Zhu, D. *et al*. (2019) ***Géotechnique Letters* 9**, 1–4, https://doi.org/10.1680/jgele.18.00252

Probabilistic stability analyses of layered excavated slopes

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The random finite-element method (RFEM) is employed to study the effect of vertical spatial soil variability on the stability of layered excavated slopes. Particular emphasis of the paper is on the critical or 'worst-case' vertical spatial correlation length, at which the probability of slope failure reaches a maximum. The RFEM results indicate that layered slopes with a relatively low mean factor of safety or a relatively high coefficient of variation of soil strength are most likely to display the 'worst-case' phenomenon. The 'worst-case' phenomenon is explained by observing the failure mechanisms in layered soils where the critical spatial correlation length optimises the number of horizontal paths of weaker soil available for the mechanism to pass through.

KEYWORDS: anisotropy; failure; slopes

NOTATION

c_u	undrained shear strength
$c_{u,FS=1}$	characteristic value of the undrained strength that would result in FS = 1
D	depth ratio
FS	factor of safety
$\overline{FS}$	mean factor of safety
H	slope height
p_f	probability of failure
p_f(max)	maximum probability of failure
v_{c_u}	coefficient of variation for c_u
β	slope angle
γ	saturated unit weight
Θ_y	dimensionless vertical spatial correlation length
θ_x	horizontal spatial correlation length
θ_y	vertical spatial correlation length
μ_{c_u}	mean of c_u
σ_{c_u}	standard deviation of c_u
$\Phi(\cdot)$	standard normal cumulative distribution function
ϕ_u	total stress friction angle (= 0)

INTRODUCTION

Traditionally, slope stability is assessed by deterministic approaches leading to a factor of safety where the soil properties are often assumed to be constant based on characteristic values. It has long been recognised, however, that soil properties exhibit spatial variability and that the classical factor of safety is not a consistent way of measuring risk, since it includes no concept of soil strength variability – for example, when comparing two slopes, the one with the higher factor of safety may also have a higher probability of failure (e.g. Lacasse *et al*., 2013). Accordingly, numerous probabilistic slope stability analysis methods incorporating soil variability have been developed (e.g. Tang *et al*., 1976; Whitman, 1984; Paice & Griffiths, 1997; Hassan & Wolff, 1999; Griffiths & Fenton, 2000, 2004). Among these approaches, the random finite-element method (RFEM) (Griffiths & Fenton, 1993; Fenton & Griffiths, 1993, 2008) has been proved to be one of the most robust and effective approaches.

In this paper, the RFEM program `rslope2d` has been modified and applied to the stability analysis of a slope excavated in layered anisotropic soil in which the horizontal spatial correlation length is significantly higher than that in the vertical direction (e.g. Phoon & Kulhawy, 1999). As a special case, only spatial variability in the vertical direction is considered in this paper, with the horizontal spatial correlation length assumed as infinite. This assumption represents a limiting case of layered soil and has previously been considered for braced excavations and slopes (Griffiths *et al*., 2009; Luo *et al*., 2012; Allahverdizadeh, 2015; Allahverdizadeh *et al*., 2015a). For conservative reliability-based design, the focus of this study will be on the influence of the vertical spatial correlation length on the probability of failure, and particularly the 'worst-case' value that might lead to a maximum probability of failure (or minimum reliability).

INPUT PARAMETERS

Figure 1 shows a typical profile of the test slope considered, with height $H = 10$ m, depth ratio $D = 2$, total undrained friction angle $\phi_u = 0$ and saturated unit weight $\gamma = 20$ kN/m^3. Four slope angles β = 26.6, 45, 60 and 90° were considered with this profile. The undrained shear strength c_u was assumed to be a log-normal random variable in the vertical direction defined by a mean μ_{c_u}, a standard deviation σ_{c_u} and a spatial correlation length θ_y. A convenient measure of variability is given by the coefficient of variation $v_{c_u} = \sigma_{c_u}/\mu_{c_u}$ and it has been suggested that v_{c_u} for undrained strength typically lies in the range $0{\cdot}1 < v_{c_u} < 0{\cdot}5$ (e.g. Lee *et al*., 1983).

