. An estimation of soil properties at unsampled locations is a common requirement during the design of a geotechnical project. Some investigators (e.g., [28;25]) used a Kriging method to predict the soil profile at unsampled locations based on limited data from site investigation. As a best linear unbiased estimation method, Kriging estimates the value of a parameter at any point based on a weighted linear average of nearby samples. Compared to other common interpolation techniques, Kriging can take the correlation function into account. Yang et al. [36] adopted the Kriging method to assess the soil properties at unsampled locations. However, Kriging is a deterministic method to estimate the soil properties. In the last two decades, the probabilistic random finite element method (RFEM) has attracted a lot of attention. The random finite element method is utilized to characterize the spatial variability of soil properties to assess the soil properties of a site by Griffiths and Fenton [10,11]. In this paper, the conditional random finite element method is utilized to generate random fields while including known data at particular locations. The implementation of conditional random fields is described in Section 3. A conditional random field is employed to ensure that the simulated random fields exactly match the soil properties at particular locations. Mapping the conditional field to each mesh, the slope stability can be determined by carrying out finite element analysis. This stability analysis is based on limited amount of information. So the stability index I_{F_c} defined below is an estimation

$$I_{F_c} = \begin{cases} 1 & \text{failure} \\ 0 & \text{non} - \text{failure} \end{cases} \tag{3}$$

Monte Carlo simulation can be used to estimate the conditional probability of failure based on obtained site investigation data by randomly simulating a sequence of realizations of conditional random field.

$$p_{f_c} \approx \frac{1}{n_{sim}} \sum^{n_{sim}} I_{F_c} \tag{4}$$

Conditional random fields can make best use of limited site investigation data while still properly characterizing the spatial variation of soil properties. The conditional simulations are able to increase the confidence in a design's success or failure (e.g., [26;27]). However, inevitable uncertainty remains at locations which have not been examined. The slope design using sampled information may be considerably over design, which is not cost-effective, or under design, which may lead to potential failures and subsequent rehabilitation. Inadequate characterization of the soil properties may contribute to two types of hypothesis errors: (1) a Type I error where the slope is actually unsafe but the slope stability analysis suggests that the slope is safe, or, (2) a Type II error where the slope is actually safe but the slope stability analysis suggests that the slope is unsafe. Hereafter, the type I and II errors are called false safe and false unsafe, respectively.

The real stability index, I_F and the estimated stability index, I_{F_c} can be determined and used to estimate the probability of false safe or false unsafe. If the “real” slope is unsafe, $I_F = 1$, while the slope stability analysis based on partial information suggested the slope is safe ($I_{F_c} = 0$), this is a false safe. Then, the probability of making a false safe is equal to the number of realizations which $I_{F_c} = 0$ divided by the total number of realizations. The probability of false safe equals the complement of estimated conditional probability of failure. Alternatively, when the “real” slope is safe, $I_F = 0$, the slope stability analysis based on partial information suggested the slope is unsafe ($I_{F_c} = 1$), this is a false unsafe. The probability of making a false unsafe is equal to the number of realizations which $I_{F_c} = 1$ divided by the total number of realizations which is the estimated conditional probability of failure. Therefore, the probabilities of false safe ($p_{\text{false safe}}$) and false unsafe ($p_{\text{false unsafe}}$)can be estimated according to

$$\begin{aligned} p_{\text{false safe}} &= 1 - p_{f_c} \quad \text{if } I_F = 1 \\ p_{\text{false unsafe}} &= p_{f_c} \quad \text{if } I_F = 0 \end{aligned} \tag{5}$$

Although risk inherent in the ground is inevitable, it can ideally be identified and mitigated by way of incorporating geotechnical investigation. Normally, the risk is defined as the product of the probability of failure and the consequence. For geotechnical engineering projects, false safe and false unsafe would result in different consequences. Therefore, the consequences of these two types of errors should be assessed individually. The risk, in this paper, is considered to be a function of the costs, including the costs of site investigation, under design and over design, associated with performing site investigations of varying scope. A definition of risk of the particular slope is

$$R_i = \left\{ \begin{matrix} p_{\text{false safe}} \times C_{\text{false safe}} + C_{SI} & \text{if } I_F = 1 \\ p_{\text{false unsafe}} \times C_{\text{false unsafe}} + C_{SI} & \text{if } I_F = 0 \end{matrix} \right\} i = 1, 2, \cdots m \tag{6}$$

where R_i is the risk of the ith slope, $C_{\text{false safe}}$ is the consequence of making a false safe which is the cost of slope failure, $C_{\text{false unsafe}}$ is the consequence of making a false unsafe which is the cost of over design, C_{SI} is the cost of the site investigation and m is the total number of “real” slopes, i.e., unconditional initially simulated slopes. The expense of site investigation is determined by the type and number of tests. False safe is the worst outcome of slope stability analysis, where an unsafe slope is deemed to be safe. This type of error can lead to slope failure. The consequence of making a false unsafe means more funds being spent on the project construction process to convert an already safe slope to a deemed safe slope. False safe have more serious consequence than false unsafe.

The risk associated with site investigation for a specific slope realization can be determined by Eq. (6). However, for a certain degree of spatial variability, the mean risk should be estimated. When simulating the “real” slope several times, it is possible to estimate the mean risk of these samples. The mean risk can be estimated as follow

$$\mu_R \approx \frac{\sum_{i=1}^{m} R_i}{m} \tag{7}$$

where μ_R is the mean risk.

Meanwhile, the mean probability of false safe and false unsafe can be assessed as follow

$$\begin{cases} \mu_{p_{\text{false safe}}} \approx \frac{\sum_{i=1}^{m}\left(1-p_{fc}\right)}{m} & \text{if } I_F = 1 \\ \mu_{p_{\text{false unsafe}}} \approx \frac{\sum_{i=1}^{m}\left(p_{fc}\right)}{m} & \text{if } I_F = 0 \end{cases} \tag{8}$$

where $\mu_{p_{\text{false safe}}}$ is the mean probability of false safe and $\mu_{p_{\text{false unsafe}}}$ is the mean probability of false unsafe.

