Technical Note

Probabilistic Analysis of Shallow Passive Trapdoor in Cohesive Soil

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Abstract: This technical note used the random finite-element method (RFEM) to investigate the influence of soil strength variability on the limit load of a shallow passive trapdoor embedded in cohesive soil. The mean undrained shear strength was held constant while the coefficient of variation and spatial correlation length were varied systematically. For trapdoors against soils with low values of the coefficient of variation, the mean limit load agreed well with the results from a uniform deterministic analysis. For higher values of the coefficient of variation, the mean limit load decreased. By interpreting the Monte Carlo simulations in a probabilistic context, the probability of failure was assessed as a function of the factor of safety based on the mean and the spatial variability of the soil. It was found, for example, that a factor of safety of 2.5 is required to avoid the probability of failure exceeding 5% for soils with strength variability within typical ranges. **DOI: 10.1061/(ASCE)GT.1943-5606.0002051.**

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Introduction

The stability of a trapdoor, originally studied by Terzaghi (1936), has become an important benchmark solution in theoretical soil mechanics. This problem has two kinds of displacement pattern, depending on whether the movement of the trapdoor is upward (passive) or downward (active). Passive failure of a trapdoor might correspond to the uplift problem of a plate anchor (e.g., Merifield et al. 2001), whereas active failure might correspond to gravitational flow of a granular material, which has been used in the design of tunnels. The trapdoor problem has received considerable attention deterministically (e.g., Koutsabeloulis and Griffiths 1989; Sloan et al. 1990; Smith 1998, 2012; Martin 2009; Keawsawasvong and Ukritchon 2017). Natural soils usually exhibit spatial variability, with the properties varying from point to point. This technical note investigated the influence of soil strength variability on the limit load of a rough rigid strip trapdoor embedded in an undrained clay by the random finite-element method (RFEM) (Griffiths and Fenton 1993, 2007). This advanced numerical method combines elastic-plastic finite-element (FE) methodologies (e.g., Smith and Griffiths 2004) with random field theory in a Monte Carlo framework. The local averaging subdivision (LAS) method (Fenton and Vanmarcke 1990) was used to generate the random fields, because this method is able to model spatial variability while properly accounting for local averaging over each finite element. The undrained shear strength c_u was assumed to be lognormally distributed, with the parameters given in Table 1.

With the purpose of nondimensionalizing the input parameters, the variability of c_u can be expressed by the coefficient of variation $v_{c_u} = \sigma_{c_u}/\mu_{c_u}$, which is a convenient measure of dispersion relative to the mean. The parameter θ_{c_u} is a dimensional spatial correlation length, which governs the distance over which properties are essentially similar, i.e., small correlation lengths lead to rapid spatial variability, whereas large correlation lengths lead to slow spatial variability. For the random field modeling, a dimensionless and isotropic spatial correlation length $\Theta_{c_u} = \theta_{c_u}/B$ was used, where B is the width of the strip trapdoor.

It has been recommended (e.g., Lee et al. 1983; Phoon and Kulhawy 1999) that typical values of the coefficient of variation for c_u are between 0.1 and 0.5. The spatial correlation length however, is rarely reported and may show anisotropy. Although the RFEM methodology has the ability to model anisotropy, for simplicity, the assumption of isotropy is made throughout this study. Site-specific refinement relating to anisotropy may be a topic for future studies.

In this study, μ_{c_u} was fixed at a value of $100\ \mathrm{kN/m^2}$, and the other two input parameters v_{c_u} and Θ_{c_u} were varied systematically.

Deterministic Analysis

The stability analysis of a shallow passive trapdoor is performed assuming the soil is a saturated undrained clay ($\phi_u = 0$) with an elastic-perfectly plastic Tresca failure criterion. The viscoplastic

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Table 1. Undrained shear strength properties

Statistical property	Symbol	Unit
Mean	μ_{c_u}	Stress
Standard deviation	σ_{c_u}	Stress
Spatial correlation length	θ_{c_u}	Length

finite-element method is used (e.g., Smith and Griffiths 2004). The initial total stresses are assumed to be essentially hydrostatic, hence they have no influence on the strength. Furthermore, the variability of the unit weight is small relative to the undrained strength (e.g., Lee et al. 1983) and is often considered to be a deterministic variable. For simplicity, this study assumed the soil to be weightless ($\gamma = 0$), with no surcharge pressure acting on the soil surface. Thus the behavior of a trapdoor is affected by undrained shear strength (c_u), Young's modulus (E), and Poisson's ratio (ν). The elastic parameters E and ν affect the calculated displacement prior to collapse (e.g., Griffiths et al. 2002), but the limit loads of the trapdoor depend mainly on soil strength parameter c_u. Although the RFEM program can model random distributions of all three parameters, only the undrained shear strength was considered to be random in this study, whereas the Young's modulus and Poisson's ratio were set to $E = 10^5$ kN/m^2 and $\nu = 0.3$, respectively.

