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ARTICLE

Characterizing site investigation performance in a two layer soil profile[1]

M.P. Crisp, M.B. Jaksa, Y.L. Kuo, G.A. Fenton, and D.V. Griffiths

Abstract: Insufficient or inappropriate soil testing can lead to a range of undesirable consequences, and yet there is no guideline for optimal investigation. This study analyses the influence of a two layer, virtual soil profile with an undulating boundary on site investigation performance. Factors investigated include the method of representing the boundary within the soil model, the stiffness ratio of the two layers, choice of test type, the pile length relative to the boundary length, and the number of boreholes and piles. The relative error contribution from the uncertainty sources of layer geology and soil variability is also examined. Investigation performance is assessed through Monte Carlo analysis in terms of total expected project cost, while implicitly incorporating the risk of damage from poor investigation. It has been shown that the optimal investigation can save in the order of AUD$1.5 million and that 2D soil models can represent 3D soils.

Key words: pile design, Monte Carlo analysis, optimization, virtual soils, site investigation.

Résumé : Lorsque insuffisantes ou inappropriées, les analyses de sol peuvent entraîner une série de conséquences indésirables, et pourtant, il n'existe pas de lignes directrices pour une étude optimale. Cette étude analyse l'influence d'un profil de sol virtuel à deux couches avec une limite ondulée sur le résultat de l'étude du site. Les facteurs étudiés comprennent la méthode de représentation de la limite dans le modèle de sol, le rapport de rigidité des deux couches, le choix du type d'essai, la longueur du pieu par rapport à la longueur de la limite et le nombre de trous de forage et de pieux. La contribution de l'erreur relative à cause des sources d'incertitude de la géologie de la couche et de la variabilité du sol est également examinée. Le résultat de l'étude est évalué au moyen de l'analyse de Monte Carlo en fonction du coût total prévu du projet, tout en intégrant implicitement le risque de dommages découlant d'une étude médiocre. Il a été démontré que l'étude optimale peut économiser de l'ordre de 1,5 million de dollars australiens et que les modèles de sol 2D peuvent représenter des sols 3D. [Traduit par la Rédaction]

Mots-clés : conception de pieux, analyse de Monte Carlo, optimisation, sols virtuels, étude de site.

1. Introduction

Subsurface ground conditions can change drastically from site to site, implying two considerations. Firstly, one should examine the ground by means of a geotechnical site investigation to adequately characterize it. Secondly, optimal investigations are site specific, with the extent dependant on subsurface conditions. However, despite these points, there is no optimal site investigation guideline relating testing to soil variability. Rather, investigations are planned by civil engineers through subjective reasoning (Baecher and Christian 2005), vague or broad rules of thumb (Simpson 2003), or are otherwise dictated by cost, comprising as little as 0.025%–0.03% of the total budget (National Research Council 1984; Jaksa 2000). It is therefore not surprising that insufficient investigations regularly occur, resulting in one or more of the following outcomes: foundation failure (Moh 2004); change orders (Loehr et al. 2015); delays of up to 33% of the total project duration (Jaksa 2000; Albatal 2013); and, most commonly but difficult to quantify, over-design (Clayton 2001). In contrast, studies have shown that there can be considerable financial benefits by conducting investigations beyond the minimal scope, and that the optimal investigation depends on the nature of the soil (Goldsworthy 2006; Crisp et al. 2018). Clearly, there is a need to develop a site investigation optimization guideline for a range of soil conditions.

The method used to determine site investigation quality is based on a framework described by Crisp et al. (2019*a*) and originally proposed by Jaksa et al. (2003). The framework is based on the random finite element method (RFEM), which is a powerful statistical technique that can generate a wide range of soil-related information (Fenton and Griffiths 1993; Griffiths and Fenton 1993). RFEM involves the use of random virtual soils combined with finite element analysis within a Monte Carlo simulation (Fenton and Griffiths 2008). In the context of the present study, RFEM is used to assess the accuracy of site investigations with regards to a set of given structures and soil conditions. The wealth of information provided by RFEM allows costs to be assigned to the soil testing and construction of each investigation. Furthermore, cost penalties are associated to various degrees of structural failure resulting from inadequate investigation, and are defined as the cost of repairing the structure to its original condition. By examining the trade-off between these costs, it is possible to recommend an optimal investigation strategy corresponding to the lowest expected total project cost.

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M.P. Crisp, M.B. Jaksa, and Y.L. Kuo. School of Civil, Environmental and Mining Engineering, University of Adelaide, Adelaide, South Australia, 5005 Australia.
G.A. Fenton. Department of Engineering Mathematics and Internetworking, Dalhousie University, Halifax, Nova Scotia, Canada.
D.V. Griffiths. Department of Civil and Environmental Engineering, Colorado School of Mines, Golden, Colorado, USA.
Corresponding author: M.P. Crisp (email: michael.p.crisp@gmail.com).

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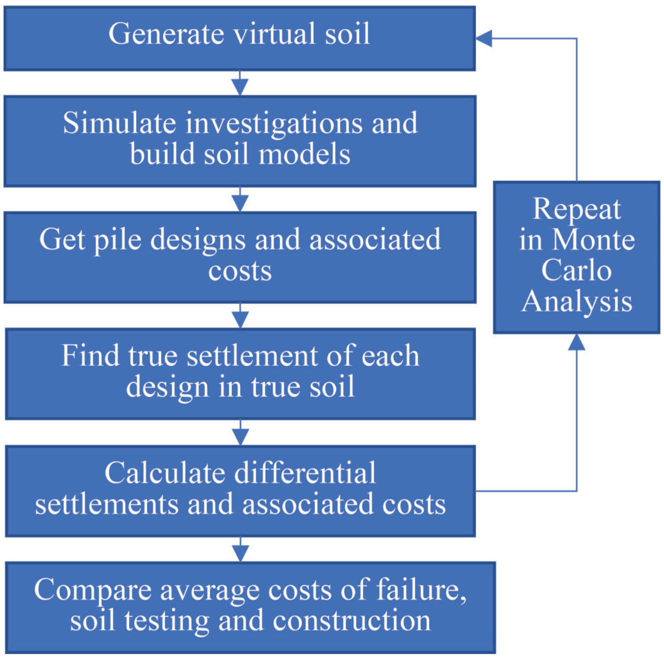
Fig. 1. Methodology flowchart for calculating total costs. [Colour online.]