The above procedure can be used to quantify the risk of a geotechnical site investigation with respect to a slope design. Although it is intuitive to expect that the risk of a design will reduce as the site investigation scope increases (i.e., additional sampling), it is not known to what degree the risk is reduced, nor whether other uncertainties have an impact on this relationship. By comparing the mean risk of different site investigation strategies, the optimal site investigation that yields a slope design with lowest risk can be assessed, which is the main objective of this paper. Fig. 1 shows the flow chart of the proposed approach.

3. Conditional random field simulation based on site investigation

3.1. Conditional random field simulation for stationary normal data

In this paper, the conditional random finite element method is used to perform slope stability based on partial information. The measurements are extracted at selected locations from the "real" slope. Then a conditional random field is employed to ensure that the simulated random fields match the soil properties at these particular locations. This indicates that, in each realization of a conditional random field, the soil properties at these particular locations are constrained, and the soil properties at the other locations are random variables. To achieve this, the Kriging method [22] is employed because it can be used to estimate unobserved locations using known (measured) locations.

A conditional random field, which preserves the known values at

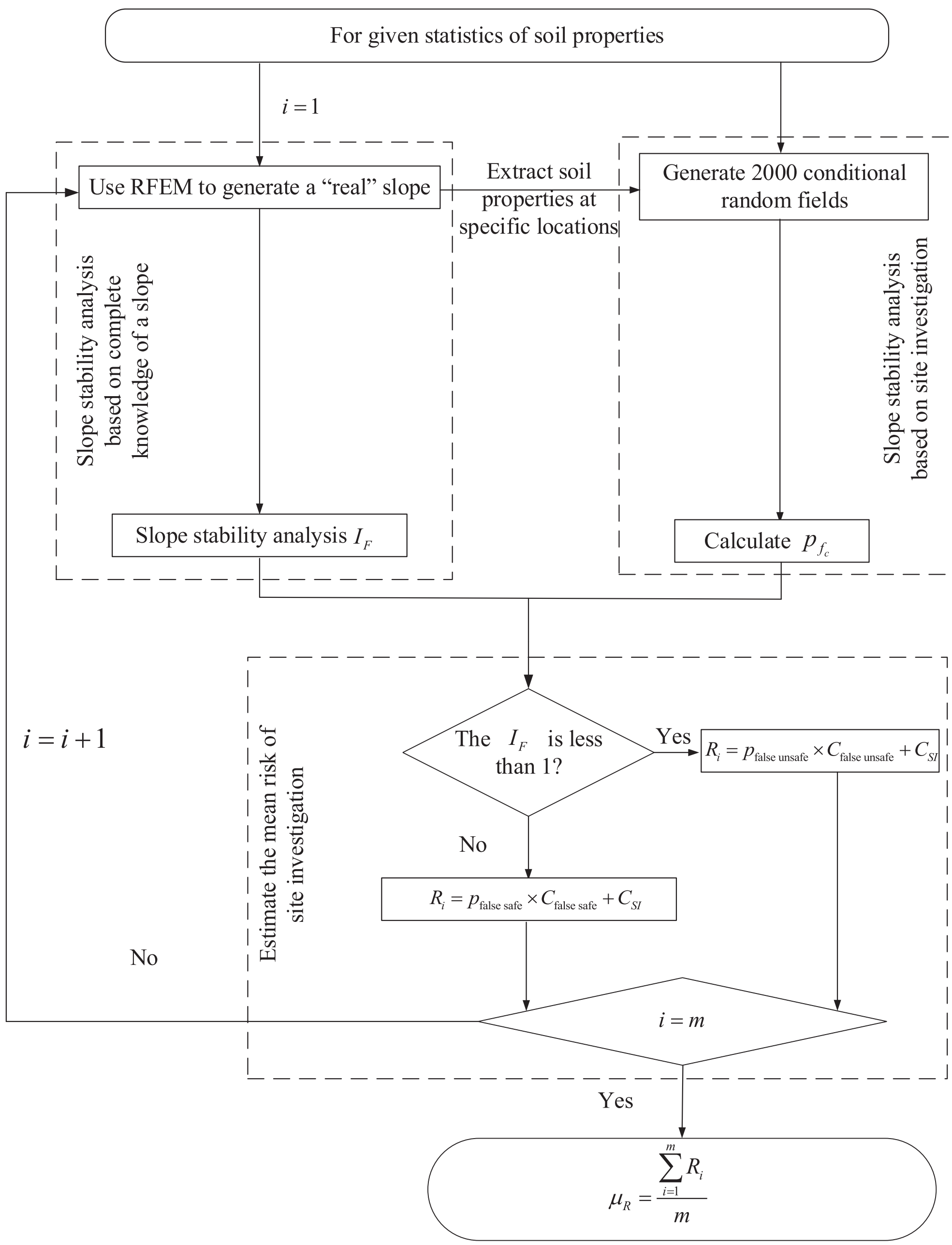

Fig. 1. Flow chart for calculating the mean risk of slope designs.

the measurement locations, can be formed from three different fields by Journel [20]

$$X_c(\mathbf{x}) = X_{uc}(\mathbf{x}) + (\widehat{X}(\mathbf{x}) - \widehat{X}_{uc}(\mathbf{x})) \tag{9}$$

where $X_c(\mathbf{x})$ is the conditional simulated random field, $X_{uc}(\mathbf{x})$ is the unconditional random field, $\widehat{X}(\mathbf{x})$ is the best estimate of field by Kriging based on measured values at known locations and $\widehat{X}_{uc}(\mathbf{x})$ is the best estimate of field by Kriging based on unconditional simulated values at the same measurement locations.

To accomplish the conditional simulation, the random field will be separated into two parts spatially (1) x_s, $s = 1, 2, \cdots, n_s$, being those points at which measurements have been taken, and at which the random field takes on deterministic values X_s, and (2) x_p, $p = 1, 2, \cdots, N - n_s$, being those points at which the random field is still random and at which we wish to simulate realizations of their possible random values. That is, s the subscript will denote known values, the subscript p will denote unknown values which are to simulated. N is the total number of points in the field.