The spatial correlation length is a measure of the distance over which properties tend to be correlated. A high spatial correlation length implies gradually varying, while a small spatial correlation length implies rapidly varying properties. In the current study, the vertical spatial correlation length θ_y has been varied systematically, while the spatial correlation length in the horizontal direction is fixed and infinite (i.e. $\theta_x = \infty$; there is no change in soil properties in the horizontal direction). Figure 2 shows two typical realisations

Manuscript received 28 December 2018; first decision 13 March 2019; accepted 27 April 2019.
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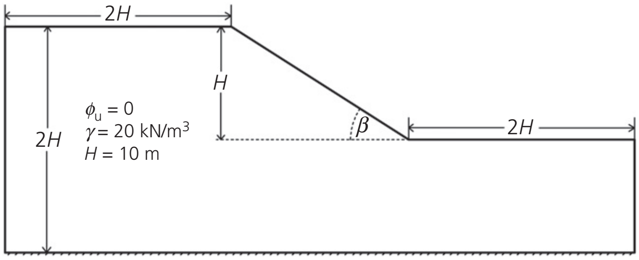

Fig. 1. Slope profile and deterministic parameters for the test slope

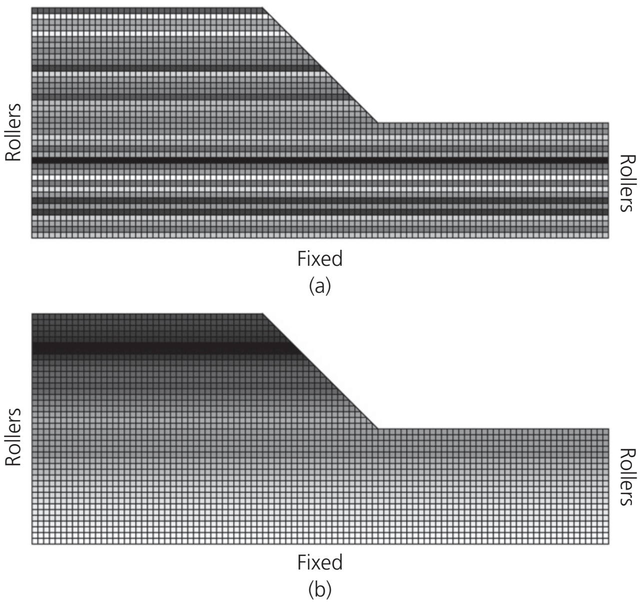

Fig. 2. Effect of vertical spatial correlation length in RFEM analysis of layered excavated slopes: (a) $\theta_y = 0{\cdot}1$ m and (b) $\theta_y = 1000$ m

of random fields corresponding to low and high vertical spatial correlation lengths ($\theta_y = 0{\cdot}1$ and $\theta_y = 1000$ m) for a slope with $\beta = 45°$. Light and dark layers mean low and high values of undrained shear strength, respectively. It can be observed from Fig. 2 that high θ_y gives a more slowly varying c_u across the mesh. A non-dimensional vertical spatial correlation length given by $\Theta_y = \theta_y/H$ is adopted, and the following values have been chosen for the current parametric study

$$\Theta_y = 0{\cdot}01,\ 0{\cdot}1, 0{\cdot}2, 0{\cdot}5, 1{\cdot}0, 2{\cdot}0, 5{\cdot}0, 10{\cdot}0, 100{\cdot}0$$

RFEM RESULTS

In conventional slope reliability analysis, the single random variable (SRV) approach has been widely used due to its simplicity. The SRV approach assumes a uniform isotropic soil with randomised strength parameters (i.e. $\theta_x = \theta_y = \infty$). The probability of failure in such cases can be derived analytically as shown in Fig. 4 in Griffiths & Fenton (2004) based on the equation

$$p_f = \Phi\left[\frac{\ln\left(1 + v_{c_u}^2\right) - 2\ln\left(\overline{\mathrm{FS}}\right)}{2\sqrt{\ln\left(1 + v_{c_u}^2\right)}}\right] \quad (1)$$

where $\overline{\mathrm{FS}}$ is defined as the mean factor of safety – that is, the factor of safety of the slope assuming a uniform soil with its strength set equal to μ_{c_u}.