There have been several studies that have examined the influence of site investigation options on total cost, including Jaksa et al. (2005), Goldsworthy (2006), and Goldsworthy et al. (2007) in the context of pad footings, and Arsyad (2009) and Crisp et al. (2018, 2019*b*) with regards to piles. However, the literature almost exclusively focuses on variable single-layer soil profiles. The exception is the study by Crisp et al. (2017) which examined a simplified 2D, 2 layer case without considering costs. This single layer assumption is generally unrealistic and unconservative, as the uncertainty of layer boundary locations is expected to contribute considerably to inadequate site investigation performance, as opposed to the uncertainty of the engineering properties within those layers. As such, the impact of layer boundary uncertainty on investigation performance is poorly understood.

As the present study is among the first to examine site investigations in multi-layer soils in detail, a wide range of factors are analyzed for their impact on investigation performance. These factors include engineering considerations such as the number of boreholes, the pile embedment depth relative to the pile layer, the selection of soil test type, and the manner in which the layer boundary is represented within the soil model. Furthermore, variables related to soil variability are assessed, such as the degree of layer undulation, the stiffness ratio of the two layers, as well as the magnitude and spatial distribution of variability within each layer. The conclusions drawn from this endeavour will serve as the basis for a future site investigation optimization guideline.

2. Methodology

2.1. Overview

The framework for determining site investigation performance is described by Crisp et al. (2019*a*), and the authors refer readers to that report for verification and a detailed account of the procedures adopted in the present study. Such verification includes sensitivity analyses for values of many parameters stated throughout the paper. For completeness, an overview is given here, with the overall process summarized in Fig. 1.

Briefly, site investigation performance, given as total expected project cost, is determined through the use of Monte Carlo analysis using 8000 realizations. Within each realization, a random, variable, two layer virtual soil profile is generated. These profiles consist of a volume of soil properties over a 3D grid of discrete elements, elaborated upon in the next section. As the properties within these soils are known, it is possible to conduct a wide variety of virtual site investigations by extracting columns of soil samples at their respective physical locations, which represent sampling boreholes or in situ test soundings. The properties of these samples are used to construct an idealized soil model from which the pile foundations are designed using 3D linear-elastic finite element analysis (FEA). The soil is idealized, in that it is a simple representation of the site as discussed later, due to the relatively limited quantity of available information. The justification for adopting a linear-elastic FEA model is also provided later.

Once the foundation is designed, it can then be assessed for differential settlement, using the aforementioned FEA model with the complete, original random virtual soil. Differential settlement (δ) has a well-defined relationship with structural damage; the cracking increases as δ increases. Therefore, by assigning repair costs to various degrees of structural damage, it is possible to assign a penalty cost to various degrees of site investigation scope and quality. This penalty is referred to as failure cost. The total expected project cost associated with a given investigation is the sum of its average failure cost, soil testing cost, and construction costs of both the foundation and superstructure.

2.2. Generation of virtual soil profile

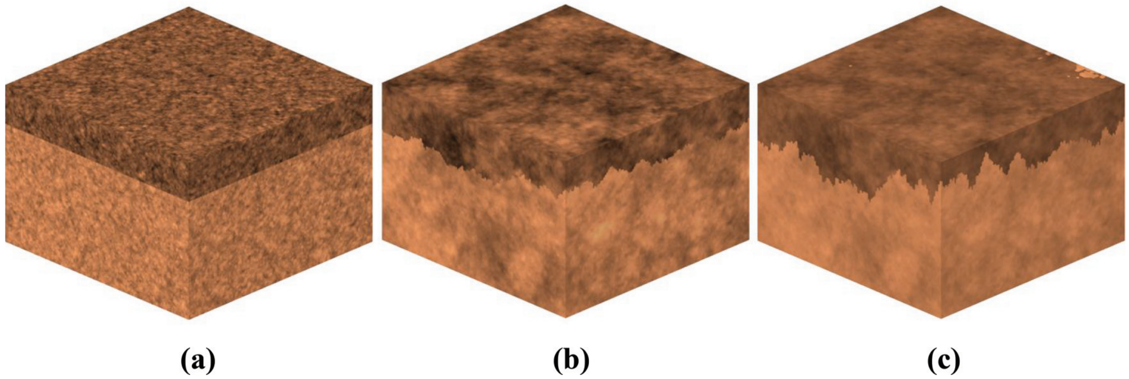
The randomly-generated virtual soil profiles, or random fields, are volumes of soil properties represented by a 3D grid of discrete elements. The fields are generated by local average subdivision (LAS) (Fenton and Vanmarcke 1990). This algorithm is commonly used in geotechnical research, is well-documented, and Fortran open source code is freely available (Fenton and Griffiths 2008). The authors refer the reader to Crisp et al. (2019*d*) for a multiple-layer implementation of the algorithm, with associated descriptions of its use. Due to space constraints and the abundance of available resources, LAS will not be described here in detail.

In practice, LAS produces fields of soil properties with a desired size and spatial variability, where the latter is statistically described by three parameters supplied as inputs: the mean, standard deviation, and the scale of fluctuation (SOF) (Vanmarcke 1983). The SOF is analogous to the range parameter in geostatistics (Jaksa et al. 1997) and is defined as the distance over which soil properties exhibit a degree of similarity. In other words, high SOF values correspond to large pockets of similar material. Mathematically, the SOF is defined by autocorrelation using an exponential Markov model (Fenton and Vanmarcke 1990). Isotropic soils, where the SOF is constant in all directions, are considered in this analysis, as they are the worst case when compared to anisotropic soils, which have a higher SOF in the horizontal direction (Naghibi et al. 2014*b*). Within this study, the standard deviation is normalized by the mean to produce the coefficient of variation (CV), which is more useful as the results can be applied to any mean parameter value.