The best estimated properties at the known spatial locations, x_s, are equal to the value at the known locations, that is, $\widehat{X}(\mathbf{x}_s) = X_s$, while $X_{uc}(\mathbf{x}_s) = \widehat{X}_{uc}(\mathbf{x}_s)$. At each measurement point, x_s, the conditional random field becomes

$$X_c(\mathbf{x}_s) = X_s \tag{10}$$

which is the measured value, as desired.

The unconditional simulation can be produced based on the statistics of the soil properties via several different algorithms, such as the Karhunen-Loeve expansion (e.g., [29]), the Cholesky decomposition (e.g., [18]) technique and the local average subdivision (LAS) (e.g., [7]). The LAS method is adopted in this study. The methodology has been described in detail in other publications (e.g., [12]).

The Kriged field based on the measured values at unknown locations, $\widehat{X}(\mathbf{x})$, is determined by

$$\widehat{X}(\mathbf{x}_p) = \mu_X + \sum_{s=1}^{n_s} \beta_s (X_s - \mu_X) \tag{11}$$

for, $p = 1, 2, \cdots, N - n_s$,where μ_X is the unconditional field mean and β_s is a weighting coefficient to be discussed shortly.

Similarly, the Kriged field of the simulation, $\widehat{X}_{uc}(\mathbf{x})$, is determined by

$$\widehat{X}_{uc}(\mathbf{x}_p) = \mu_X + \sum_{s=1}^{n_s} \beta_s (X_{uc}(\mathbf{x}_s) - \mu_X) \tag{12}$$

for $p = 1, 2, \cdots, N - n_s$. The only substantial difference between $\widehat{X}(\mathbf{x})$ and $\widehat{X}_{uc}(\mathbf{x})$ is that the former is based on observed values, X_s, while the latter is based on unconditional simulation values at the same locations. The difference appearing in Eq. can be computed more efficiently and directly as

$$\widehat{X}(\mathbf{x}_p) - \widehat{X}_{uc}(\mathbf{x}_p) = \sum_{s=1}^{n_s} \beta_s (X_s - X_{uc}(\mathbf{x}_s)) \tag{13}$$

The weighting coefficients, β_s, are determined from

$$\beta = \boldsymbol{C}^{-1}\mathbf{b} \tag{14}$$

where β is the vector of weighting coefficients β_s, and is the $n_s \times n_s$ matrix of covariances between the unconditional random field values at the known points. The matrix $\boldsymbol{C}$ has components

$$C_{jk} = Cov[X_{uc}(\mathbf{x}_j), X_{uc}(\mathbf{x}_k)] \tag{15}$$

for $j, k = 1, 2, \cdots, n_s$. Finally, $\mathbf{b}$ is a vector of length n_s containing the covariances between the unconditional random field values at the known points and the prediction point, $X_{uc}(\mathbf{x}_\eta)$. It has components

$$b_s = Cov[X_{uc}(\mathbf{x}_s), X_{uc}(\mathbf{x}_p)] \tag{16}$$

for $s = 1, 2, \cdots, n_s$.

Since $\boldsymbol{C}$ is dependent on the covariances between the known points, it only needs to be inverted once and can be used repeatedly in Eq. (14) to produce the vector of weights β for each of the $N - n_s$ best linear unbiased estimates (Eq. (13)).

3.2. Conditional random field simulation for lognormal data

However Eq. (9) can only be used directly for normal data. If the random variable is assumed normal distributed, both negative and positive values are possible, which is not acceptable for non-negative geotechnical parameters. To avoid negative values a non-Gaussian distribution is desirable. Elishakoff et al. [4] and Chilès and Delfiner [3] proposed a conditional simulation method for non-Gaussian fields by transforming the actual data to Gaussian data.

Let $Z(\mathbf{x})$ be a lognormal distribution with mean μ_Z and standard deviation σ_Z. The following steps described the transformation to a Gaussian (normal) distribution

(1) Transfer the lognormal data into Gaussian data by $X(\mathbf{x}) = \ln(Z(\mathbf{x}))$. The associated statistics in normal space are $\sigma_X = \sqrt{\ln\left(1 + \frac{\sigma_Z^2}{\mu_Z^2}\right)}$ and $\mu_X = \ln \mu_Z - \frac{1}{2}\sigma_X^2$.
(2) Generate an unconditional random field $X_{uc}(\mathbf{x})$ with transferred point statistics and correlation structure, and extract the values of $X_{uc}(\mathbf{x})$ at the measurement locations.
(3) Calculate the difference between known data and values extracted from unconditional random field $X(\mathbf{x}) - X_{uc}(\mathbf{x})$.
(4) Do Kriging for the difference $\widehat{X}(\mathbf{x}) - \widehat{X}_{uc}(\mathbf{x})$.
(5) Generate conditional random field $X_c(\mathbf{x}) = X_{uc}(\mathbf{x}) + (\widehat{X}(\mathbf{x}) - \widehat{X}_{uc}(\mathbf{x}))$.
(6) Transfer $Z_c(\mathbf{x}) = \exp^{X_c(\mathbf{x})}$.

The conditionally simulated process passes through the known data and has the same mean and covariance with the unconditional

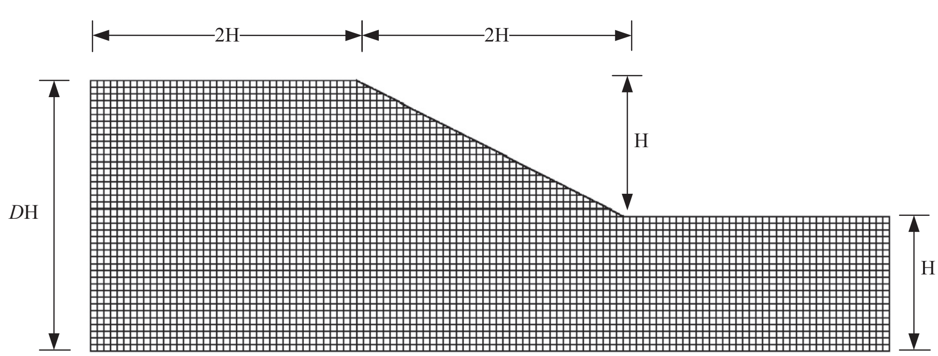

Fig. 2. Finite element mesh.

simulation.