The SRV approach and equation (1) have, however, been shown to be potentially unconservative (e.g. Griffiths *et al.*, 2007; Allahverdizadeh *et al.*, 2015b; Zhu *et al.*, 2019) where a maximum and conservative probability of failure have been identified at intermediate critical or 'worst-case' spatial correlation lengths. In the current study, to systematically search for the critical *vertical* spatial correlation length in the reliability analysis of anisotropic layered excavated slopes, the test slope with $\beta = 26{\cdot}6°$ shown in Fig. 1 was selected. For this test slope, traditional deterministic approaches (e.g. stability chart of Taylor, 1937) give $c_{u,FS=1}/(\gamma H) = 0{\cdot}17$, where $c_{u,FS=1}$ is the characteristic value of the undrained shear strength that would result in a factor of safety of FS = 1. Due to the linear relationship between the factor of safety and the undrained shear strength in a uniform slope, the mean factor of safety $\overline{\mathrm{FS}}$ is given by

$$\overline{\mathrm{FS}} = \frac{\mu_{c_u}/(\gamma H)}{0{\cdot}17} \quad (2)$$

Following RFEM parametric studies, Fig. 3 shows the relationship between Θ_y and p_f for three different mean factors of safety using equation (2) with $\beta = 26{\cdot}6°$ and $v_{c_u} = 0{\cdot}5$. It can be observed from Fig. 3 that there exists a pronounced worst-case scenario occurring at about $\Theta_y = 0{\cdot}2$ for $\overline{\mathrm{FS}} = 1{\cdot}2$. For higher values of $\overline{\mathrm{FS}}$, the value of Θ_y corresponding to the maximum p_f increases, but the maximum also becomes less pronounced. For the case of $\overline{\mathrm{FS}} = 1{\cdot}4$, the maximum is barely noticeable. In summary, the results shown in Fig. 3 indicate that for layered excavated slopes with $v_{c_u} = 0{\cdot}5$, the SRV approach and equation (1) give unconservative solutions if the mean factor of safety is relatively low (i.e. $\overline{\mathrm{FS}} < 1{\cdot}4$). As might be expected, as Θ_y increases, the RFEM outcomes converge asymptotically from above for equation (1) solutions.

Figure 4 shows the effect of Θ_y on p_f for three different coefficients of variation with $\beta = 26{\cdot}6°$ and $\overline{\mathrm{FS}} = 1{\cdot}3$. The result for the case of $v_{c_u} = 0{\cdot}5$ corresponds to the middle plot in Fig. 3. It can be observed that, with the decrease of v_{c_u}, the 'worst-case' phenomenon gets less noticeable. For the case of $v_{c_u} = 0{\cdot}3$, the analytical solution is greater than all RFEM results and may be considered conservative. Figure 4 indicates that, for layered excavated slopes with $\overline{\mathrm{FS}} = 1{\cdot}3$, the SRV approach may give unconservative solutions, but only for a relatively high coefficient of variation ($v_{c_u} > 0{\cdot}3$).

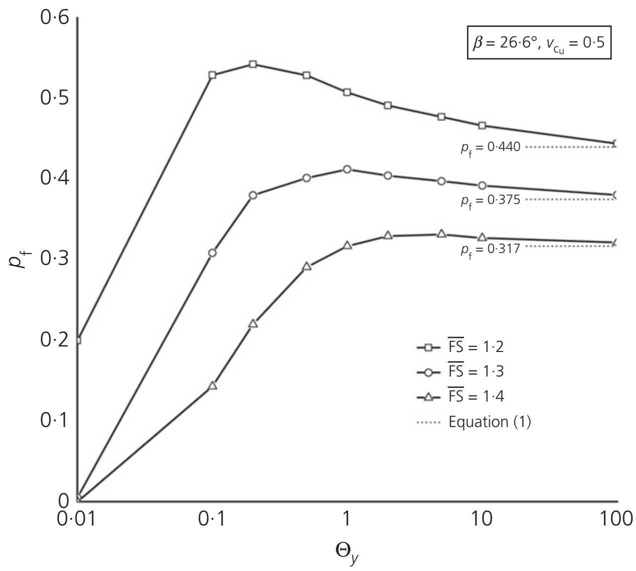

Fig. 3. Influence of Θ_y on p_f with different mean factors of safety

Figure 5 shows the effect of Θ_y on p_f for four different slope angles with $\overline{FS} = 1{\cdot}3$ and $v_{c_u} = 0{\cdot}5$. Similarly, the result for the case of $\beta = 26{\cdot}6°$ is the same as those with circle symbols in Figs 3 and 4. It can be observed that the critical Θ_y occurs at the same position for all cases; however, the $\beta = 60°$ result gives the most obvious maxima in p_f. Figure 6 shows the effect of β on the maximum p_f for these selected slope angles. Apparently, for the horizontally layered soils under consideration, the 60° slope allows more paths (deep and shallow) to failure over a suite of Monte-Carlo simulations than the 45° or even the 90° case. An advantage of stability analysis by RFEM is its inherent ability to 'seek out' the critical failure mechanism. Taylor (1948) indicated that 53° slopes represented the transition between deep and shallow critical failure mechanism for uniform undrained slopes. It seems logical that in random-layered slopes, this transition would occur at steeper slope angles (e.g. van den Eijnden & Hicks, 2018), because to go deep, the critical failure mechanism has to cut through layers of varied strength in the series (see Fig. 7). Since the probability of failure is defined as the proportion of RFEM analyses which failed, more paths to failure would result in higher p_f. As shown in Fig. 5, when the vertical spatial correlation length increases towards infinity ($\Theta_y \to \infty$), the RFEM results for all cases approach the analytical solution of $p_f = 0{\cdot}375$ from equation (1). This is expected, because the probability of failure given by equation (1) as $p_f = f(\overline{FS}, v_{c_u})$ does not depend on the slope angle β.