As linear-elastic FEA is used, two soil properties are required. This includes Young's modulus (E), which is randomly generated by LAS, and Poisson's ratio (v), which is constant at 0.3. The deterministic treatment of v is due to this parameter's spatial variability having a relatively insignificant effect on settlement (Paice et al. 1996; Naghibi et al. 2014*a*). The soil properties themselves are generated according to the lognormal distribution, which has been found to be appropriate for geotechnical engineering probabilistic studies (Fenton and Griffiths 1993; Griffiths and Fenton 1993), and ensures that stiffness values are strictly non-negative.

The undulating layer boundary between the two layers is represented as a 2D, normally-distributed random field, as described by Crisp et al. (2019*d*). As such, the layer is described as an undulating boundary with a specified mean and standard deviation, with the latter parameter denoted as bSD. The impact of varying

Fig. 2. Example soils generated using LAS with a mean layer depth of 10 m and stiffness ratio of 1:9, with parameters (*a*) CV 80%, SOF 1 m, bSD 0 m; (*b*) CV 80%, SOF 8 m, bSD 2 m; (*c*) CV 40%, SOF 8 m, bSD 4 m. [Colour online.]



CV, SOF, and bSD on a virtual soil is shown in Fig. 2 for a 2-layer soil with a mean boundary depth of 10 m, and a layer 1 to layer 2 stiffness ratio of 1:9. The 2D boundary SOF is set at 100 m, which is consistent with the minimal literature available on layer boundary statistics (Crisp et al. 2019*a*).

2.3. Site description

The standard structural configuration in the present study is a six-storey, 20 m × 20 m structure supported by 9 piles that are evenly spaced at 10 m in a grid pattern, as shown in Fig. 3. A 4-pile case with 20 m spacing is also considered. The piles are designed according to a differential settlement of 0.0025 m/m which equates to a settlement tolerance of 25 mm and 50 mm for the 9 and 4 pile cases respectively, based on their spacing (Sowers 1962; Salgado 2008).

Each floor is subject to a dead load of 5 kPa and a live load of 3 kPa, without load factoring applied, as is typical in settlement calculations. Load is distributed to each pile based on tributary area. Therefore, in the 4-pile case, each pile supports 25% of the building load; 4800 kN. In the 9-pile case, the corner, edge and central piles support 1200, 2400, and 4800 kN, respectively. The pile is modelled as a rigid 0.5 m square prism, with a maximum length of 20 m.

The random field used in this study consists of 240 × 240 × 160 elements, where the elements are 0.25 m cubes. Therefore, the physical dimensions of the field are 60 m × 60 m × 40 m in the *x*, *y*, and *z* directions, respectively. This field size was selected to accommodate sufficient distance between piles and the FEA mesh boundaries, as discussed later. The 0.25 m element size was found to be ideal for distinguishing between various test types (Crisp et al. 2019*a*). The mean soil stiffness is chosen independently for each structural configuration and soil profile to achieve a desired average pile length. This is achieved by iteratively decreasing soil stiffness until the desired average pile length is reached. It is deemed useful to specify a pile length as opposed to soil stiffness to aid in the examination of the influence of pile embedment relative to the mean layer boundary depth. As such, the relationship between these variables is assessed by varying the average pile length, while maintaining the average layer depth fixed at 10 m.

2.4. Cost calculations

The four components of total project cost are those of geotechnical testing, foundation construction, superstructure construction, and structural failure. These failure costs were interpreted from a series of differential settlement thresholds for various magnitudes of failure, as suggested by Day (1999), and correlated with repair costs given by Rawlinsons (2016). It was found that failure costs are well-represented by a linear function of differential settlement, bounded at a minimum of \$0, where no damage occurs at 0.003 m/m, and a maximum of \$6 536 000 at 0.009 m/m, approximating the process of demolishing and rebuilding the superstructure. Construction costs of the superstructure itself add up to \$6 158 000, with pile construction cost set at \$200 per metre, per pile (Crisp et al. 2019*a*). All costs are given in Australian dollars. The site investigation costs are given in the next section.

Fig. 3. Standard structural configuration. [Colour online.]

2.5. Site investigation

The boreholes in the present study are extended to a depth of 20 m, and are regularly spaced in a grid pattern. The tests used are the standard penetration test (SPT) and cone penetration test (CPT). Furthermore, the performance of perfectly accurate discrete and continuous sampling has also been determined, with these cases being denoted as 'Disc.' and 'Cont.' tests, respectively. The four test types differ in three ways: the sampling cost per metre, the sampling frequency, and accuracy, as given in Table 1. As such, the investigation is carried out by sampling the virtual soil at discrete locations, extracting a column of values, and applying random errors.

Table 1. Test type information.

			Uncertainty measures as CV (%)		
				Measurement	
Test type	Sampling interval (m)	Cost ($/m)	Transformation model	Bias	Random
SPT	1.5	156	25	20	40
CPT	0.25	76.6	15	15	20
Disc.	1.5	156	0	0	0
Cont.	0.25	76.6	0	0	0

The tests are subject to three sets of errors, comprised of: random bias per borehole (based on each borehole's mean), random error per sample, and random global bias (based on the global mean). These are applied in the given order, where the former two components represent sampling error, and the latter represents model transformation error in converting the test results to engineering design parameters. These errors are expressed as unit-mean, lognormal variables, with their CVs given in Table 1. Testing errors are treated in greater detail by Crisp et al. (2019*a*).