For non-stationary conditional random field simulation, the above method can still be used by removing the trend first (e.g., [2]), and then transform non-stationary random field into stationary random field.

4. Example

In order to illustrate the proposed method, an undrained slope is considered with profile shown in Fig. 2. The slope is inclined to the horizontal at angle $\alpha = 26.6°$ (2:1 slope), with height $H = 10$ m, and depth ratio to a lower firm layer $D = 2$. The slope has soil unit weight $\gamma_{sat}\,(or\ \gamma) = 20.0$ kN/m^3. The undrained shear strength is assumed to be lognormally distributed with the mean $\mu_{c_u} = 50$ kPa and standard deviation $\sigma_{c_u} = 25$ kPa. The spatial correlation length is assumed to be isotropic with $\theta_{c_u} = 10$ m. A mesh size of 0.5 m$\times$ 0.5 m is chosen for the example. $\sigma_{c_u} = 25$ kPa. Based on the parameters given above, two thousand RFEM simulations are performed and p_f is found to be 0.21. The CPU time runs about 10 min on a X5675@3.07 GHz laptop for this example.

To give a good approximation to the mean risk of slope designs for a given degree of spatial variability, a sufficiently large number of samples should be chosen. Hogg and Tanis [14] suggest that the sample size would have to be at least 30. For a well behaved (almost normal) distribution, $m \geqslant 30$ yields a reasonably good approximation to the population mean. For poorly behaved distributions, a larger sample size is needed. In this paper 100 "real" slopes (i.e., $m = 100$) are simulated to estimate the mean risk of slope designs. Two typical slopes are selected from the 100 "real" slopes, one of them being unstable, and the other stable. The strength reduction method is performed for these two particular slopes to show clearly the two types of errors.

For the unstable slope, if the properties of the whole slope are "known" as shown in Fig. 3(a), the stability analysis suggests that the *FS* is 0.86, which means the slope is unsafe. The failure mechanism can be seen from Fig. 3(d). Fig. 3 depicts the variation of undrained shear strength where the dark and light regions depict "strong" and "weak" soils, respectively. To explore the subsurface soil properties 5 CPTs are performed to the depth of 20.0 m (i.e., to the bottom of the slope model) from the natural ground surface. The relative locations of the CPTs are plotted in Fig. 3(a). As Yang et al. [36] suggested the optimal sampling should be conducted between slope toe and crest. Five columns of undrained shear strengths are obtained from every element along a vertical boring as indicated. The conditional random field method using Kriging is then used to estimate the undrained shear strengths of the whole slope. Two thousand conditional simulations are carried out of the conditional random field giving a probability of failure of about 0.96. The prediction of the slope stability based on the conditional simulations can result in two outcomes; the slope is safe or is unsafe, i.e., the prediction is either correct or gives a false safe. A typical conditional simulation which makes a correct prediction of the stability of the slope is shown in Fig. 3(b) and (e). Fig. 3(c) and (f) indicate that the slope stability analysis based on partial information suggests the slope is safe while the slope is unsafe which makes a false safe. The probability of making a false safe is about 0.04 according to Eq. (5). Even though the stability analysis results for these two conditional simulations are opposite, the measurements at sample locations remain same from simulation to simulate. Fig. 3(b) and (c) show the grey scale of the estimated undrained shear strengths derived from Kriging with the constraint that the undrained shear strengths between the red[1] lines are fixed and based on the virtual samples taken from the "real" slope in Fig. 3(a).

For a stable slope as shown in Fig. 4(a) and (d), 5 CPTs are performed to characterize the soil properties. Conditional simulation is adopted to predict the undrained shear strengths at unsampled locations. The slope stability analysis will make a correct prediction in Fig. 4(e) or a false unsafe in Fig. 4(f).

4.1. Calculation of risk of slope stability analysis

In order to perform risk analysis, it is necessary to assign consequences representative of each of the component due to inadequate information. In this study, only consider the costs for site investigation tests, making a false safe and making a false unsafe.

The assumed costs for site investigation in this paper are adopted from common industry rates in South Australia, as described by Goldsworthy et al. [8], i.e., 1 CPT is assumed to cost $AUD 5000. It is assumed the consequence of making a false safe cost $AUD 1,000,000 and the cost of making a false unsafe is $AUD 150,000.

4.2. Influence of number of cone penetration test on mean risk

In this subsection, the method is used to explore the optimal number of site investigation tests for minimizing risk in the stability analysis of slopes. Suppose site investigations have been conducted based on CPTs to obtain the soil properties. Five different sampling schemes involving from 1 CPT sounding to 5 are applied to 100 slopes to determine the optimal site investigation strategy. The conditional random field is employed to assess the reliability of slopes based on the obtained information. It can be seen from Fig. 5 that the risk would be very high when inadequate site investigation information is used. The mean probability of false safe and false unsafe of these 100 "real" slopes can be calculated by Eq. (8). The mean probability of making either false safe or false unsafe is decreased with additional CPTs as shown in Figs. 6 and 7. The probabilities of false safe and false unsafe are significantly reduced for the first 3 sampling. However, little benefit is evident for the probabilities of false safe and false unsafe when the fourth and fifth CPT is conducted.

Fig. 8 shows the influence of number of CPTs on the mean risk. Each point on the plot is obtained using 100 "real" slopes. It can be seen from Fig. 8 that as the number of CPTs increases, the mean risk of slope stability analysis first decreases and then rises. Increasing site investigation expenditure at first causes to the mean risk noticeably reduce, reaching a minimum value when 3 tests are conducted. An increase in site investigation expenditure from $AUD 5000 (1 sampling location) to $AUD 15,000 (3 sampling locations) yields an expected total cost saving of approximately $AUD 80,000. In this example, further CPTs beyond the optimal value of 3 lead to designs with a higher risk due to the increased cost of the site investigation.