This study used a typical two-dimensional FE mesh consisting of 1,200 eight-noded square elements, with 60 columns and 20 rows (Fig. 1). The side length of each element was 0.05 m and the width of the strip trapdoor B was 1 m, which occupied 20 elements. The height of soil above the trapdoor, H, was assumed to be equal to B, so with $H/B = 1$ the trapdoor was considered to be shallow (i.e., $H/B \leq 2$) (e.g., Sloan et al. 1990; Martin 2009). The trapdoor was assumed to be rigid and rough, and displacement control was employed. The trapdoor was incrementally displaced vertically into the soil, whereas the horizontal movement of the nodes which represent the trapdoor were fixed at zero. After each increment, the average vertical stress σ_y in the row of Gauss points just above the displaced nodes was calculated. Collapse of the trapdoor was said to have occurred when the average σ_y became essentially constant. Finally, the limit load was given by

$$F_{\mathrm{p}} = \sigma_y B \tag{1}$$

Martin (2009) proposed a slip-line solution of the shallow trapdoor stability problem. For passive failure (Fig. 1), Martin's analytical solution is given by

$$F_{\mathrm{p}}/B = N_c c_u \tag{2}$$

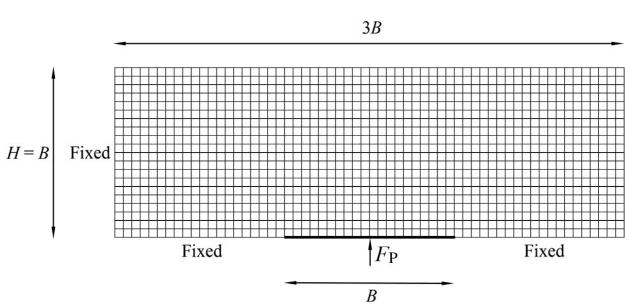

Fig. 1. Mesh used in probabilistic stability analysis of a shallow passive trapdoor.

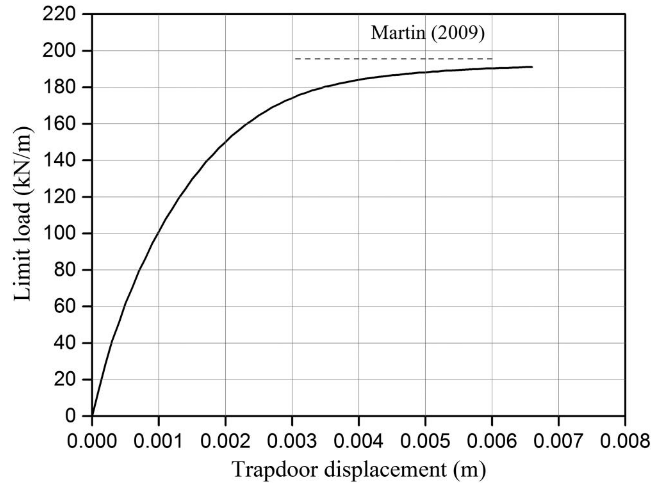

Fig. 2. Deterministic stability analysis of the trapdoor.

where N_c = dimensionless stability factor, which is proportional to the cover ratio H/B, and is given by

$$N_c = 1.956H/B \tag{3}$$

For small cover ratios, Davis (1968) and Gunn (1980) derived an upper bound $N_c = 2H/B$ by the vertical slip mechanism and a three-parameter rigid block mechanism, respectively, whereas Sloan et al. (1990) and Merifield et al. (2001) obtained quite close bounds on the solution in the form of charts using finite-element limit analysis (FELA). The slip-line solution of Martin (2009) was later compared favorably with FELA solutions of Martin and Randolph (2006) and Makrodimopoulos and Martin (2007) for cover ratios $H/B \leq 1.3$ and was used as the benchmark solution in this study. For cover ratio $H/B = 1$ and undrained shear strength $c_u = 100$ kN/m^2, Martin's analytical solution [Eq. (3)] gives the limit load for passive failure as $F_p = N_c c_u B = (1.956)(1)(100)(1) = 195.6$ kN/m.