There are two main steps in constructing a soil model from site investigation results. First is interpreting the aggregate of soil testing data from each layer into a single set of representative material properties. The interpretation used in the present study is the method of taking one geometric standard deviation ($\sigma_{\ln}$) below the geometric mean ($\mu_{\ln}$), which consistently produced the optimal results. The reduced representation of Young's modulus (E_{SD}) from n soil samples in a given layer is defined as follows, where x is an arbitrary sample:

$$\mu_{\ln} = \exp\left(\frac{1}{n}\sum_{i=1}^{n}\ln(x_i)\right) \tag{1}$$

$$\sigma_{\ln} = \exp\left(\sqrt{\frac{1}{n}\sum_{i=1}^{n}\ln\left(\frac{x_i}{\mu_{\ln}}\right)^2}\right) \tag{2}$$

$$E_{SD} = \frac{\mu_{\ln}}{\sigma_{\ln}} \tag{3}$$

The second consideration is the manner by which layer boundaries are represented in the soil models. Historically, practicing engineers have represented these boundaries as an idealized horizontal interface, at a depth equal to the average of the layer depths encountered by each borehole. More recently, with the increases in computing power and the improved usability and feature sets of FEM software such as Plaxis, it is becoming increasingly common to linearly interpolate layer boundaries between boreholes (Plaxis 2018). Both the horizontal average (HA) and full interpolation (FI) layer representations are analyzed to determine if there is a notable advantage with the latter, more sophisticated technique.

It should be noted that, in the context of this research, analysis of a horizontal boundary, such as in the HA case, requires significantly less time to process. This is because, if the soil properties within each layer are uniform, as is the case with the soil model, and if the layer boundary is perfectly horizontal, the 3D FEM mesh can be replaced with a 2D axisymmetric mesh without loss of accuracy (Crisp et al. 2019*a*). As such, a third interpretation is considered, the weighted horizontal (WH) case, which for a given pile, uses a constant layer depth taken as a weighted average of the fully-interpolated layer. The weights are calculated using the inverse square of the distance between the pile and a given depth value. Essentially, this WH method reflects the behaviour that soil that is closer to a pile has more impact on its performance than soil that is further away (Crisp et al. 2019*b*).

The layer boundary depths, as encountered by the boreholes, are known exactly. Depth uncertainty, due to soil material ambiguity, is not incorporated in the analysis. As such, if the soil was consistently sampled with a continuous test type, then the layer would be recreated exactly in the 3D soil model. In the case of discrete tests, such as the SPT, when the borehole encounters the 2nd layer, the layer depth is interpreted as being mid-way between the first sample taken from that layer, and the previous, higher-up sample. In other words, when a change of layer is detected, it is assumed to be at the average distance between samples where the change occurred. As such, the maximum deviation the true layer can have from the interpreted depth is 0.75 m.

2.6. Settlement model

The 3D and 2D linear-elastic settlement models used were adapted from Programs 5.6 and 5.1 by Smith et al. (2014), respectively. The respective element types are hexahedral and quadrilateral with a length of 0.25 m, increasing in width with distance from the pile as a performance measure (Crisp et al. 2019*a*). The mesh boundaries are a minimum of 20 m from the pile to minimize boundary effects.

A linear-elastic FEA model is used, as it is currently considered the most practical model in the context of this research, while retaining an appropriate degree of accuracy (Crisp et al. 2019*a*). As millions of FEA simulations are required, it is not feasible, from a computational time perspective, to use more sophisticated models, such as elastic-plastic, at this time. Goldsworthy (2006) compared a range of pad footing settlement models and found there to be little difference in terms of relative site investigation performance. Indeed, Naghibi et al. (2014*b*) stated that the settlement model only changes the pile design, not the probability of failure, and that the Smith et al. (2014) models are the best available to capture the effects of soil spatial variability in the context of this research. In other words, since the same settlement model is being compared, with both the true soil and the soil model, any settlement model inaccuracy largely cancels itself out, leaving soil variability as the sole source of error. Furthermore, Leung et al. (2010) investigated the choice of linear versus nonlinear models with respect to the settlement of pile groups, and concluded that the linear model was sufficient when the pile spacing is greater than 2.5 times the diameter.

This methodology relies on the assumption that differential settlement is the primary cause of structural damage, which is often the case (Zhang and Ng 2004). However, it should be noted that while design, as opposed to the aforementioned damage, is typically governed by elastic settlement in coarse-grained soils, this is less likely to be the case in fine-grained soils.

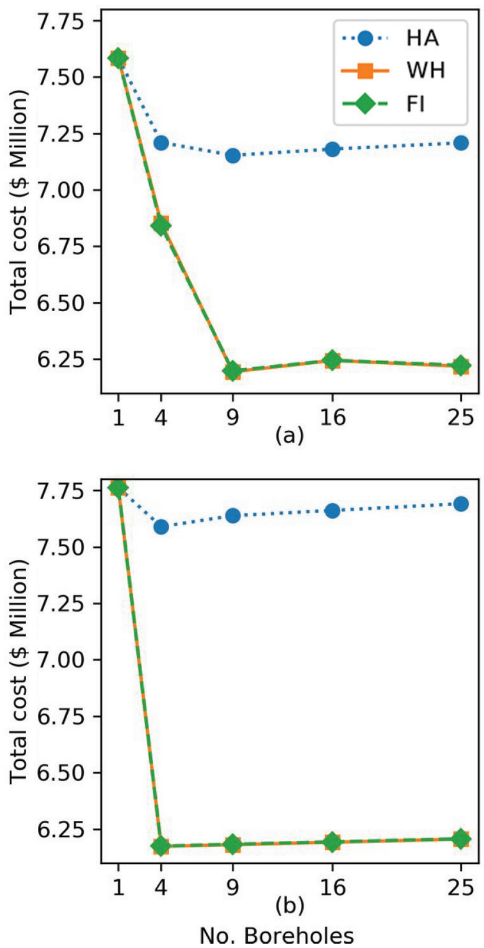
3. Results and discussion

3.1. Comparison of layer boundaries and number of piles

An analysis is conducted to determine the impact of layer boundaries on site investigation performance. Two sets of piles are assessed: 4 and 9 arranged in a regular grid pattern. An average pile length of 12 m is specified so that the pile is embedded below the average layer depth, so as to maximize the layer boundary's influence and yield meaningful trends. The layer's standard deviation of depth is 4 m, with a stiffness ratio of 1:9. The total expected project cost for these cases is given in Fig. 4 for an increasing number of boreholes. Continuous sampling (Cont.) is used, which provides test results with perfect accuracy, as shown previously in Table 1.