5. Parametric studies

5.1. Influence of standard deviation on the mean risk

The previous results are based on a certain level of spatial correlation and soil variability (i.e., $\theta_{c_u} = 10$ m, $\sigma_{c_u} = 25$ kPa and $\mu_{c_u} = 50$ kPa). In this subsection, the influence of σ_{c_u} (i.e., σ_{c_u}=10 kPa, 20 kPa, 25 kPa and 50 kPa) on the mean risk is investigated. All other statistics remain the same as described in previous section. Two thousand simulations are used for most cases while twenty thousand simulations are used for low standard deviations ($\sigma_{c_u} \leqslant 20$ kPa). When σ_{c_u} is 10 kPa, 20 kPa, 25 kPa and 50 kPa, the corresponding p_f obtained by unconditional simulations is $<1/20,000$, 0.02, 0.21 and 0.735, respectively. The influence of σ_{c_u} on mean risk is shown in Fig. 9. It can be seen from Fig. 9 that the mean risk first decreases and then increases as the number of CPTs increases when σ_{c_u}=20 kPa, 25 kPa and 50 kPa. There is an optimal site investigation scope, i.e., 3 CPTs as observed in Fig. 8. However, when σ_{c_u} is low (i.e., σ_{c_u}=10 kPa), the optimal site investigation scheme is to conduct 1 CPT. This is because when the

[1] For interpretation of color in Fig. 3, the reader is referred to the web version of this article.

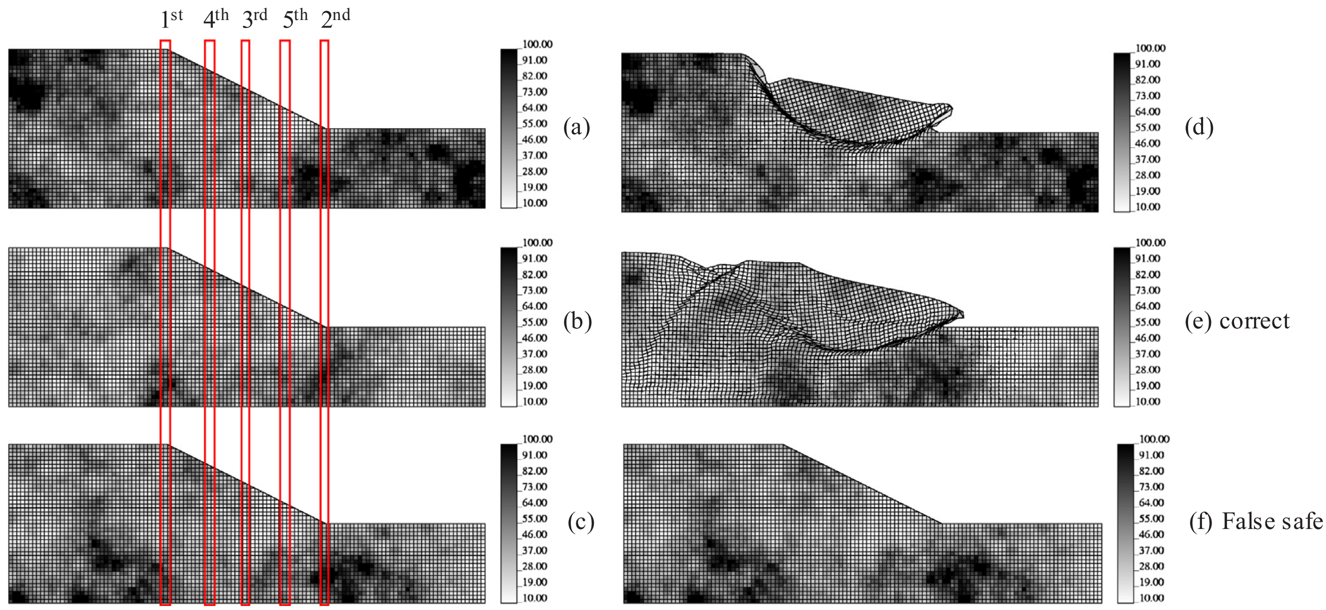

Fig. 3. The prediction of the unstable slope based on obtained measurements results in correct predictions of slope stability or false safe. (a) “Real” slope. (b) One typical conditional simulation based on 5 CPTs. (c) Another typical conditional simulation based on 5 CPTs. (d) Slope stability based on knowing all properties indicated that the slope is unstable (FS = 0.86). (e) Slope stability based on the conditional simulation indicated that the slope is unstable which means that this is a correct prediction. (f) Slope stability based on the conditional simulation indicated that the slope is safe which makes a false safe.