The results of deterministic analysis by the finite-element method are shown in Fig. 2, giving a limit load of about 191.1 kN/m, which is 2.3% lower than Martin's analytical solution of $F_p = 195.6$ kN/m. The slightly lower values may be due to mesh coarseness, especially at the trapdoor edge, and may also be due to the measurement of stress at the first row of Gauss points, which are slightly above the actual trapdoor depth. Table 2 compares Martin's analytical solution with the finite-element analysis using 30×10, 60×20, and 120×40 elements. As the number of elements increased, the difference between the FE and analytical solutions decreased slightly (Table 2), indicating that the mesh in Fig. 1 represents a reasonable compromise between accuracy and computational efficiency. In the discussion that follows, and for consistency, the mean limit load is standardized by the deterministic value from FE analysis. The influence of this slightly lower

Table 2. Limit load by finite-element and Martin's analytical solutions (kN/m)

Number of elements	FE solution	Martin's solution
30×10	190.2	195.6
60×20	191.1	
120×40	191.9	

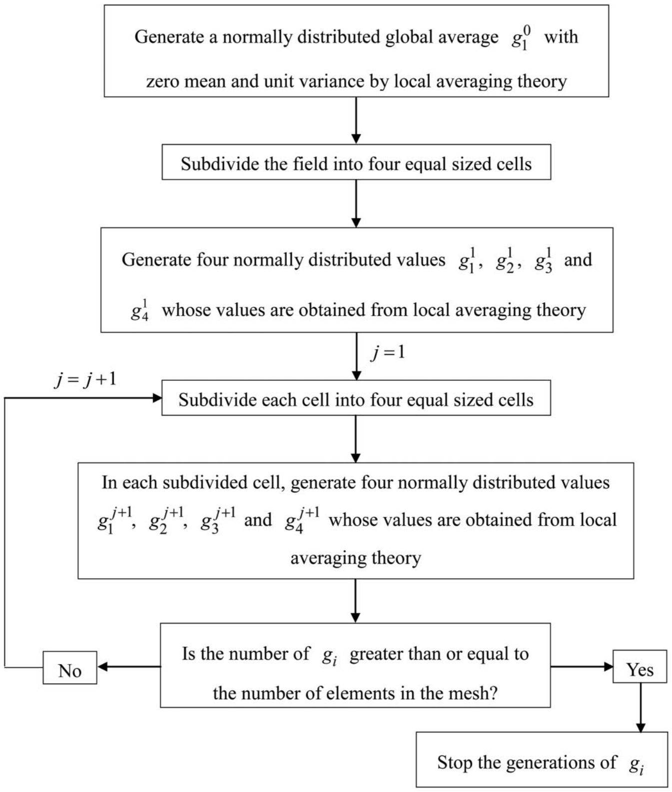

Fig. 3. Flow chart for generating the local averages g_i.

prediction is relatively unimportant, and in this study the deterministic limit load is referred to as F_{p_d}, that is, $F_{p_d} = 191.1$ kN/m.

Brief Description of RFEM

The random undrained shear strength was obtained via the LAS approach using the following relationship

$$c_{u_i} = \exp(\mu_{\ln c_u} + \sigma_{\ln c_u} g_i) \tag{4}$$

where c_{u_i} = undrained shear strength allocated to the ith element; g_i = local average of standard normal random field g over the region of the ith element; and $\mu_{\ln c_u}$ and $\sigma_{\ln c_u}$ = mean and standard deviation of $\ln c_u$, respectively. Realizations of the local averages g_i were generated in a top-down fashion (Fig. 3).

In the context of a Monte Carlo analysis, each realization of an RFEM analysis of the problem in Fig. 1 involves generation of the c_u random field and the succeeding FE analysis of the trapdoor stability. In each realization, the underlying random field properties v_{c_u} and Θ_{c_u} are the same, however, the spatial pattern of c_u over the region of the FE mesh is quite different, leading each time to a different value of the trapdoor limit load. Following each suite of 1,000 Monte-Carlo simulations, the calculated limit loads were subjected to statistical analysis.