Upon inspection of Fig. 4, it is clear that in all cases the optimal investigation is one where there is a borehole located at each pile. This conclusion applies to both 4 and 9 piles, which require 4 and 9 boreholes respectively, and applies across the layer generation methods. This is because, as mentioned previously, soil that is in close proximity to a pile has the greatest impact on its perfor-

Fig. 4. Comparison of full interpolation (FI), weighted horizontal (WH), and horizontal average (HA) soil model layer representations for (*a*) 9 piles and (*b*) 4 piles. [Colour online.]



mance. Therefore, if the layer depths at each pile are known, there is little benefit in increasing layer accuracy elsewhere, and so additional boreholes do not provide notable improvement. The recommendation of placing boreholes specifically at the pile centers is reinforced by the case of 9 piles using the FI method. Here, 16 boreholes provide a notably higher failure cost when compared with 9 boreholes, despite providing additional boundary information. However, the majority of the 16 boreholes do not coincide with piles, and the boundary is interpolated at the pile locations which results in errors, and an increased probability of failure.

Comparing the HA and FI methods, the latter provides significantly better site investigation performance, saving roughly $1.5 million and $1 million in the cases of 4 and 9 piles, respectively. This is because with the FI method, adding additional boreholes to an existing set always results in increased layer accuracy. However, in the case of the HA method, while averaging the depths encountered at each pile improves the method's performance as a whole, it reduces the piles' individual accuracy. When additional sampling away from the piles is undertaken, as in the case of 16 and 25 boreholes, the failure cost increases due to this incorporation of insignificant data.

Comparing the WH and FI methods, which should theoretically be near-identical, there is indeed a negligible discrepancy between the cost curves seen in Fig. 4. Upon visual inspection, the curves appear to overlap almost exactly. Given that the WH method is very accurate with respect to full interpolation, and that it is two orders of magnitude faster to compute, the WH method is used for the remaining analyses in the present paper.

Lastly, comparing the case of 4 and 9 piles directly, the 9-pile foundation has significantly better performance in terms of total cost. This is because, while the average pile length is identical in both cases, the individual bases of the piles are offset from the boundary depth of 10 m due to the variation in applied loads. By comparison, each pile is subject to the same load in the 4-pile case, and are therefore all designed to a 12 m average length, which increases the probability and magnitude of differential settlement due to the layer boundary's proximity.

3.2. Comparison of test type and degree of layer boundary variability

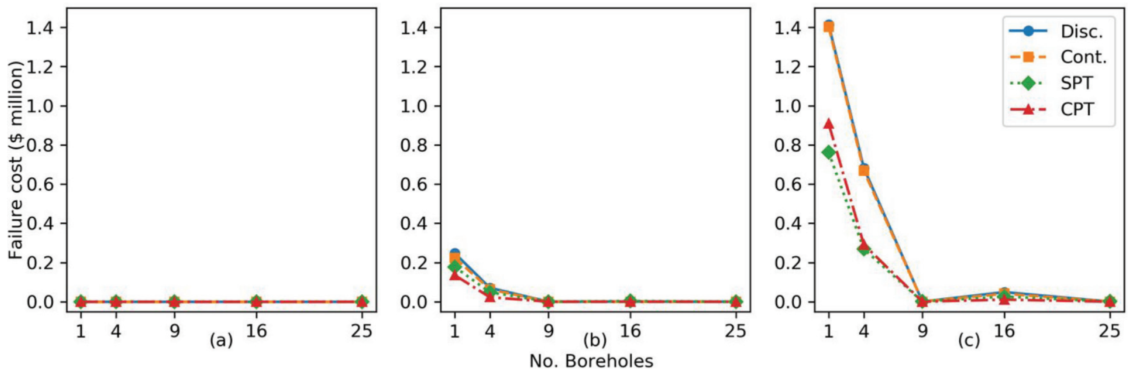
The second analysis examines the influence of test type and layer boundary on site investigation performance. As such, plots comparing the 4 tests in soil with a 1:9 stiffness ratio, 12 m average layer depth, and layer depth standard deviations of 0, 2, and 4 m are given in Fig. 5. Again, the piles are designed to an average 10 m depth. Contrary to the other analyses in this study, the failure cost itself is inspected, as opposed to the total cost. This was done so that the accuracy of the investigations may be examined more directly, removing discrepancies caused by differences in testing costs.

It is immediately apparent upon inspection of Fig. 5, that there is no or negligible difference between continuous and discrete sampling. This is most evident in Fig. 5*c* where the Disc. and Cont. tests have largely overlapping failure costs, despite the high degree of layer undulation. Given that both tests have perfect accuracy in terms of determining material properties, this means that boundary inaccuracy due to discrete sampling is not a notable source of error. This suggests that the 1.5 m sampling interval may be sufficiently frequent to determine the layer boundary, due to the 0.75 m maximum discrepancy, as discussed above. Alternatively, the layer depth error encountered by discrete sampling may cancel itself out to some degree, as the error results from slight underestimation, resulting in a bias in the depth interpretations. It should be noted that differential settlement results from relative variation between the soil at the pile locations; therefore, a reasonably constant bias would not necessarily affect this metric. It is possible that if the layer boundary were made ambiguous, such as through the mixing of material at the interface, then there may be a higher discrepancy between the discrete and continuous tests. This is beyond the scope of the present paper and future analysis is required in this area.

Comparing the full set of tests, the average failure cost for the SPT and CPT is typically lower than that of the perfectly accurate tests. Theoretically, the improved SPT and CPT performance should not be possible given the test's relative inaccuracy. However, it should be noted that the errors associated with the CPT and SPT produce a distribution of values, which in combination with the conservative SD reduction method, can result in lower and more conservative properties in the soil model, compared to the artificial tests (i.e., Disc. and Cont.). This material property conservatism is sufficient to occasionally compensate for the error due to the inaccurate soil layer boundaries.