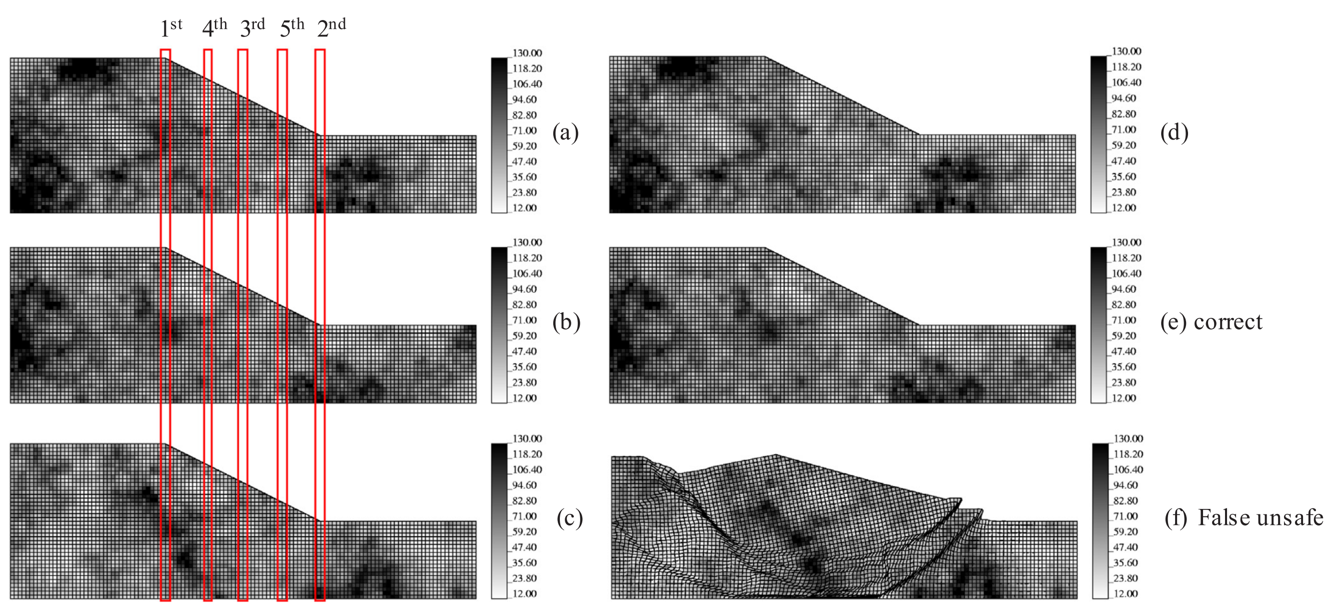

Fig. 4. The prediction of the stable slope based on obtained measurements results in correct predictions of slope stability or false unsafe. (a) “Real” slope. (b) One typical conditional simulation based on 5 CPTs. (c) Another typical conditional simulation based on 5 CPTs. (d) Slope stability based on knowing all properties indicated that the slope is safe (FS = 1.42). (e) Slope stability based on the conditional simulation indicated that the slope is safe which means that this is a right prediction. (f) Slope stability based on the conditional simulation indicated that the slope is failed which makes a false unsafe.

degree of variability in soil properties is relatively low, only 1 CPT is enough to characterize the slope. The probabilities of false safe and false unsafe are largely independent of site investigation scope, resulting in minimal testing being sufficient. It appears that the mean risk linearly increases as the number of CPTs increases.

5.2. Influence of spatial correlation length on the mean risk

The influence of spatial correlation length on the mean risk is investigated by varying the spatial correlation length (i.e., θ_{c_u}=1 m, 10 m, 20 m, 40 m and 100 m) but maintain all other parameters constant (i.e., σ_{c_u} = 25 kPa and μ_{c_u} = 50 kPa). Two thousand simulations are used for most cases while twenty thousand simulations are used for low spatial correlation length (θ_{c_u} = 1 m). When θ_{c_u} is 1 m, 10 m, 20 m, 40 m and 100 m the corresponding p_f obtained by unconditional simulations is <1/20, 000, 0.21, 0.25, 0.26 and 0.28. Figs. 10 and 11 show the effect of spatial correlation length on the mean risk of slope designs. As shown in Figs. 10 and 11, the mean risk increases at first and then decreases as the spatial correlation length increases. A worst case spatial correlation length of 20 m is evident where the greatest mean risk occurs for all the five site investigation scopes. The above results can be explained as follows. The spatial correlation length is defined as the distance within which points are significantly correlated. When the correlation length is small, the field tends to be rough. At the lower limit, when θ_{c_u} tends to

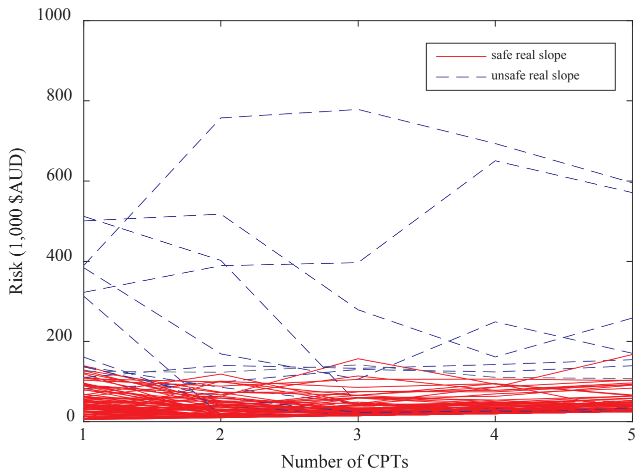

Fig. 5. The influence of number of CPTs on the risk of slope designs.

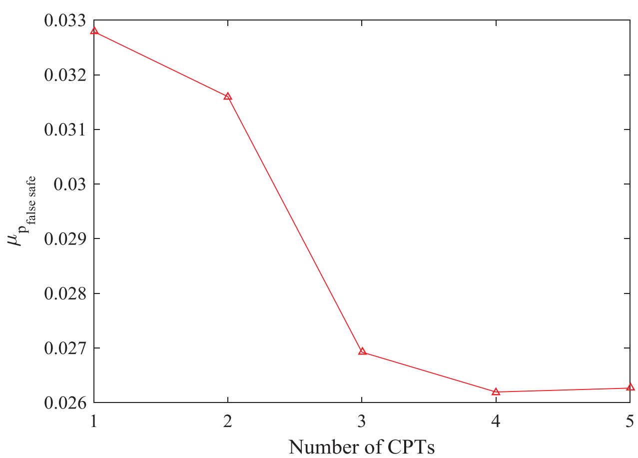

Fig. 6. Influence of number of CPTs on the mean probability of false safe.