Results and Discussions

Parametric Analyses

Parametric analyses were carried out by adopting the mesh in Fig. 1, with the following input values:

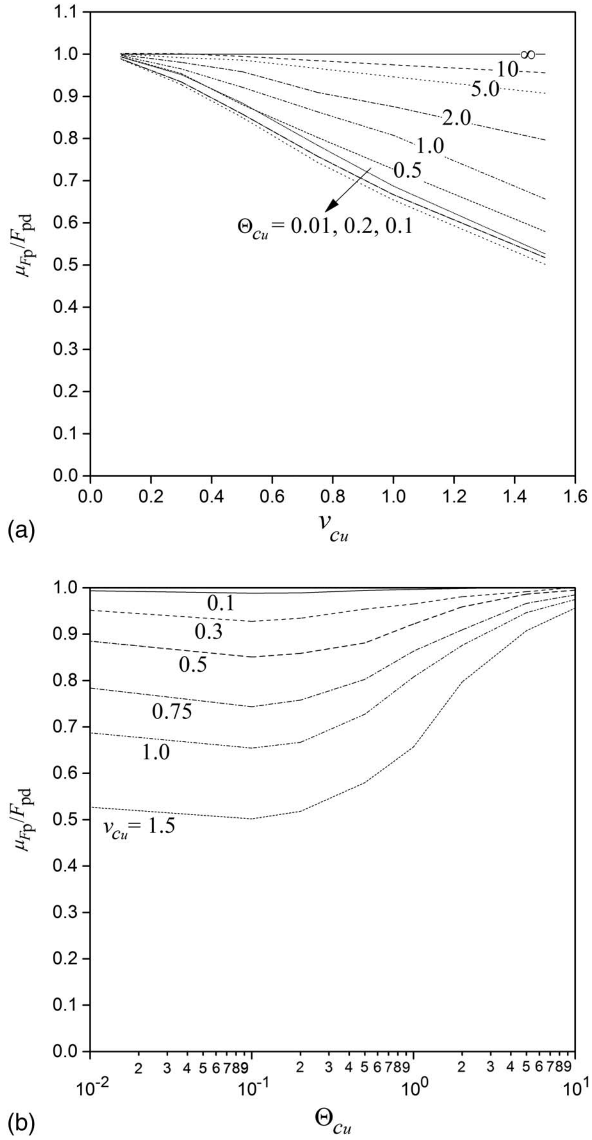

Fig. 4. Estimated mean limit load as a function of undrained shear strength statistic: (a) v_{c_u}; and (b) Θ_{c_u} with $H/B = 1$.

$$\Theta_{c_u} = 0.01,\ 0.1,\ 0.2,\ 0.5,\ 1.0,\ 2.0,\ 5.0,\ \text{and}\ 10.0$$

$$v_{c_u} = 0.1,\ 0.3,\ 0.5,\ 0.75,\ 1.0,\ \text{and}\ 1.5$$

For different parametric combinations of Θ_{c_u} and v_{c_u}, the mean and standard deviation of the trapdoor limit load were calculated following 1,000 Monte-Carlo simulations.

Fig. 4 shows how the mean limit load, normalized by the deterministic value from FE analysis, changes with Θ_{c_u} and v_{c_u}. As might be expected, for low values of v_{c_u}, the mean limit load μ_{F_p} tended to the deterministic value. As the value of v_{c_u} increased, the mean limit load decreased, and this phenomenon is particularly obvious for a spatial correlation length of $\Theta_{c_u} \approx 0.1$. For example, for a highly variable soil with $\Theta_{c_u} = 0.1$ and $v_{c_u} = 1.5$, the mean limit load decreased to about 50% of the deterministic value [Fig. 4(a)]. A minimum mean limit load or worst case was reached when $\Theta_{c_u} \approx 0.1$ [Fig. 4(b)]. At the next lower value of Θ_{c_u}, μ_{F_p} started to increase for all values of v_{c_u}. As $\Theta_{c_u} \to 0$, local averaging causes the random field of shear strength to be essentially uniform, with a value set at the median of the lognormal distribution.