A special case occurs in Fig. 5*a*, where the soil properties are constant, and the boundary is perfectly horizontal. The conditions are ideal, in that a single borehole will provide an accurate representation of the complete site conditions. Furthermore, as the soil properties do not vary with horizontal location, it is expected that there would be no differential settlement between the piles. However, despite this, failure still occurs as a result of the testing errors involved. A single SPT borehole, for example, will result in a failure cost of $12 700, through to a minimum of $400. A nonzero failure cost may seem counter intuitive, however in this scenario differential settlement can still occur because the piles are designed to different lengths. Therefore, if one or more

Fig. 5. Comparison of various test types in a soil with layer depth standard deviation of (*a*) 0 m, (*b*) 2 m, and (*c*) 4 m. [Colour online.]



piles are incorrectly designed and they interact with the layer boundary in a manner that is contrary to the others, some level of failure may indeed occur. As such, the benefit of additional samples in this case is due solely to overcoming the inherent inaccuracy of the SPT itself. This high error in an ideal, zero-variability scenario demonstrates the significant impact that testing errors have on foundation performance, and how it is important to conduct multiple boreholes, even in the simplest of soil profiles. Furthermore, the authors recommend avoiding the SPT if possible, due to the aforementioned inaccuracy. While the average failure costs are relatively modest, the maximum potential failure costs are significantly higher.

An additional test comparison is given in Fig. 6, which is identical to the situation in Fig. 5*c*, except that the soil in each layer is variable with an SOF of 8 m and CV of 80%. Here, it can be seen that, when the soil is variable in each layer, as opposed to uniform, the CPT and SPT result in a failure cost that is typically equal to or greater than their perfectly accurate equivalents. The CPT possesses a generally similar performance to the Cont. and Disc. tests, with the SPT being up to $200 000 more expensive. It should be noted that the SPT and CPT's inferior performances compared to the perfectly accurate tests are due to both a degradation in the former's performance, and an improvement in the latter's. Both changes are due to the conservative reduction method used.

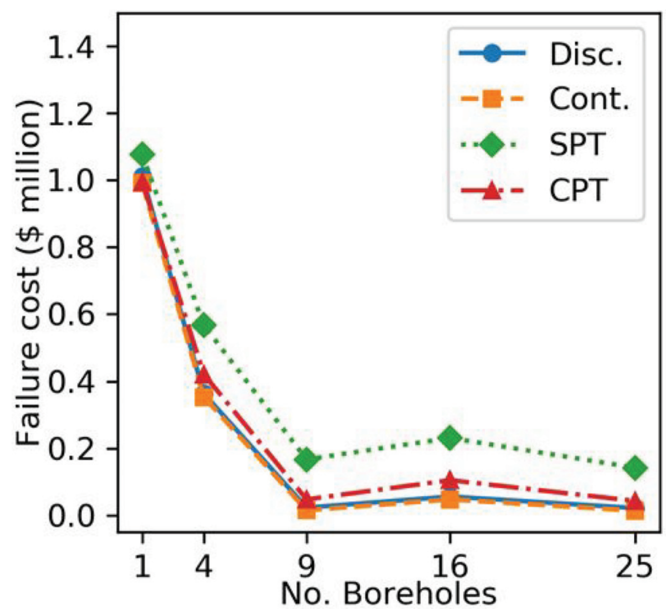
The CPT will be used for the remainder of the study, as it has an intermediate accuracy compared to the others, and it is ubiquitous in practice. Because the sampling frequency appears to have a minimal impact on test performance, the authors refer the reader to Crisp et al. (2019*c*) for further details on the performance of various tests, albeit in a single layer soil.

3.3. Scale of fluctuation, CV and pile length comparisons

The following analysis investigates the impact of SOF on site investigation performance, in a soil with an average layer depth of 10 m, standard deviation of 4 m, stiffness ratio of 1:9, and a CV of 80%. A range of SOF values are considered: 1, 8, and 24 m, with an infinite SOF represented by the soil being uniform within each layer. In other words, the infinite SOF case is a soil with CV of 0%, which allows for comparison between best and worst-case CV conditions. The results are given in Fig. 7 for average pile lengths of 5, 10, and 15 m.

It is immediately obvious in Fig. 7 that the more intermediate SOFs of 8 m and 24 m have considerably higher failure costs than the others. This is because of the presence of moderately-sized pockets of distinct soil properties that are detrimental to both site investigation performance and differential settlement of piles. Of the cases examined, the 24 m SOF appears to be the worst case, as it has a consistently higher failure cost of up to $250 000 over the 8 m case, and has a nominally better relative cost saving of drilling 9 boreholes over one. This result is contrary to previous studies which suggest the worst case SOF of a similar order of magnitude

Fig. 6. Comparison of various test types in a soil with layer depth standard deviation of 4 m, a soil CV of 80% and SOF of 8 m. [Colour online.]



to the center-to-center pile spacing (Fenton and Griffiths 2005; Goldsworthy 2006; Crisp et al. 2019*c*). The discrepancy is likely due to the higher SOF providing a stronger distinction between the two layers, as a high CV increases the overlap of soil properties between the two layers, therefore resulting in them being more similar overall.

An important observation is that three distinct categories of SOF can be defined based on the overall trend of the cost curves seen in Fig. 7; low, intermediate, and infinite. The cost trend of low (1 m) and infinite SOFs are relatively unique. For the more intermediate SOFs, such as 8 m and 24 m, however, the results are quite similar, showing the same overall trend within each pile length case. Since this framework optimizes investigations according to relative performance, it is the cost trend, as opposed to the absolute total cost, that is important. Therefore, a wide range of SOF values can be represented by analysis of a single intermediate SOF value, i.e., 16 m. By extension, all soils can be described by the low, intermediate, and infinite SOF categories.

This classification is beneficial for two reasons. Firstly, the use of representative categories reduces the number of computationally-intensive simulations required, as a smaller number of variables are considered. Secondly, a future site investigation optimization guideline derived from this work will be more useful for practic-

Fig. 7. Comparison of scales of fluctuation for an average pile length of (*a*) 5 m, (*b*) 10 m, and (*c*) 15 m. [Colour online.]