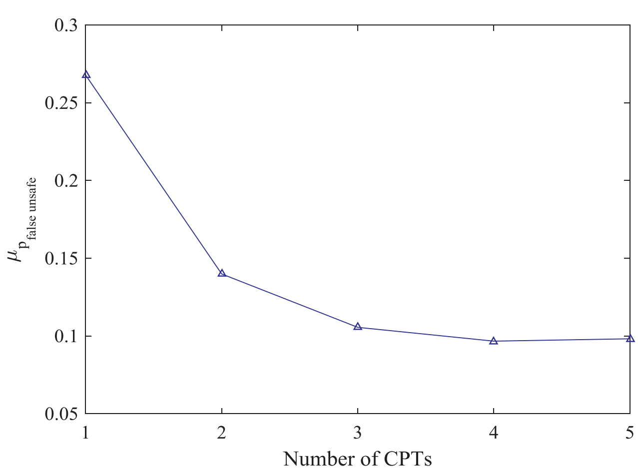

Fig. 7. Influence of number of CPTs on the mean probability of false unsafe.

zero, all points in the field become uncorrelated with each other, which means that the random field is white noise (e.g., [11]). It implies that two points arbitrarily close to one another will have independent values. The local average of lognormal undrained shear strength would consist of an infinite number of independent values whose mean tends to the median (a non-random constant value). Hence, only one sample is sufficient to represent the undrained shear strength of the entire slope. That is, the probability of making either false safe or false unsafe

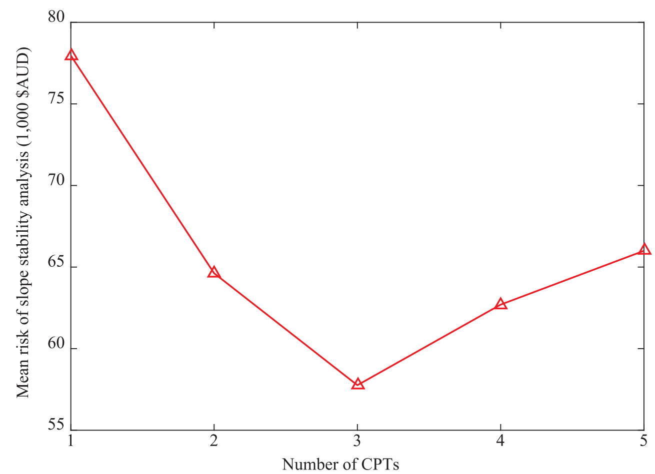

Fig. 8. Influence of number of CPTs on mean risk.

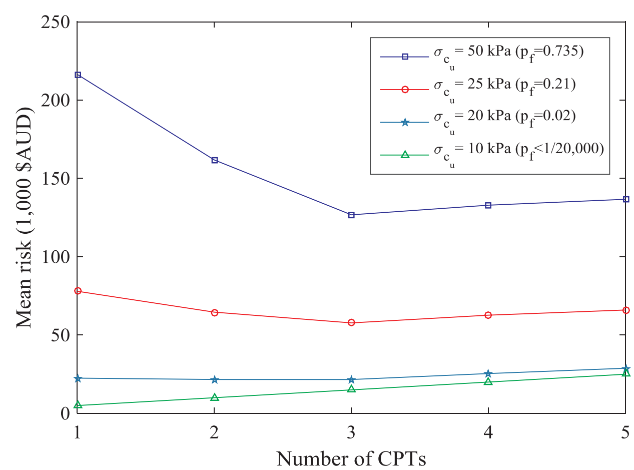

Fig. 9. Influence of standard deviation on mean risk.

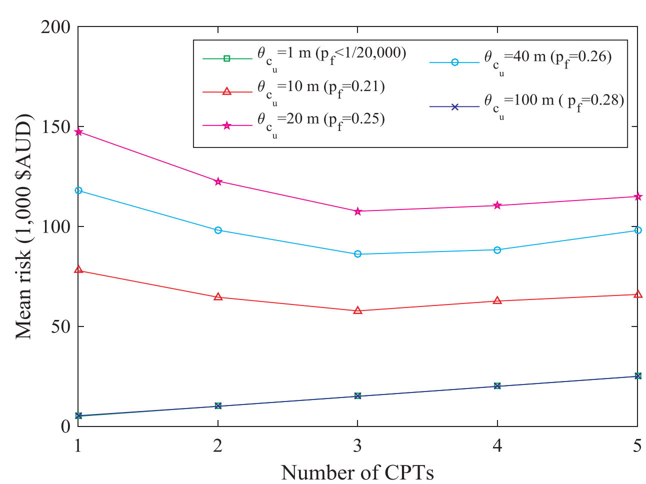

Fig. 10. Influence of spatial correlation length on mean risk.

will be zero based on any amount of information. Conversely, when the correlation length becomes large, the field becomes smoother. The random field becomes completely uniform but different from realization to realization when θ_{c_u} tends to infinity. Each realization is composed of a single random value so that one sample is enough to predict the soil properties of the whole field. In this case, the risk would be zero on the basis of one sample. Therefore, the growth in mean risk is caused

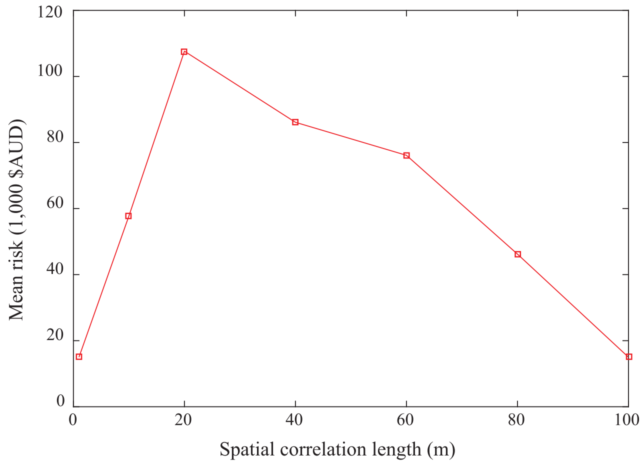

Fig. 11. Influence of spatial correlation length on mean risk based on 3 CPTs.

by the increasing site investigation expenditure when θ_{c_u}=1 m or 100 m. At intermediate spatial correlation lengths, there must be a worst case scenario where the mean risks are at a maximum. For example, when θ_{c_u}=20 m, the mean risk of the slope design based on 3 sample locations is less than that for designs based on 2 or 4 sampling tests.