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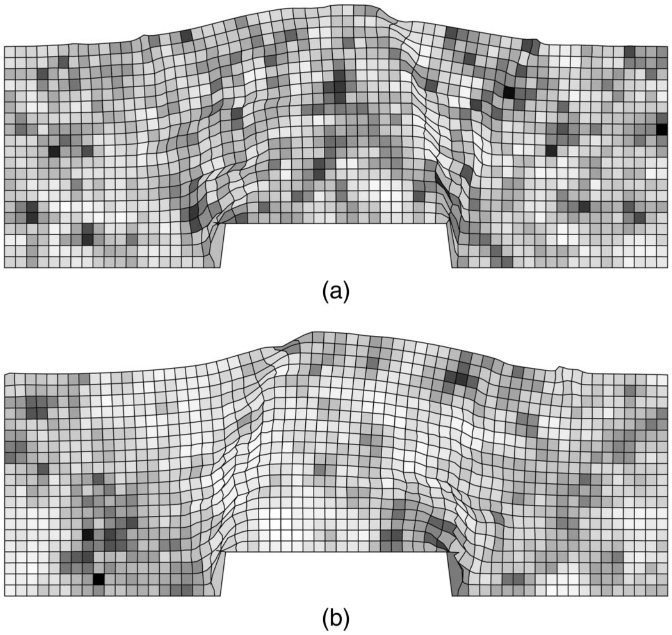

Fig. 5. Two typical deformed meshes at failure with (a) $v_{c_u} = 0.75$ and $\Theta_{c_u} = 0.1$; and (b) $v_{c_u} = 0.75$ and $\Theta_{c_u} = 0.2$.

Fig. 5 shows two representative deformed meshes at failure above the trapdoor with parametric combinations indicated in the figure caption. The deformed mesh is shaded, with dark and light regions indicating higher and lower soil strengths, respectively. Due to the spatial variability of the soil, the failure mechanism is of course not symmetrical.

Probabilistic Interpretation

For practical applications, it is interesting to predict the probability of design failure. Failure is defined here to have occurred if the calculated limit load is less than the deterministic solution based on the mean value of c_u, reduced by an appropriate factor of safety FS, that is

$$\text{Design failure if } F_p < F_{p_d}/\text{FS} \tag{5}$$

The probability of design failure is defined as the probability that the calculated limit load is less than the factored deterministic value

$$p_f = P[F_p < F_{p_d}/\text{FS}] \tag{6}$$

Fig. 6 can be used to choose a required factor of safety to satisfy the desired probability of failure. For example, if a target probability of failure no greater than 5% is desired for an undrained clay with $v_{c_u} = 0.1$ [e.g., a lower end of the recommended range from Lee et al. (1983)], a factor of safety of at least FS = 1.2 is required [Fig. 6(a)]. For higher values of $v_{c_u} = 0.3$ or $v_{c_u} = 0.5$, the required factor of safety needs to be at least 1.7 or 2.5, respectively [Fig. 6(c)]. The choice of p_f for design projects is entirely project-dependent and is beyond the scope of this technical note. The choice of $p_f = 5\%$ is arbitrary in order to show how the methodology can make a link between a probability of failure and a factor of safety. A very similar process is applicable for other values of p_f.