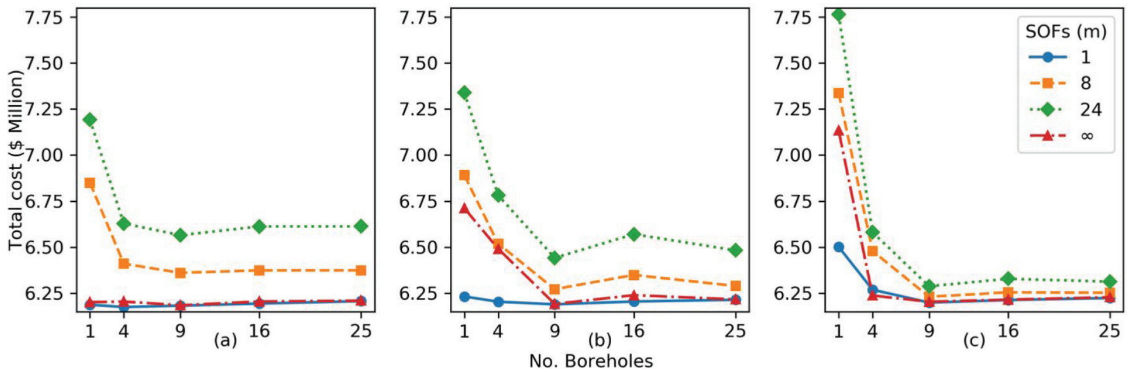
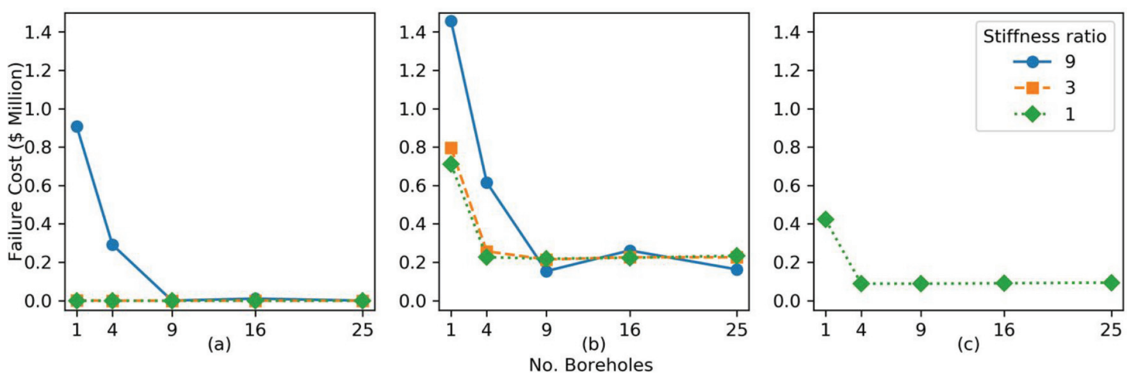


Fig. 8. Comparison of different stiffness ratios and (*a*) two layers with 0% CV, (*b*) two layers with 80% CV, (*c*) one layer with 80% CV. [Colour online.]



ing engineers, as there will be a simple choice of generalized conditions to align to their particular site. It should be noted that there may be occasions when an engineer has insufficient information regarding their site to ascertain the most appropriate SOF category. In these cases, it is recommended that the intermediate category be selected by default, as it represents the worst-case scenario. Furthermore, infinite SOFs are not typically found in practice, which simplifies an engineer's choice to the lower two categories.

As pile length increases, the failure cost decreases, assuming sufficient investigation. In the case of the 5 m average length, failure is largely dominated by local soil variability within the top layer, as seen in Fig. 7*a*. This conclusion is related to how failure occurs solely with the intermediate SOFs, for reasons discussed earlier. In contrast, the infinite and low SOF cases have a minimal failure cost, as the soil within each layer appears uniform at a macro scale. Therefore, there is no dominant mechanism for differential settlement, as the piles are based at a notable distance from the layer boundary. As the average pile length increases to 10 m (the average boundary depth), the infinite SOF case appears to resemble that of the intermediate SOFs, as seen in Fig. 7*b*. This similarity suggests a diminished influence of the soil variability within each layer, as the infinite SOF is analogous to a CV of 0%. One reason for this is that the longer piles distribute stress over a larger volume of soil, diminishing the importance of individual pockets of soil that are distinct from the soil model. The influence of the layer boundary could be argued as being at a maximum, as there is notable improvement with 9 boreholes over 4 with the infinite SOF, as opposed to longer or shorter piles. For a 15 m average pile case, as seen in Fig. 7*c*, the overall costs are closest when the soil is sufficiently sampled with 9 or more boreholes. As the influence of both the layer boundary and soil variability is diminished, it is possible to achieve good foundation reliability with adequate information. However, if the information is inadequate, as is the case for one borehole, then the failure costs can be higher than for shorter piles, due to the possibility that the borehole grossly mischaracterizes the soil profile.

With regards to the optimal number of boreholes, again it can be seen that 9 is best in the majority of cases, which is consistent with the previous conclusion for a 9-pile foundation. The cost savings of using 9 boreholes over a single one can be as high as $1.5 million in the case of a 80% CV and intermediate SOF, where the pile is embedded in the lower layer. The exceptions to this are the infinite SOF case with short or long piles, as well as a low SOF in short piles. Here, 4 boreholes are recommended, as they provide either significantly improved costs over fewer tests, or otherwise do not notably increase the total cost.