5.3. Influence of the consequence of making a false safe on the mean risk

In previous discussions, it is assumed that consequences of making a false safe for different levels of p_f are the same (i.e., $C_{\text{false safe}}$=$AUD 1,000,000). In reality, the consequences of making a false safe are different for different levels of p_f. Baecher and Christian [2] reported a probability-consequence chart which shows the corresponding consequences for different levels of p_f. In this chart, the total cost of failure could reach $AUD 1 billion when p_f is 0.001. If p_f decreases by one order of magnitude, the total cost of failure would increase one order of magnitude. In this subsection, $C_{\text{false safe}}$ for different p_f is assumed to be different. The costs of conducting 1 CPT and making a false unsafe remain the same (i.e., C_{SI}=$AUD 5000 and $C_{\text{false unsafe}}$=$AUD 150,000). When p_f is <1/20, 000, 0.02, 0.21, and 0.735, $C_{\text{false safe}}$ is assumed to be $AUD 10,000,000, $AUD 10,000,000, $AUD 1,000,000 and $AUD 285,000, respectively. Based on these assumptions on $C_{\text{false safe}}$, the results in Fig. 9 are replotted in Fig. 12. It can be seen from Fig. 12 that the optimal number of CPTs is still 3 when p_f = 0.21 (σ_{c_u}= 25 kPa) and p_f = 0.735(50 kPa). When p_f = 0.21 (σ_{c_u}= 25 kPa), $C_{\text{false safe}}$ is not change. Therefore, the optimal number of CPTs is still 3. When p_f = 0.735 (σ_{c_u}=50 kPa), $C_{\text{false safe}}$ is decreased from $AUD 1,000,000 to $AUD 285,000. It is expected that the optimal number of CPTs would be less than 3 due to the decrease of $C_{\text{false safe}}$. However, the third CPT leads to a significantly reduction in probability of false safe. Therefore, the expense of the third CPT is outweighed by risk reduction. The optimal number of CPTs is still 3. When p_f = 0.02 (σ_{c_u}=20 kPa), $C_{\text{false safe}}$ increases tenfold from $AUD 1,000,000 to $AUD 10,000,000. As expected, the mean risk is decreased as the scope of the site investigation is increased. This is because the cost of conducting additional CPT is negligible in comparison to the consequence of false safe when the probability of failure is low. This reinforces the belief that engineers should invest in more thorough investigations if the consequence of slope failure is significant. When p_f < 1/20, 000 (σ_{c_u}=10 kPa), the optimal number of CPT is still 1. This is because false safe is unlikely to occur for reasons discussed previously in Section 5.1.

The results shown in Fig. 10 assume that the values of $C_{\text{false safe}}$ are the same for different levels of p_f. Fig. 10 can also be replotted by assigning different $C_{\text{false safe}}$ for different levels of p_f. However, the change of p_f is insignificant when θ_{c_u} is increased from 10 m to 100 m. Therefore, $C_{\text{false safe}}$ remains the same and the results shown in Fig. 10 remain the same.

6. Conclusion

As site investigations play a vital role in any geotechnical engineering design, it is important that such investigations are adequately planned to characterize the subsurface conditions. These results will assist geotechnical engineers in planning a site investigation in a more rational manner with knowledge of the associated risks. Based on the results obtained in this paper, the following conclusions can be drawn:

1. For a certain level of spatial variability, the risk of slope design is reduced significantly by increased site investigation tests. However, there appears to be an optimal number of site investigation tests, where the expected total cost of the slope design is a minimum.
2. Conditional random field is employed to assess the soil properties at unsampled location. The conditional simulations make better use of the measurements and decreasing the simulation variance of the random fields.
3. The influence of spatial correlation length on the mean risk associated with site investigation is investigated. It is shown that there is a worst case spatial correlation length where the mean risk of slope designs are maximum. At the limit values of spatial correlation length, only one CPT sounding is sufficient to predict the stability of the slopes.
4. The influence of the standard deviation on the mean risk is also investigated. The results show that increased site investigation expenditure has a greater influence when the soil standard deviation is large. When the degree of variability for soil properties is relatively small, less information is needed to characterize the slope.
5. The influence of consequence of making a false safe on the mean risk is also investigated. The results show that more thorough site investigations are needed when the consequence of making a false safe is high.

In this paper, it is assumed that the mean, standard deviation and spatial correlation length of soil properties are known. Based on the assumptions, this paper proposes a framework which can be used to find the optimal site investigation scope. Parametric studies of the optimal site investigation scope for different degrees of spatial variability have been conducted. In practice, if engineers do not know the covariance function, the standard deviation or the spatial correlation length of soil properties, they can refer to the literature or similar projects to determine these statistics. Then, the optimal site investigation scope can be determined based on the estimated statistics and the parametric studies performed in this study.

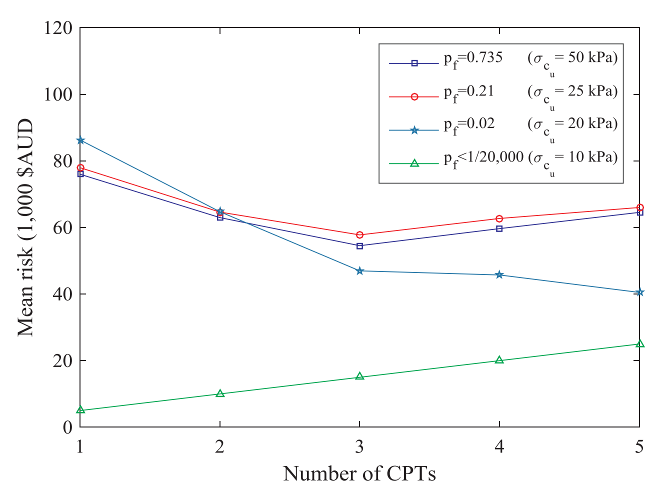

Fig. 12. Influence of consequence of making a false safe error on mean risk.

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