An explanation of the 'worst-case' phenomenon might lie in an observation of the failure mechanisms for layered excavated slopes shown in Fig. 7. The figure shows three typical failure simulations (shallow, toe and deep failure mechanisms) from the Monte-Carlo suite corresponding to the 'worst-case' properties of $\Theta_y = 0{\cdot}2$, $\beta = 26{\cdot}6°$, $v_{c_u} = 0{\cdot}5$

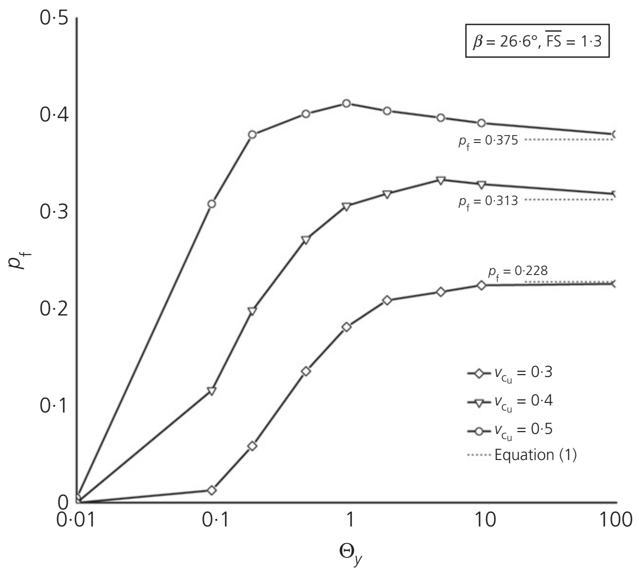

Fig. 4. Influence of Θ_y on p_f with different coefficients of variation

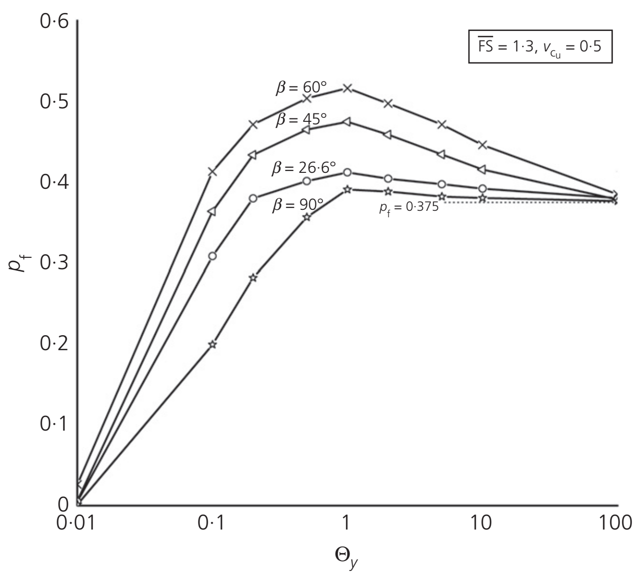

Fig. 5. Influence of Θ_y on p_f with different slope angles

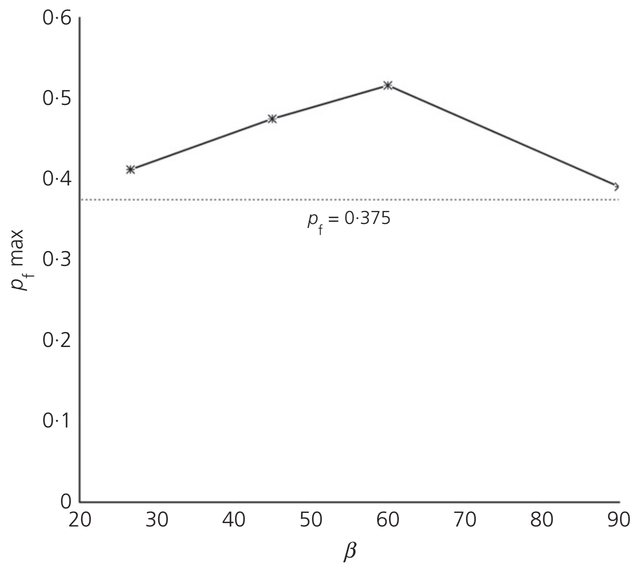

Fig. 6. Influence of β on maximum p_f

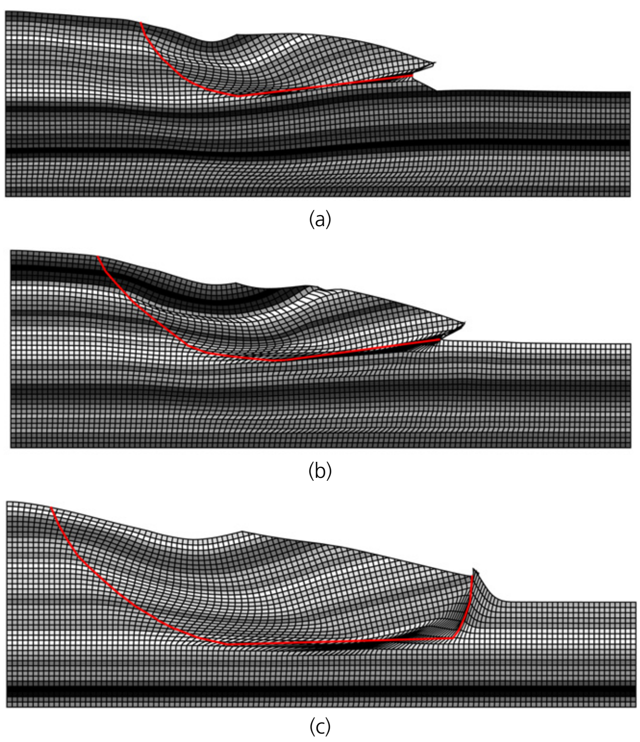

Fig. 7. Typical failure mechanisms for a slope with $\beta = 26{\cdot}6°$, $v_{c_u} = 0{\cdot}5$, $\overline{FS} = 1{\cdot}2$ and $\Theta_y = 0{\cdot}2$: (a) shallow failure mechanism; (b) toe failure mechanism and (c) deep failure mechanism

and $\overline{FS} = 1{\cdot}2$. The failure mechanisms follow the path of least resistance through the slope and involve a quite complicated integral of soil strengths. On the uphill side, the mechanism has to cut in series through multiple layers of varying strength, while on the downhill side, the mechanism is attracted to a weaker layer, and runs parallel to the stratification. The 'worst-case' correlation length appears to offer more paths for the mechanism to follow than higher or lower values, and hence delivers a higher probability of failure. In the realm of probabilistic geotechnical analysis, the 'worst-case' correlation length is an interesting topic of ongoing investigations (e.g. Zhu *et al*., 2019).