3.4. Layer stiffness ratio comparison

The final analysis examines the effect of layer stiffness ratio with a layer bSD of 4 m, average pile length of 12 m, SOF of 24 m, and soil CVs of 0% and 80%. The results are shown in Fig. 8 for stiffness ratios of 1:1, 1:3, and 1:9, in comparison with a single-layer soil with the same SOF and a CV of 80%. The two layer 1:1 stiffness ratio is effectively a single layer profile with an artificial undulating layer imagined to exist in the soil model. Again, failure costs are used as opposed to total costs, to better reflect the uncertainty in the soil model. Note that where two layers are present, the lower layer is always stiffer than the upper one. This is because the linear-elastic model results in pile settlements increasing monotonically as pile length increases, even as the pile increases into softer soil. This is not strictly accurate, as soil in the soft lower layer is more likely to yield, resulting in an increase in pile settlement contrary to what the model suggests. As a result, as softer lower layers cannot be examined with confidence, they are excluded from this analysis.

For a CV of 0%, as shown in Fig. 8*a*, it can be seen that foundation failure only occurs in the 1:9 stiffness ratio case. In other words, for smaller ratios, differential settlement between the piles is never sufficient to cause structural damage, and so no failure cost is applied, as detailed in Section 2.4. This result implies that in relatively uniform soils, failure does not occur unless the lower layer is roughly an order of magnitude stiffer. However, it is possible failure begins to occur between the 1:3 and 1:9 ratios, and this requires future analysis. For a CV of 80% seen in Fig. 8*b*, failure occurs in all cases.

By comparing the uniform multi-layer soils in Fig. 8*a* with the variable single layer soil in Fig. 8*c*, it is possible to examine independently the relative influence of soil property variability within a layer and the undulating layer boundary on site investigation performance, as seen in Fig. 8*b*. For example, the total costs of the 1:3 stiffness ratio soil with 80% CV would theoretically be the sum of the single-layer 80% CV and the 1:3 0% CV cases. However, the latter has a failure cost of $0. Therefore, as the 1:3 stiffness ratio with 80% CV has a higher total cost than the single layer, it can be said that the combination of errors and uncertainly from soil and geological variabilities is greater than the sum of individual sources in isolation. This trend is evident across all observed stiffness ratios, even the 1:1 ratio, implying that the mere presence of a layer boundary in the soil model can have a notable impact, increasing total cost by up to $100 000.

Although it is not shown here due to space constraints, the above analysis was repeated for a horizontal soil boundary with no undulation. In all two layer cases, the total costs strongly converge to that of the 1:1 stiffness ratio. This is logical, since the layer boundary uncertainty would not directly contribute to soil model discrepancies, meaning that the stiffness ratio has negligible impact. However, the converged costs of the 80% CV case are still higher than that of the single layer, as discussed above. Therefore, the additional failure cost is due to incorrect estimates of Young's modulus in each layer, rather than the layer undulation. This reinforces the point that layer undulation only contributes to uncertainty when high stiffness ratios are present between layers.

The combined variability and geology failure being an order of magnitude higher than a variable single layer reinforces that previous single-layer studies are quite conservative with regards to their recommendations of optimal investigations. Therefore, any suggested optimal numbers of boreholes from previous studies should certainly be considered as a strict minimum, as the true optimal is always higher in the presence of multiple layers.

4. Conclusion

A variety of site investigations, soil conditions, and investigation data interpretations have been assessed. The results illustrate the optimal number of tests for the specific soil cases examined. Furthermore, the study draws several conclusions about the processing and interpretation of multi-layered site investigation data, as well as the relative sensitivity of the examined parameters to investigation performance.

Regarding the processing of site investigation data, it was found that one can develop simplified 2D soil models for piles, representing complex 3D soils, without compromising reliability. This was achieved by using a horizontal layer boundary derived through a weighted average of layer depths, where the weights are the inverse of the distance between the pile and each soil element. This procedure was found to produce analogous results to a 3D soil model where the layer boundaries are linearly interpolated between each borehole. Therefore, for piles embedded in multi-layer soils where the material properties are uniform within each layer, 2D axisymmetric linear-elastic FEA can be used instead of 3D. This simplification results in numerical computation that is two orders of magnitude faster.

Furthermore, the manner of interpreting layer boundary information can have a significant impact on failure costs. Linearly interpolating layer depths recorded at borehole locations in a 3D soil model, as opposed to using a constant, horizontal layer depth taken as the simple average of recorded depths. The former produces a cost saving of $1.5 million over the latter, in the case of high layer undulation and stiffness ratio.

The optimal number of boreholes was observed to be affected by many factors, particularly the number of piles, the stiffness ratio between layers, and the degree of layer undulation. To a lesser extent, the pile length, the soil's coefficient of variation and scale of fluctuation also demonstrated some influence. The interaction between these variables, in terms of recommended investigations, is complex, with some variables only having an influence under certain conditions. Future work can investigate these variables, with more extensive combinations, to create a site investigation optimization guideline for use by practicing engineers. Further work could also incorporate varying structural configurations, as well as the impact of additional layers and soil lenses.

In general, it can be concluded that the greatest reliability, if not lowest cost, can be achieved by drilling a borehole at each pile location. In the majority of cases with a layer stiffness ratio of 1:9, 9 boreholes is optimal for 9 piles. For other soil cases, and for 4 piles, 4 boreholes or less is optimal. The savings, in terms of total expected project cost, can be as high as $1.5 million. The SPT performed consistently worse than the CPT, potentially by up to $150 000 in failure costs. It has been demonstrated that this is due to the SPT's inherent inaccuracy, as opposed to the discrete nature of the test. When discrete and continuous tests with perfect accuracy are compared, the differences in failure costs, due to the layer boundary uncertainty, is negligible. The SPT's inaccuracy is such that an average failure cost of $12 700 is possible in a uniform soil with a perfectly horizontal layer; a simple case where a single borehole should provide complete site characterization. As such, the authors recommend avoiding use of the SPT, if possible.

A comparison has shown that the expected failure cost for a two layer profile with undulating boundary and variable soil, is greater than the sum of these two individual sources. Again, the interaction between these parameters is complex, with the undulating layer having a greater influence in the case of high layer stiffness ratios. This reinforces the point that a site investigation optimization guideline must incorporate both sources. Treating them individually, as has been the case with the various single-layer studies in this field to date, has been insufficient. Therefore, all recommended numbers of boreholes from such studies should be taken as a minimum, as the true optimal number is always higher.

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