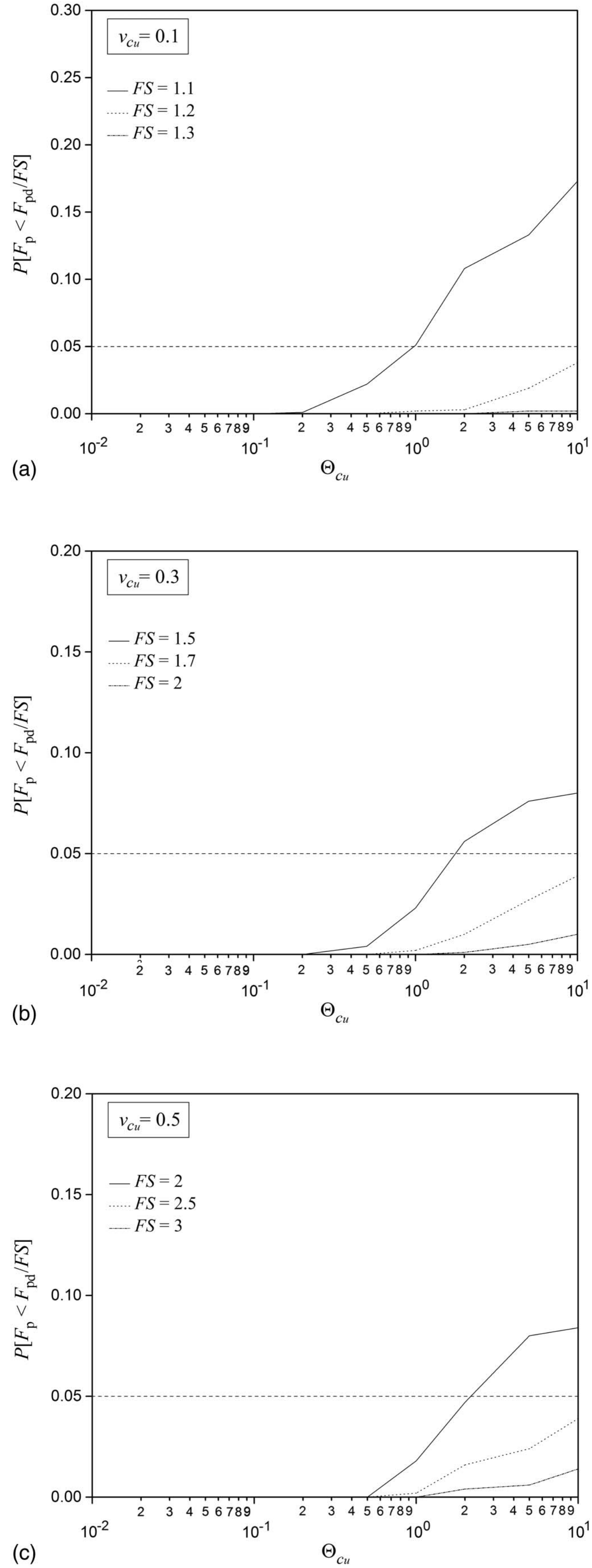

Fig. 6. Probability of design failure as a function of Θ_{c_u} for different factors of safety with $H/B = 1$ and (a) $v_{c_u} = 0.1$; (b) $v_{c_u} = 0.3$; and (c) $v_{c_u} = 0.5$.

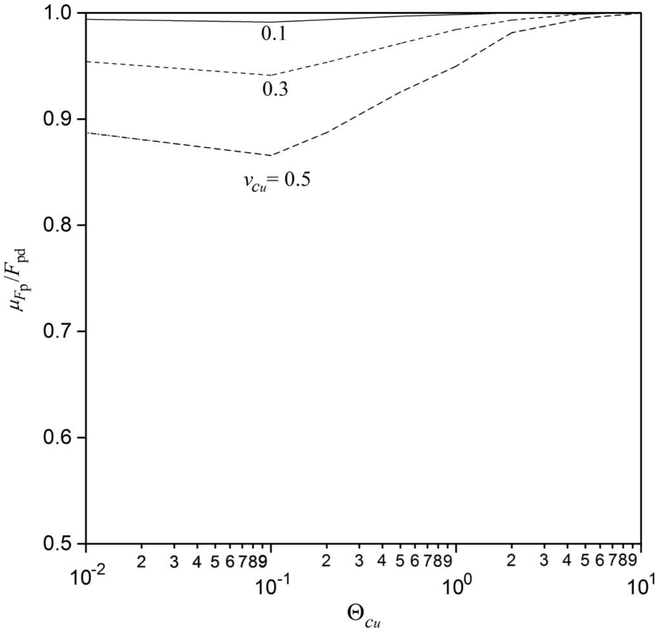

Fig. 7. Estimated mean limit load as a function of Θ_{c_u} with $H/B = 0.5$.

Influence of Trapdoor Cover Ratio

The previous discussion related to a trapdoor with a cover ratio of $H/B = 1$; this subsection considers a shallower trapdoor with $H/B = 0.5$. For this value of the cover ratio, and with other parameters the same as before, the limit load for passive failure by Martin's slip-line solution is $F_p = N_c c_u B = (1.956)(0.5)(100)(1) = 97.8$ kN/m. The deterministic FE result is $F_p = 94.6$ kN/m, which is 3.3% lower than Martin's proposed analytical solution. In this subsection, the deterministic limit load was fixed at $F_{p_d} = 94.6\,\mathrm{kN/m}$.

Fig. 7 shows the mean limit load, normalized by the deterministic value from FE analysis, plotted against Θ_{c_u} with $v_{c_u} = 0.1$, 0.3, and 0.5. For $H/B = 0.5$, the minimum mean limit load was also reached when $\Theta_{c_u} \approx 0.1$. Comparison of Figs. 6 and 8 indicates that the minimum factors of safety needed to achieve a target probability of failure no greater than 5% for different values of v_{c_u} is essentially unchanged from the results observed in the $H/B = 1$ case.