CONCLUSION

This paper investigated the reliability of layered excavated slopes by the RFEM. Parametric studies used varied vertical spatial correlation length while the horizontal spatial correlation length was fixed to infinity. The emphasis of the paper is to investigate the 'worst-case' vertical spatial correlation length leading to the maximum probability of failure (or minimum reliability). Similar overall conclusions were reported by Zhu *et al*. (2019) for reliability analysis of slopes with isotropic spatial correlation lengths. The results in this paper show that the 'worst-case' phenomenon is most obvious when the $\overline{FS}$ is relatively low (e.g. $\overline{FS} < 1{\cdot}4$) and the v_{c_u} is relatively high (e.g. $v_{c_u} > 0{\cdot}3$). Results also indicate a 'worst-case' slope angle of around 60° (with all other parameters held constant), leading to higher probabilities of failure than flatter or even steeper slopes.

RANDOM FINITE-ELEMENT PROGRAMS

The source code for the RFEM programs covering a range of geotechnical applications can be downloaded from DOEMI (n.d.).

ACKNOWLEDGEMENTS

This work was supported by the National Natural Science Foundation of China (grant number 51808484), the Natural Science Foundation of the Jiangsu Province (grant number BK20180934), the Natural Science Foundation of the Jiangsu Higher Education Institutions of China (grant number 18KJB170022), the China Postdoctoral Science Foundation (grant number 2018M642336) and the Jiangsu Planned Projects for Postdoctoral Research Funds (grant number 2018K137C).

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