Concluding Remarks

The following conclusions can be made:

1. The mean limit load for a shallow passive trapdoor embedded in a spatially varying undrained clay is always less than the deterministic value calculated using the mean shear strength. This is due to the possibility that low strengths may be generated above the trapdoor and the failure mechanism is attracted toward the weak elements. This important observation confirms that low strengths dominate computed limit loads.
2. A minimum mean limit load or worst case is observed for higher values of v_{c_u} when the spatial correlation length is approximately $\Theta_{c_u} \approx 0.1$. It is thought that this spatial correlation length facilitates the development of failure paths above the trapdoor more readily than higher or lower values of Θ_{c_u}.
3. The RFEM analyses enabled the probability of failure to be compared with the design factor of safety based on the mean strength for different Θ_{c_u} and v_{c_u}. For example, the results

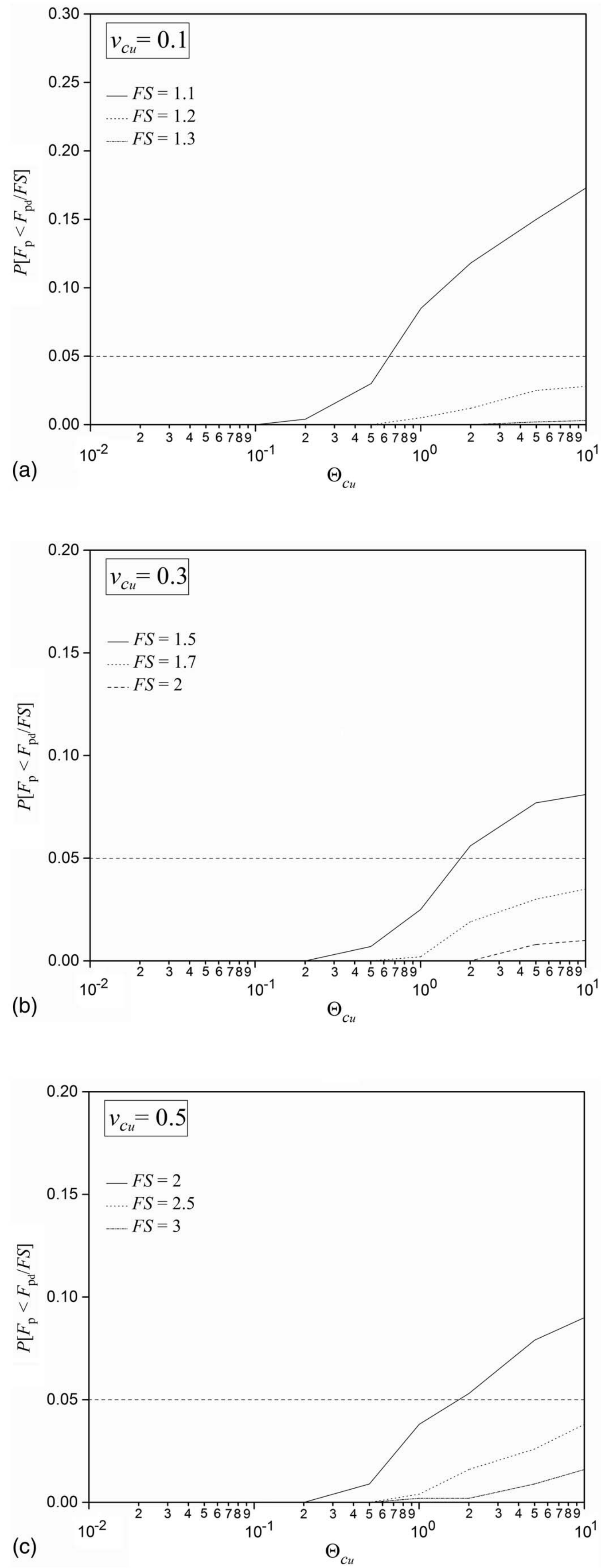

Fig. 8. Probability of design failure as a function of Θ_{c_u} for different factors of safety with $H/B = 0.5$ and (a) $v_{c_u} = 0.1$; (b) $v_{c_u} = 0.3$; and (c) $v_{c_u} = 0.5$.

showed that the required factor of safety needs to be at least 2.5 to avoid the probability of design failure exceeding 5% for soils with $v_{c_u} \leq 0.5$.

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